

Discrete Structures and Optimization Unit - 1

As per updated syllabus

DIWAKAR EDUCATION HUB

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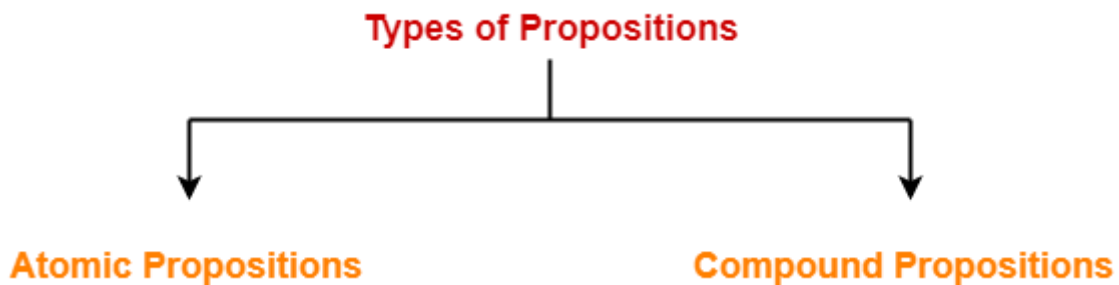
Propositional Logic

The rules of logic give precise meaning to mathematical statements. These rules are used to distinguish between valid and invalid mathematical arguments.

Propositions

A proposition is a declarative sentence that is either true or false, but not both.

- Propositional logic the area of logic that deals with propositions.
- Propositional variables that represent propositions: p, q, r, s .
- Truth values T, F
- Proposition is a declarative statement that is either true or false but not both.
- Connectives are used to combine the propositions.

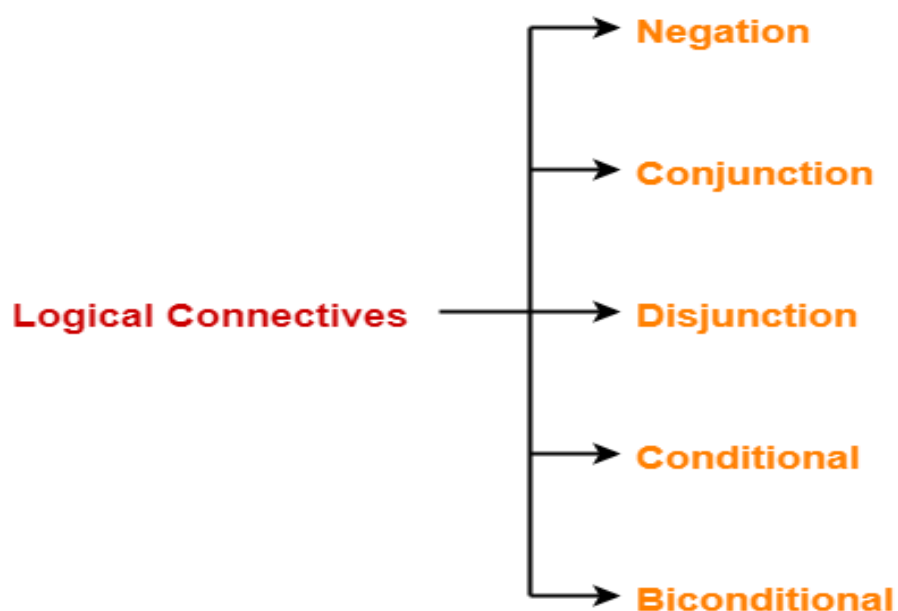


About Connectives in Propositional logic-

Logical Connectives-

Connectives are the operators that are used to combine one or more propositions.

In propositional logic, there are 5 basic connectives-



Name of Connective	Connective Word	Symbol
Negation	Not	] or $\sim$ or ' or –
Conjunction	And	$\wedge$
Disjunction	Or	$\vee$
Conditional	If-then	$\rightarrow$
Bi-conditional	If and only if	$\leftrightarrow$

Negation-

If p is a proposition, then negation of p is a proposition which is-

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- True when p is false
- False when p is true.

Truth Table-

p	$\sim p$
F	T
T	F

Example-

If p : It is raining outside.

Then, Negation of p is-

$\sim p$: It is not raining outside.

Conjunction-

If p and q are two propositions, then conjunction of p and q is a proposition which is-

- True when both p and q are true
- False when both p and q are false

Truth Table-

p	q	$p \wedge q$

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F	F	F
F	T	F
T	F	F
T	T	T

Example-

If p and q are two propositions where-

- $p : 2 + 4 = 6$
- $q : \text{It is raining outside.}$

Then, conjunction of p and q is-

$p \wedge q : 2 + 4 = 6$ and it is raining outside

3. Disjunction-

If p and q are two propositions, then disjunction of p and q is a proposition which is-

- True when either one of p or q or both are true
- False when both p and q are false

Truth Table-

p	q	$p \vee q$
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F	F	F
F	T	T
T	F	T
T	T	T

Example-

If p and q are two propositions where-

- $p : 2 + 4 = 6$
- $q : \text{It is raining outside}$

Then, disjunction of p and q is-

$p \vee q : 2 + 4 = 6$ or it is raining outside

Conditional-

If p and q are two propositions, then-

- Proposition of the type “If p then q” is called a conditional or implication proposition.
- It is true when both p and q are true or when p is false.
- It is false when p is true and q is false.

Truth Table-

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p	q	$p \rightarrow q$
F	F	T
F	T	T
T	F	F
T	T	T

Examples-

If $a = b$ and $b = c$ then $a = c$.

- If I will go to Australia, then I will earn more money.

Biconditional-

If p and q are two propositions, then-

- Proposition of the type “ p if and only if q ” is called a biconditional or bi-implication proposition.
- It is true when either both p and q are true or both p and q are false.
- It is false in all other cases.

Truth Table-

p	q	$p \leftrightarrow q$

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F	F	T
F	T	F
T	F	F
T	T	T

Examples-

- He goes to play a match if and only if it does not rain.
- Birds fly if and only if sky is clear.

The common binary connectives are:

$p \wedge q$	Conjunction	p and q
$p \vee q$	Disjunction	p or q
$p \rightarrow q$	Implication	if p then q
$p \leftrightarrow q$	Bi-conditional	p if and only if q
$p \oplus q$	Exclusive or	p x-or q
$p \downarrow q$	not or	p nor q
$p \mid q$ or $p \uparrow q$	not and	p n-and q

In the conditional $p \rightarrow q$, p is the **antecedent** and q is the consequent. The conditional $p \rightarrow q$ is often read informally as “p implies q”.

Converse, Contra-positive and Inverse

- The proposition $q \rightarrow p$ is called the **converse** of $p \rightarrow q$. The **contra-positive** of $p \rightarrow q$ is the proposition $\neg q \rightarrow \neg p$. The proposition $\neg p \rightarrow \neg q$ is called the inverse of $p \rightarrow q$.
- We will see that of these three conditional statements formed from $p \rightarrow q$, only the **contra-positive** always has the same truth value as $p \rightarrow q$.

EXAMPLE: What are the **contra-positive**, the converse, and the inverse of the conditional- statement? **“The home team wins whenever it is raining?”**

Solution:

“If it is raining, then the home team wins.”

The **contra-positive** of this conditional statement is

“If the home team does not win, then it is not raining.”

The **converse** is - “If the home team wins, then it is raining.”

The **inverse** is - “If it is not raining, then the home team does not win.”

Bi-conditionals

DEFINITION 4

Let p and q be propositions. The **bi-conditional** statement $p \leftrightarrow q$ is the proposition “ p if and only if q .” The **bi-conditional** statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise. **Bi-conditional** statements are also called bi-implications.

The Truth Table for the Bi-conditional $p \leftrightarrow q$

Truth Table for the Bi-conditional $p \leftrightarrow q$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F

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F	T	F
F	F	T

The statement $p \leftrightarrow q$ is true when both the conditional statements are $p \rightarrow q$ and $q \rightarrow p$ are true and is false otherwise.

Example: Let p be the statement “You can take the flight,” and let q be the statement “You buy a ticket.” Then $p \leftrightarrow q$ is the statement

Answer “You can take the flight if and only if you buy a ticket.”

Precedence of Logical Operator

Operator precedence is an ordering of logical operators designed to allow the dropping of parenthesis in logical expression.

Precedence of Logical Operators

Operator	Precedence
$\neg$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

Logic and Bit Operations

- A bit is a symbol with two possible values, 0 (zero) and 1 (one).
- A bit can be used to represent a truth value, because there are two truth values, true and false.
- A variable is called a Boolean variable if its value is either true or false.

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Truth	Bit	Bit Operators OR, AND, and XOR				
Value		x	y	$x \vee y$	$x \wedge y$	$x \oplus y$
T	1	0	0	0	0	0
F	0	0	1	1	0	1
		1	0	1	0	1
DEFINITION		1	1	1	1	0

A bit string is a sequence of zero or more bits. The length of this string is the number of bits in the string.

EXAMPLE Find the bitwise OR, bitwise AND, and bitwise XOR of the bit strings 01 1011 0110 and 11 0001 1101.

Solution: The bitwise OR, bitwise AND, and bitwise XOR are:

01 1011 0110
11 0001 1101

11 1011 1111 **Bitwise OR**

01 0001 0100 **Bitwise AND**

10 1010 1011 **Bitwise XOR**

Propositional Equivalences

Tautologies and contradictions

A compound proposition that is always true, regardless of the truth values of the individual propositions involved, is called a tautology.

Example: $p \vee \neg p$ is a tautology.

A compound proposition that is always false, regardless of the truth values of the individual propositions involved, is called a contradiction.

Example: $p \wedge \neg p$ is a contradiction.

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A compound statement that is neither a tautology nor a contradiction is called a contingency.

Logical Equivalences

- Compound propositions that have the same truth values in all possible cases are called logically equivalent.
- The compound propositions p and q are called logically equivalent if $p \leftrightarrow q$ is a tautology. The notation $p \equiv q$ denotes that p and q are logically equivalent.

Logical equival ence	Examples of Tautology & Contradiction.				De Morgan's Laws.
	p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$	
	T	F	T	F	$\neg(p \wedge q) \equiv \neg p \vee \neg q$
	F	T	T	F	$\neg(p \vee q) \equiv \neg p \wedge \neg q$

- The two algebraic expressions equal if they have the same value for each possible value of the input variables.
- For example, for all real numbers x , the left side and the right side have the same value.

$$x^2 - 1 = (x + 1)(x - 1)$$
- The two compound statements p and q "equal" if they always share the same truth value. $p \equiv q$ means that $p \leftrightarrow q$ is a tautology.
- In these equivalences, T denotes the compound proposition that is always true and F denotes the compound proposition that is always false.

Logical Equivalences	
Equivalence	Name
$p \wedge T \equiv p$	Identity laws
$p \vee F \equiv p$	

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$p \vee T \equiv T$ $p \wedge F \equiv F$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv T$ $p \wedge \neg p \equiv F$	Negation laws

Logical Equivalences involving
Conditional Statements.

$$p \rightarrow q \equiv \neg p \vee q$$

$$p \rightarrow q \equiv \neg q \rightarrow \neg p$$

$$p \vee q \equiv \neg p \rightarrow q$$

$$p \wedge q \equiv \neg(p \rightarrow \neg q)$$

$$\neg(p \rightarrow q) \equiv p \wedge \neg q$$

$$(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$$

$$(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$$

$$(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$$

$$(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$$

Logical Equivalences Involving

Bi-conditional Statements.

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

$$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$$

$$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$$

$$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$$

Using De Morgan's Laws

The two logical equivalences known as **De Morgan's laws** are particularly important. In particular, the equivalence $\neg(p \vee q) \equiv \neg p \wedge \neg q$ tells us that the negation of a disjunction is formed by taking the conjunction of the negations of the component propositions.

Example Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent by developing a series of logical equivalences.

Sol $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg(\neg p \wedge q)$ 2nd De Morgan law

$$\equiv \neg p \wedge [\neg(\neg p) \vee \neg q]$$
 1st De Morgan law

$$\equiv \neg p \wedge (p \vee \neg q)$$
 double negation law

$$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$$
 2nd distributive law

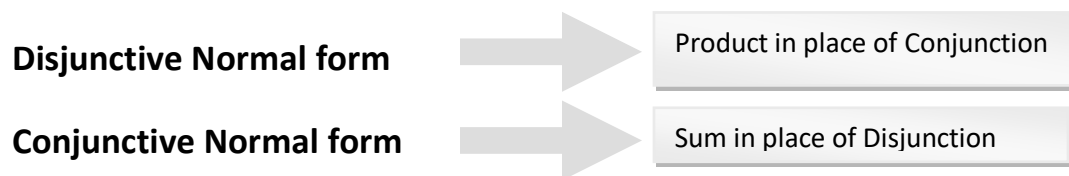
$$\equiv F \vee (\neg p \wedge \neg q)$$
 because $\neg p \wedge p \equiv F$

$$\equiv (\neg p \wedge \neg q) \vee F$$
 commutative law for disjunction

$$\equiv \neg p \wedge \neg q$$
 identity law for F

Normal Form

- Suppose, $A(P_1, P_2, \dots, P_n)$ is a statements formula where $P_1, P_2, \dots, P_n$ are the atomic variables if we consider all possible assignments of the truth value to $P_1, P_2, \dots, P_n$ and obtain the resulting truth values of the formula A then we get the truth table for A , such a truth table contains 2^n rows.
- The formula may have the truth value **T** for all possible assignments of the truth values to the variables $P_1, P_2, \dots, P_n$. In this case, A is called identically true or tautology.
- If A has the truth value **T** for at least one combination of truth values assigned to $P_1, P_2, \dots, P_n$ then A is called Satisfiable.



Types of Normal form

1) Disjunctive Normal form

A product of the variable and their negations in a formula is called an elementary product. Similarly, a sum of the variables and their negations is called as an elementary sum.

Some statements hold for elementary sums and product:

1. For any elementary product, the necessary condition is false is when it contains at least one pair of a factor in which one is the negation of the other.
2. For any elementary sum, the necessary condition is true when it contains at least one pair of factors in which one is the negations of the other.

2) Conjunctive Normal Form

The conjunctive normal form of a given formula is the one which contains the product of elementary sums (that formula is equivalent in the given formula).

3) Principle Disjunctive normal form

Suppose, **P** and **Q** is two variables. We construct all possible formulas which consist of a conjunction of **P** or in negation and conjunction of **Q** or its negation. Now of the formulas should contain both a variable and its negation. For two variables **P** and **Q** there is 2^2 such formula.

These formulas are called min-terms or Boolean conjunction of **P** and **Q** from the truth tables of theses min-terms, it is clear that no two min-terms are equivalent. Each min-term has the truth value **T** for exactly one combination of the truth value of the variables **P** and **Q**.

For a given formula an equivalent formula consisting of a disjunction of **min-terms** only is known as its principle disjunction normal form. Such a normal form is also said to be the sum-product canonical form.

4) Principle conjunctive normal form

Given a number of variables max-term of these variables is a formula which consists of disjunction in which each variable or its negations but not both appear only once.

Observe that the **max-term** are the duals of **min-terms**. Therefore each of the **max-term** has the truth value **F** for exactly one combination of the truth values of the variables.

The **principle of conjunctive normal form** or the **product-sum canonical form**, the equivalent formula consists of only the conjunction of the **max-terms** only.

Predicates and Quantifiers

Predicates

A predicate is an expression of one or more variables defined on some specific domain. A predicate with variables can be made a proposition by either assigning a value to the variable or by quantifying the variable.

For Example: Let $P(x)$ denote the statement " $x > 3$." What are the truth values of $P(4)$ and $P(2)$?

Solution: We obtain the statement $P(4)$ by setting $x = 4$ in the statement " $x > 3$." Hence, $P(4)$, which is the statement " $4 > 3$," is true. However, $P(2)$, which is the statement " $2 > 3$," is false.

The following are some examples of predicates –

- Let $E(x, y)$ denote " $x = y$ "
- Let $X(a, b, c)$ denote " $a + b + c = 0$ "
- Let $M(x, y)$ denote " x is married to y "

Well Formed Formula

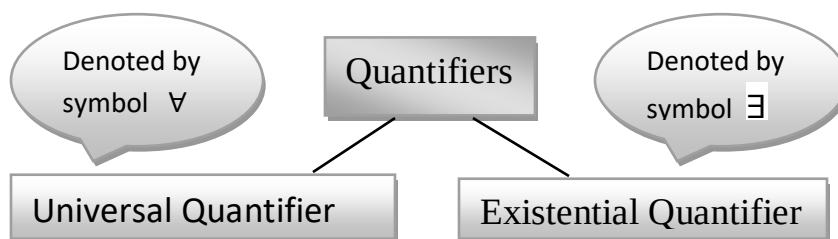
Well Formed Formula is a predicate holding any of the following –

- All propositional constants and propositional variables are WFF.
- If x is a variable and Y is a WFF, $\forall x Y$ and $\exists x Y$ are also WFF. Truth value and false values are WFF.

Quantifiers

The variable of predicates is quantified by quantifiers.

There are two types of quantifier in predicate logic – **Universal Quantifier** and **Existential Quantifier**.



Universal Quantifier

Universal quantifier states that the statements within its scope are true for every value of the specific variable. It is denoted by the symbol $\forall$.

$\forall x P(x)$ is read as for every value of x , $P(x)$ is true.

Statement When True?

When False?

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$\forall x P(x)$	$P(x)$ is true for every x	There is an x for which $P(x)$ is false.
$\exists x P(x)$	There is an x for which $P(x)$ is true.	$P(x)$ is false for every x .

Example Let $P(x)$ be the statement “ $x + 1 > x$.” What is the truth value of the quantification $\forall x P(x)$, where the domain consists of all real numbers?

Solution: Because $P(x)$ is true for all real numbers x , the quantification $\forall x P(x)$ is True.

Existential Quantifier

Existential quantifier states that the statements within its scope are true for some values of the specific variable. It is denoted by the symbol $\exists$.

$\exists x P(x)$ is read as for some values of x , $P(x)$ is true.

Example

Let $P(x)$ be the statement “ $x > 3$ ”. What is the truth value of the quantification $\exists x P(x)$, where the domain consists of all real numbers?

Solution: $x > 3$ is true, when $x = 4$ —the existential quantification of $P(x)$, which is $\exists x P(x)$, is true.

Precedence of Quantifiers

The quantifier's $\forall$ and $\exists$ have higher precedence than all logical operators from propositional calculus.

For example, $\forall x P(x) \vee Q(x)$ is the disjunction of $\forall x P(x)$ and $Q(x)$. In other words, it means $(\forall x P(x)) \vee Q(x)$ rather than $\forall x (P(x) \vee Q(x))$.

Negating Quantified Expressions

Consider the negation of the statement

“Every student in your class has taken a course in calculus.”

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This statement is a universal quantification, namely, $\forall x P(x)$, where $P(x)$ is the statement “ x has taken a course in calculus” and the domain consists of the students in your class.

- The negation of this statement is “It is not the case that every student in your class has taken a course in calculus.”
- This is equivalent to “There is a student in your class who has not taken a course in calculus.”
- And this is simply the existential quantification of the negation of the original propositional function, namely, $\exists x \neg P(x)$.

De Morgan's laws for quantifiers

Negation	Equivalent Statement	When Is Negation True?	When False?
$\neg \exists x P(x)$	$\forall x \neg P(x)$	For every x , $P(x)$ is false.	There is an x for which $P(x)$ is true.
$\neg \forall x P(x)$	$\exists x \neg P(x)$	There is an x for which $P(x)$ is false.	$P(x)$ is true for every x .

Example What are the negations of the statements “There is an honest politician”.

Solution: Let $H(x)$ denote “ x is honest.” Then the statement “There is an honest politician” is represented by $\exists x H(x)$, where the domain consists of all politicians.

The negation of the statement is $\neg \exists x H(x)$, which is equivalent to $\forall x \neg H(x)$. “Every politician is dishonest.”

Nested Quantifiers

Nested quantifiers, where one quantifier is within the scope of another, such as: $\forall x \exists y (x + y = 0)$. Everything within the scope of a quantifier can be thought of as a propositional function.

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For example, $\forall x \exists y (x + y = 0)$ is the same thing as $\forall x Q(x)$, where $Q(x)$ is $\exists y P(x, y)$, where $P(x, y)$ is $x + y = 0$.

Understanding Statements Involving Nested Quantifiers

For example, assume that the domain for the variables x and y consists of all real numbers.

- The statement $\forall x \forall y (x + y = y + x)$ says that $x + y = y + x$ for all real numbers x and y . (**Commutative law**)
- Likewise, the statement $\forall x \exists y (x + y = 0)$ says that for every real number x there is a real number y such that $x + y = 0$. (**Additive Inverse**)
- Similarly, the statement $\forall x \forall y \forall z (x + (y + z) = (x + y) + z)$ (**Associative law**)

Example: Let $Q(x, y)$ denote " $x + y = 0$." What are the truth values of the quantifications $\exists y \forall x Q(x, y)$ and $\forall x \exists y Q(x, y)$, where the domain for all variables consists of all real numbers?

Solution: The quantification $\exists y \forall x Q(x, y)$ denotes the proposition. "There is a real number y such that for every real number x , $Q(x, y)$."

Quantification of two variables

Statement	When True?	When False?
$\forall x \forall y P(x, y)$ $\forall y \forall x P(x, y)$	$P(x, y)$ is true for every pair x, y .	There is a pair x, y for which $P(x, y)$ is false.
$\forall x \exists y P(x, y)$	For every x there is a y for which $P(x, y)$ is true.	There is an x such that $P(x, y)$ is false for every y .
$\exists x \forall y P(x, y)$	There is an x for which $P(x, y)$ is true for every y .	For every x there is a y for which $P(x, y)$ is false.
$\exists x \exists y P(x, y)$	There is a pair x, y for which $P(x, y)$ is true.	$P(x, y)$ is false for every pair x, y

Translating Mathematical Statements into Statements

Involving Nested Quantifiers

Mathematical statements expressed in English can be translated into logical expressions.

Example

Translate the statement “The sum of two positive integers is always positive” into a logical expression.

Solution

“For all positive integers x and y , $x + y$ is positive.” Consequently, we can express this statement as: $\forall x \forall y((x > 0) \wedge (y > 0) \rightarrow (x + y > 0))$, where the domain for both variables consists of all integers.

Translating from Nested Quantifiers into English

Example Translate the statement $\forall x(C(x) \vee \exists y(C(y) \wedge F(x, y)))$ into English, where $C(x)$ is “ x has a computer,” $F(x, y)$ is “ x and y are friends,” and the domain for both x and y consists of all students in your school.

Solution Every student in your school has a computer or has a friend who has a computer.

Every Theorem in Mathematics, or any subject for that matter, is supported by underlying proofs. These proofs are nothing but a set of arguments that are conclusive evidence of the validity of the theory.

The arguments are chained together using Rules of Inferences to deduce new statements and ultimately prove that the theorem is valid.

Important Definitions :

- 1. Argument** – A sequence of statements, **premises**, that end with a conclusion.
- 2. Validity** – A deductive argument is said to be valid if and only if it takes a form that makes it impossible for the premises to be true and the conclusion nevertheless to be false.
- 3. Fallacy** – An incorrect reasoning or mistake which leads to invalid arguments.

Structure of an Argument :

As defined, an argument is a sequence of statements called premises which end with a conclusion.

Premises - $p_1, p_2, p_3, \dots, p_n$
Conclusion - q

$if(p_1 \wedge p_2 \wedge p_3 \wedge \dots \wedge p_n) \rightarrow q$ is a tautology,
then the argument is termed valid otherwise termed as invalid.

The argument is written as –

First Premise
Second Premise
Third Premise
.
.
Nth Premise

 $\therefore Conclusion$

Rules of Inference :

Simple arguments can be used as building blocks to construct more complicated valid arguments. Certain simple arguments that have been established as valid are very important in terms of their usage. These arguments are called Rules of Inference.

The most commonly used Rules of Inference are tabulated below

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Rule of Inference	Tautology	Name
$\frac{p \quad p \rightarrow q}{\therefore q}$	$(p \wedge (p \rightarrow q)) \rightarrow q$	Modus Ponens
$\frac{\neg q \quad p \rightarrow q}{\therefore \neg p}$	$(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$	Modus Tollens
$\frac{p \rightarrow q \quad q \rightarrow r}{\therefore p \rightarrow r}$	$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$	Hypothetical syllogism
$\frac{\neg p \quad p \vee q}{\therefore q}$	$(\neg p \wedge (p \vee q)) \rightarrow q$	Disjunctive Syllogism
$\frac{p}{\therefore (p \vee q)}$	$p \rightarrow (p \vee q)$	Addition
$\frac{(p \wedge q) \rightarrow r}{\therefore p \rightarrow (q \rightarrow r)}$	$((p \wedge q) \rightarrow r) \rightarrow (p \rightarrow (q \rightarrow r))$	Exportation
$\frac{p \vee q \quad \neg p \vee r}{\therefore q \vee r}$	$((p \vee q) \wedge (\neg p \vee r)) \rightarrow q \vee r$	Resolution

Similarly, we have Rules of Inference for quantified statements –

Rule of Inference	Name
$\frac{\forall x P(x)}{\therefore P(c)}$	Universal instantiation
$\frac{P(c) \text{ for an arbitrary } c}{\therefore \forall x P(x)}$	Universal generalization
$\frac{\exists x P(x)}{\therefore P(c) \text{ for some } c}$	Existential instantiation
$\frac{P(c) \text{ for some } c}{\therefore \exists x P(x)}$	Existential generalization

how Rules of Inference can be used to deduce conclusions from given arguments or check the validity of a given argument.

Example : Show that the hypotheses

“It is not sunny this afternoon and it is colder than yesterday”,

“We will go swimming only if it is sunny”,

“If we do not go swimming, then we will take a canoe trip”, and

“If we take a canoe trip, then we will be home by sunset”

lead to the conclusion

“We will be home by sunset”.

The first step is to identify propositions and use propositional variables to represent them.

p — “It is sunny this afternoon”

q — “It is colder than yesterday”

r — “We will go swimming”

s — “We will take a canoe trip”

t — “We will be home by sunset”

The hypotheses are –

$\neg p \wedge q, r \rightarrow p, \neg r \rightarrow s$, and $s \rightarrow t$.

The conclusion is –

t

To deduce the conclusion we must use Rules of Inference to construct a proof using the given hypotheses.

Step	Reason
1. $\neg p \wedge q$	Hypothesis
2. $\neg p$	Simplification
3. $r \rightarrow p$	Hypothesis
4. $\neg r$	Modus Tollens using (2) and (3)
5. $\neg r \rightarrow s$	Hypothesis
6. s	Modus Ponens using (4) and (5)
7. $s \rightarrow t$	Hypothesis
8. t	Modus Ponens Using (6) and (7)

Resolution Principle :

To understand the Resolution principle, first we need to know certain definitions.

1. **Literal** – A variable or negation of a variable. Eg- $p, \neg q$
2. **Sum** – Disjunction of literals. Eg- $p \vee \neg q$
3. **Product** – Conjunction of literals. Eg- $p \wedge \neg q$
4. **Clause** – A disjunction of literals i.e. it is a sum.
5. **Resolvent** – For any two clauses C_1 and C_2 , if there is a literal L_1 in C_1 that is complementary to a literal L_2 in C_2 , then removing both and joining the remaining clauses through a disjunction produces another clause C . C is called the resolvent of C_1 and C_2

For example,

$$\begin{aligned}C_1 &= p \vee q \vee r \\C_2 &= \neg p \vee \neg s \vee t\end{aligned}$$

Here, $\neg p$ and p are complementary to each other. Removing them and joining the remaining clauses with a disjunction gives us-

$$q \vee r \vee \neg s \vee t$$

We could skip the removal part and simply join the clauses to get the same resolvent.

$$p \vee \neg p \equiv T \text{ and, } T \vee q \equiv q$$

This is also the Rule of Inference known as Resolution.

Theorem – If C is the resolvent of C_1 and C_2 , then C is also the logical consequence of C_1 and C_2 .

The Resolution Principle – Given a set S of clauses, a (resolution) deduction of C from S is a finite sequence $C_1, C_2, \dots, C_k$ of clauses such that each C_i is either a clause in S or a resolvent of clauses preceding C and $C_k = C$.

We can use the resolution principle to check the validity of arguments or deduce conclusions from them. Other Rules of Inference have the same purpose, but Resolution is unique. It is complete by it's own. You would need no other Rule of Inference to deduce the conclusion from the given argument.

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To do so, we first need to convert all the premises to clausal form. The next step is to apply the resolution Rule of Inference to them step by step until it cannot be applied any further.

For example, consider that we have the following premises –

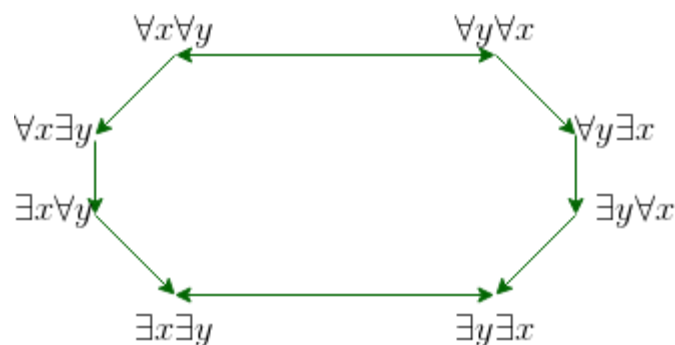
$$\begin{array}{l} p \rightarrow (q \vee r) \\ s \rightarrow \neg r \\ p \wedge s \end{array}$$

The first step is to convert them to clausal form –

$$\begin{array}{l} C_1 : \neg p \vee q \vee r \\ C_2 : \neg s \vee \neg r \\ C_3 : p \\ C_4 : s \end{array}$$

From the resolution of C_1 and C_2 , $C_5 : \neg p \vee q \vee s$
From the resolution of C_5 and C_3 , $C_6 : q \vee \neg s$
From the resolution of C_6 and C_4 , $C_7 : q$
Therefore, the conclusion is q .

Note: Implications can also be visualised on octagon as,



It shows how implication changes on changing order of there exists and for all symbols.

Set and Relations

Set

Sets are used to group objects together.

DEFINITION 1 A set is an unordered collection of objects, called elements or members of the set. A set is said to contain its elements. We write $a \in A$ to denote that a is an element of the set A . The notation $a \notin A$ denotes that a is not an element of the set A .

It is common for sets to be denoted using uppercase letters. Lowercase letters are usually used to denote elements of sets.

EXAMPLE 1 The set V of all vowels in the English alphabet can be written as $V = \{a, e, i, o, u\}$.

EXAMPLE 2 The set O of odd positive integers less than 10 can be expressed by $O = \{1, 3, 5, 7, 9\}$.

DEFINITION 2 Two sets are equal if and only if they have the same elements. Therefore, if A and B are sets, then A and B are equal if and only if $\forall x(x \in A \leftrightarrow x \in B)$. We write $A = B$ if A and B are equal sets.

EXAMPLE The sets $\{1, 3, 5\}$ and $\{3, 5, 1\}$ are equal, because they have the same elements.

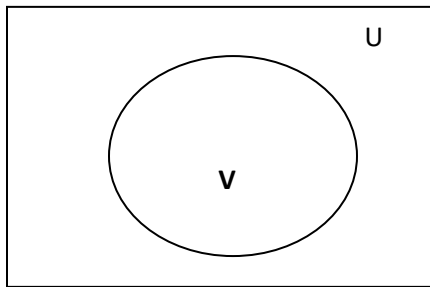
THE EMPTY SET

There is a special set that has no elements. This set is called the empty set, or null set, and is denoted by $\emptyset$.

Venn diagrams

In Venn diagrams the universal set U , which contains all the objects under consideration, is represented by a rectangle.

EXAMPLE 7 Draw a Venn diagram that represents V , the set of vowels in the English alphabet.



Venn diagram for the Set of Vowels

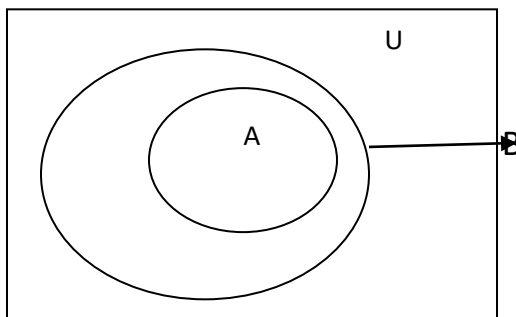
Subsets

It is common to encounter situations where the elements of one set are also the elements of a second set.

DEFINITION 3: The set A is a subset of B if and only if every element of A is also an element of B . We use the notation $A \subseteq B$ to indicate that A is a subset of the set B .

We see that $A \subseteq B$ if and only if the quantification $\forall x(x \in A \rightarrow x \in B)$ is true.

- **Showing that A is a Subset of B** To show that $A \subseteq B$, show that if x belongs to A then x also belongs to B .
- **Showing that A is Not a Subset of B** To show that $A \not\subseteq B$, find a single $x \in A$ such that $x \notin B$.



Venn diagram Showing that A is a Subset of B .

THEOREM 1 For every set S , (i) $\emptyset \subseteq S$ and (ii) $S \subseteq S$.

Proof: Let S be a set. To show that $\emptyset \subseteq S$, we must show that $\forall x(x \in \emptyset \rightarrow x \in S)$ is true. Because the empty set contains no elements, it follows that $x \in \emptyset$ is always false. It follows that the conditional statement $x \in \emptyset \rightarrow x \in S$ is always true,

because its hypothesis is always false and a conditional statement with a false hypothesis is true. Therefore, $\forall x(x \in \emptyset \rightarrow x \in S)$ is true.

Showing Two Sets are Equal

To show that two sets A and B are equal, show that $A \subseteq B$ and $B \subseteq A$.

The Size of a Set

DEFINITION 4 Let S be a set. If there are exactly n distinct elements in S where n is a non negative integer, we say that S is a finite set and that n is the cardinality of S. The cardinality of S is denoted by $|S|$.

EXAMPLE Let A be the set of odd positive integers less than 10. Then $|A| = 5$.

Power Sets

DEFINITION 6 Given a set S, the power set of S is the set of all subsets of the set S. The power set of S is denoted by $P(S)$.

EXAMPLE What is the power set of the set $\{0, 1, 2\}$?

Solution: The power set $P(\{0, 1, 2\})$ is the set of all subsets of $\{0, 1, 2\}$. Hence, $P(\{0, 1, 2\}) = \{\emptyset, \{0\}, \{1\}, \{2\}, \{0, 1\}, \{0, 2\}, \{1, 2\}, \{0, 1, 2\}\}$.

Cartesian Products

The order of elements in a collection is often important. Because sets are unordered, a different structure is needed to represent ordered collections. This is provided by ordered n-tuples.

DEFINITION 7 The ordered n-tuple $(a_1, a_2, \dots, a_n)$ is the ordered collection that has a_1 as its first element, a_2 as its second element... and a_n as its nth element.

DEFINITION 8 Let A and B be sets. The Cartesian product of A and B, denoted by $A \times B$, is the set of all ordered pairs (a, b) , where $a \in A$ and $b \in B$. Hence,

$$A \times B = \{(a, b) \mid a \in A \wedge b \in B\}.$$

EXAMPLE What is the Cartesian product of $A = \{1, 2\}$ and $B = \{a, b, c\}$?

Solution: The Cartesian product $A \times B$ is $A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}$.

Cartesian products $A \times B$ and $B \times A$ are not equal, unless $A = \emptyset$ or $B = \emptyset$ or $A = B$.

DEFINITION 9 The Cartesian product of the sets $A_1, A_2, \dots, A_n$, denoted by $A_1 \times A_2 \times \dots \times A_n$, is the set of ordered n -tuples $(a_1, a_2, \dots, a_n)$, where a_i belongs to A_i for $i = 1, 2, \dots, n$.

In other words, $A_1 \times A_2 \times \dots \times A_n = \{(a_1, a_2, \dots, a_n) \mid a_i \in A_i \text{ for } i = 1, 2, \dots, n\}$.

EXAMPLE What is the Cartesian product $A \times B \times C$, where $A = \{0, 1\}$, $B = \{1, 2\}$, and $C = \{0, 1, 2\}$?

Solution: $A \times B \times C = \{(0, 1, 0), (0, 1, 1), (0, 1, 2), (0, 2, 0), (0, 2, 1), (0, 2, 2), (1, 1, 0), (1, 1, 1), (1, 1, 2), (1, 2, 0), (1, 2, 1), (1, 2, 2)\}$.

Operations on Sets

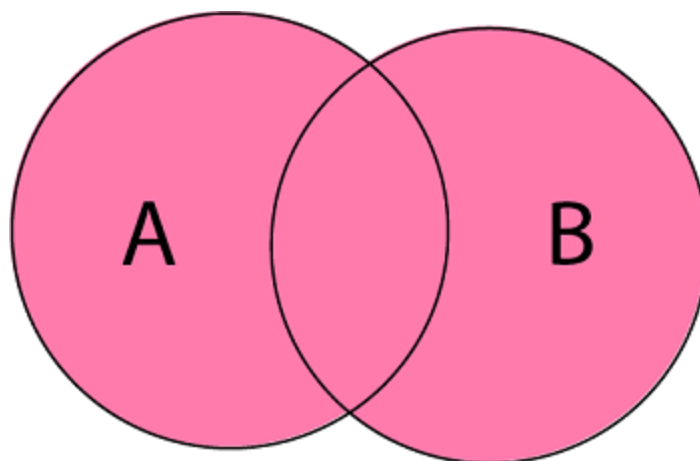
The basic set operations are:

1. Union of Sets: Union of Sets A and B is defined to be the set of all those elements which belong to A or B or both and is denoted by $A \cup B$.

$$1. A \cup B = \{x: x \in A \text{ or } x \in B\}$$

Example: Let $A = \{1, 2, 3\}$, $B = \{3, 4, 5, 6\}$

$$A \cup B = \{1, 2, 3, 4, 5, 6\}.$$

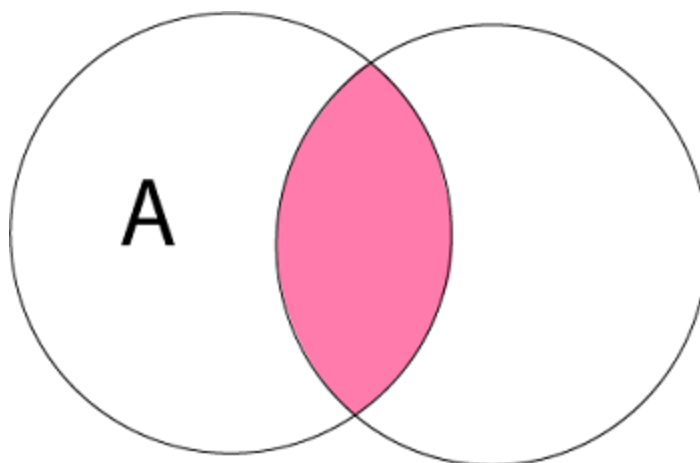


2. Intersection of Sets: Intersection of two sets A and B is the set of all those elements which belong to both A and B and is denoted by $A \cap B$.

1. $A \cap B = \{x: x \in A \text{ and } x \in B\}$

Example: Let $A = \{11, 12, 13\}$, $B = \{13, 14, 15\}$

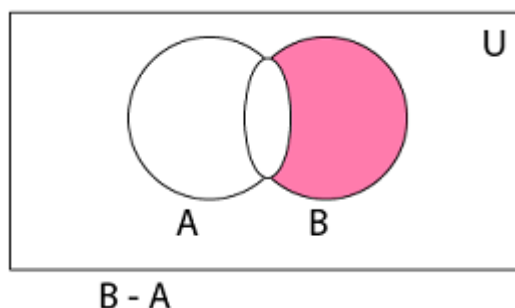
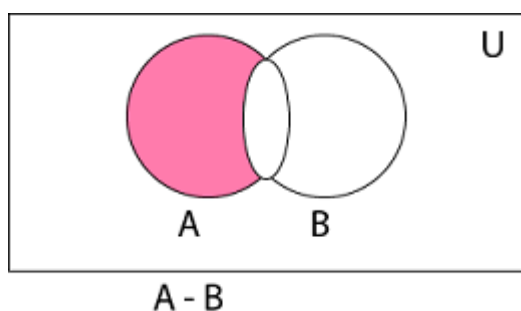
$A \cap B = \{13\}$.



3. Difference of Sets: The difference of two sets A and B is a set of all those elements which belongs to A but do not belong to B and is denoted by $A - B$.

1. $A - B = \{x: x \in A \text{ and } x \notin B\}$

Example: Let $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$ then $A - B = \{1, 2\}$ and $B - A = \{5, 6\}$



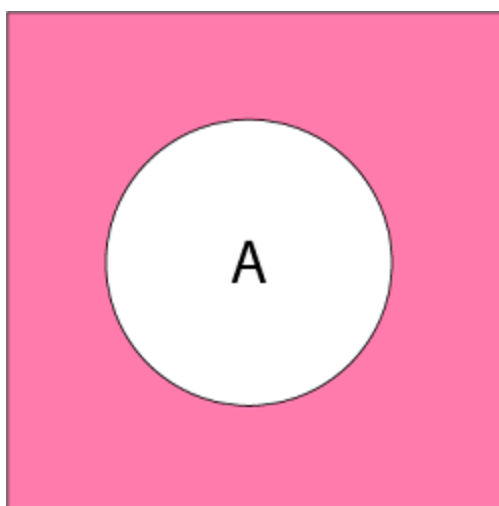
4. Complement of a Set: The Complement of a Set A is a set of all those elements of the universal set which do not belong to A and is denoted by A^c .

$$A^c = U - A = \{x: x \in U \text{ and } x \notin A\} = \{x: x \notin A\}$$

Example: Let U is the set of all natural numbers.

$$A = \{1, 2, 3\}$$

$$A^c = \{\text{all natural numbers except } 1, 2, \text{ and } 3\}.$$



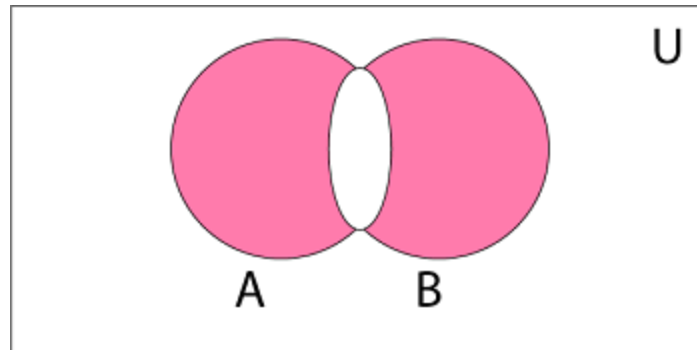
5. Symmetric Difference of Sets: The symmetric difference of two sets A and B is the set containing all the elements that are in A or B but not in both and is denoted by $A \oplus B$ i.e.

$$1. A \oplus B = (A \cup B) - (A \cap B)$$

Example: Let $A = \{a, b, c, d\}$

$B = \{a, b, l, m\}$

$A \oplus B = \{c, d, l, m\}$



Set Identities

Idempotent laws:	$A \cup A = A$	$A \cap A = A$
Associative laws:	$(A \cup B) \cup C = A \cup (B \cup C)$	$(A \cap B) \cap C = A \cap (B \cap C)$
Commutative laws:	$A \cup B = B \cup A$	$A \cap B = B \cap A$
Distributive laws:	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Identity laws:	$A \cup \emptyset = A$ $A \cap U = A$	$A \cap U = A$ $A \cup \emptyset = A$
Complement laws:	$A \cup A' = U$ $U' = \emptyset$	$A \cap A' = \emptyset$ $\emptyset' = U$
Involution laws:	$(A')' = A$	
De Morgan's laws:	$(A \cup B)' = A' \cap B'$	$(A \cap B)' = A' \cup B'$

EXAMPLE Use set builder notation and logical equivalences to establish the first De Morgan law $A \cap B = (A \cup B)'$.

Solution: $A \cap B = \{x \mid x \in A \cap B\}$ definition of complement

$= \{x \mid \neg(x \in (A \cup B))\}$ definition of does not belong symbol

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$$= \{x \mid \neg(x \in A \wedge x \in B)\} \quad \text{definition of intersection}$$

$$= \{x \mid \neg(x \in A) \vee \neg(x \in B)\} \quad \text{first De Morgan law}$$

$$= \{x \mid x \notin A \vee x \notin B\} \quad \text{by definition of does not belong symbol}$$

$$= \{x \mid x \in A \vee x \in B\} \quad \text{definition of complement}$$

$$= \{x \mid x \in A \cup B\} \quad \text{definition of union}$$

$$= A \cup B$$

A membership table for the distributive property

A	B	C	$B \cup C$	$A \cap (B \cup C)$	$A \cap B$	$A \cap C$	$(A \cap B) \cup (A \cap C)$
1	1	1	1	1	1	1	1
1	1	0	1	1	1	0	1
1	0	1	1	1	0	1	1
1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0

EXAMPLE Let A, B, and C be sets. Show that

$$A \cup (B \cap C) = (C \cup B) \cap A.$$

Solution: $A \cup (B \cap C) = A \cap (B \cap C)$ First De Morgan law

$$= A \cap (B \cup C) \quad \text{Second De Morgan law}$$

$$= (B \cup C) \cap A \quad \text{Commutative law}$$

$$= (C \cup B) \cap A \quad \text{by the commutative law for unions.}$$

Generalized Unions and Intersections

Unions and intersections of sets satisfy associative laws, the sets $A \cup B \cup C$ and $A \cap B \cap C$ are well defined; that is, the meaning of this notation is unambiguous when A , B , and C are sets.

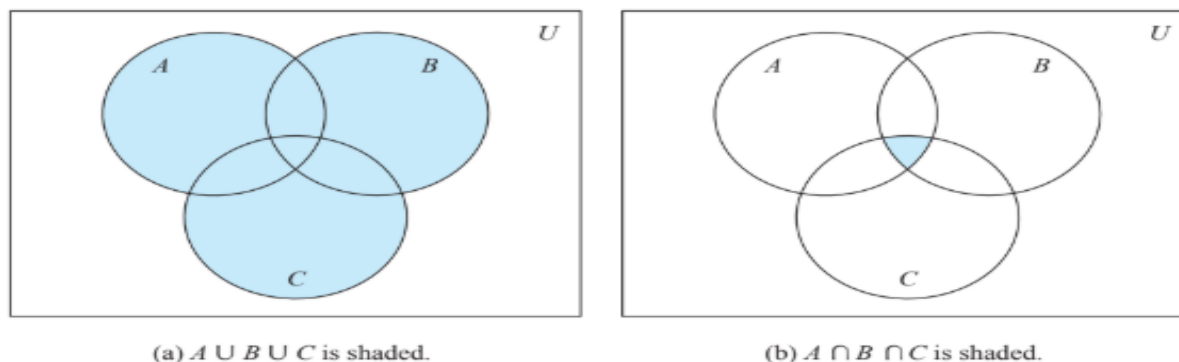


FIGURE The Union and Intersection of A , B , and C .

DEFINITION 6 The union of a collection of sets is the set that contains those elements that are members of at least one set in the collection.

DEFINITION 7 The intersection of a collection of sets is the set that contains those elements that are members of all the sets in the collection. $A_1 \cap A_2 \cap \dots \cap A_n$ to denote the intersection of the sets $A_1, A_2, \dots, A_n$.

Relations

Introduction

An ordered pair of elements a and b , where a is designated as the first element and b as the second element, is denoted by (a, b) . In particular, $(a, b) = (c, d)$ if and only if $a = c$ and $b = d$. Thus $(a, b) = (b, a)$ unless $a = b$.

Definition 1 Let A and B be sets. A binary relation or, simply, relation from A to B is a subset of $A \times B$.

(i) $(a, b) \in R$; we then say “ a is R -related to b ”, written $a R b$.

(ii) $(a, b) \notin R$; we then say “ a is not R -related to b ”, written $a \not R b$.

If R is a relation from a set A to itself, that is, if R is a subset of $A^2 = A \times A$, then we say that R is a relation on A .

EXAMPLE Set inclusion $\subseteq$ is a relation on any collection of sets. For, given any pair of set A and B , either $A \subseteq B$ or $A \not\subseteq B$.

Inverse Relation

Let R be any relation from a set A to a set B . The inverse of R , denoted by R^{-1} , is the relation from B to A which consists of those ordered pairs which, when reversed, belong to R ; that is,

$$R^{-1} = \{(b, a) \mid (a, b) \in R\}$$

For example, let $A = \{1, 2, 3\}$ and $B = \{x, y, z\}$. Then the inverse of

$$R = \{(1, y), (1, z), (3, y)\} \text{ is } R^{-1} = \{(y, 1), (z, 1), (y, 3)\}$$

Representation of Relations

Relations can be represented in many ways. Some of which are as follows:

1. Relation as a Matrix: Let $P = [a_1, a_2, a_3, \dots, a_m]$ and $Q = [b_1, b_2, b_3, \dots, b_n]$ are finite sets, containing m and n number of elements respectively. R is a relation from P to Q . The relation R can be represented by $m \times n$ matrix $M = [M_{ij}]$, defined as

$$M_{ij} = \begin{cases} 0 & \text{if } (a_i, b_j) \notin R \\ 1 & \text{if } (a_i, b_j) \in R \end{cases}$$

Example

1. Let $P = \{1, 2, 3, 4\}$, $Q = \{a, b, c, d\}$
2. and $R = \{(1, a), (1, b), (1, c), (2, b), (2, c), (2, d)\}$.

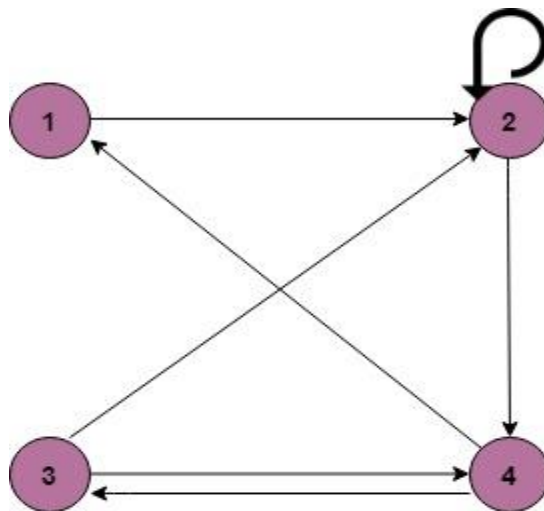
The matrix of relation R is shown as fig:

$$M_R = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \left\{ \begin{matrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix} \right\} \end{matrix}$$

2. Relation as a Directed Graph: There is another way of picturing a relation R when R is a relation from a finite set to itself.

Example

1. $A = \{1, 2, 3, 4\}$
2. $R = \{(1, 2) (2, 2) (2, 4) (3, 2) (3, 4) (4, 1) (4, 3)\}$



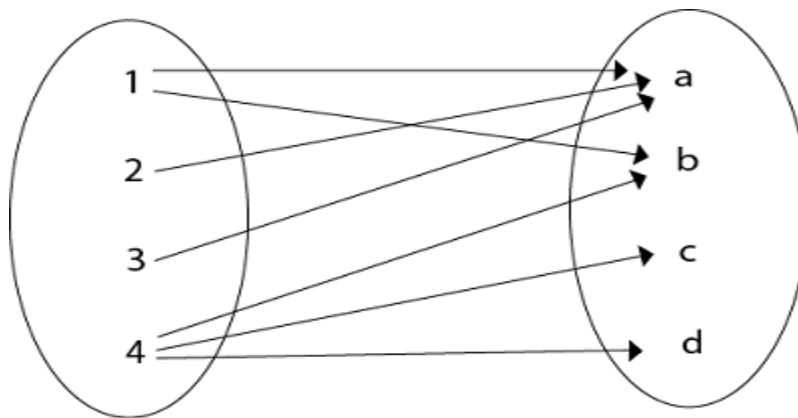
3. Relation as an Arrow Diagram: If P and Q are finite sets and R is a relation from P to Q . Relation R can be represented as an arrow diagram as follows.

Draw two ellipses for the sets P and Q . Write down the elements of P and elements of Q column-wise in three ellipses. Then draw an arrow from the first ellipse to the second ellipse if a is related to b and $a \in P$ and $b \in Q$.

Example

1. Let $P = \{1, 2, 3, 4\}$
2. $Q = \{a, b, c, d\}$
3. $R = \{(1, a), (2, a), (3, a), (1, b), (4, b), (4, c), (4, d)\}$

The arrow diagram of relation R is shown in fig:



4. Relation as a Table: If P and Q are finite sets and R is a relation from P to Q . Relation R can be represented in tabular form.

Make the table which contains rows equivalent to an element of P and columns equivalent to the element of Q . Then place a cross (X) in the boxes which represent relations of elements on set P to set Q .

Example

1. Let $P = \{1, 2, 3, 4\}$
2. $Q = \{x, y, z, k\}$
3. $R = \{(1, x), (1, y), (2, z), (3, z), (4, k)\}$.

The tabular form of relation as shown in fig:

	x	y	z	k
1	x	x		
2			x	
3			x	
4				x

Composition of Relations

Let A , B , and C be sets, and let R be a relation from A to B and let S be a relation from B to C . That is, R is a subset of $A \times B$ and S is a subset of $B \times C$. Then R and S give rise to a relation from A to C indicated by $R \circ S$ and defined by:

1. $a (R \circ S)c$ if **for** some $b \in B$ we have aRb and bSc .
2. is,
3. $R \circ S = \{(a, c) \mid \text{there exists } b \in B \text{ for which } (a, b) \in R \text{ and } (b, c) \in S\}$

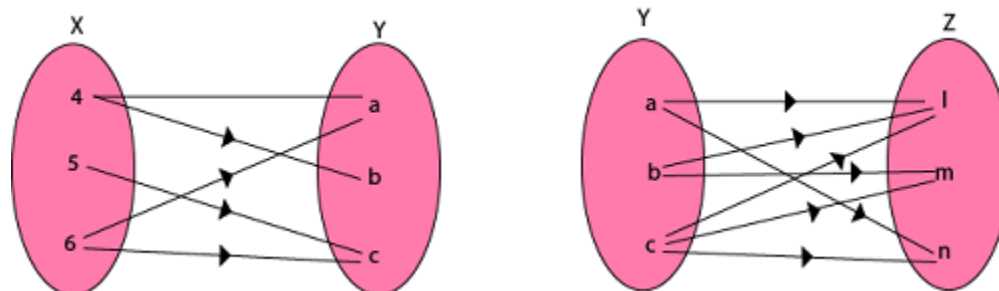
The relation $R \circ S$ is known the composition of R and S ; it is sometimes denoted simply by RS .

Let R is a relation on a set A , that is, R is a relation from a set A to itself. Then $R \circ R$, the composition of R with itself, is always represented. Also, $R \circ R$ is sometimes denoted by R^2 . Similarly, $R^3 = R^2 \circ R = R \circ R \circ R$, and so on. Thus R^n is defined for all positive n .

Example1: Let $X = \{4, 5, 6\}$, $Y = \{a, b, c\}$ and $Z = \{l, m, n\}$. Consider the relation R_1 from X to Y and R_2 from Y to Z .

$$R_1 = \{(4, a), (4, b), (5, c), (6, a), (6, c)\}$$

$$R_2 = \{(a, l), (a, n), (b, l), (b, m), (c, l), (c, m), (c, n)\}$$



Find the composition of relation (i) $R_1 \circ R_2$ (ii) $R_1 \circ R_1^{-1}$

Solution:

(i) The composition relation $R_1 \circ R_2$ as shown in fig:

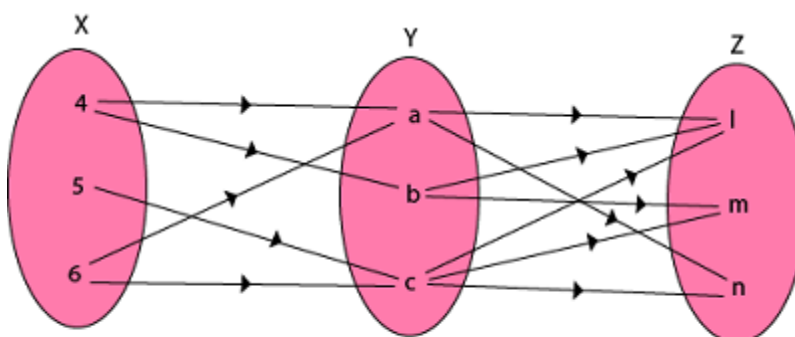


Fig : $R_1 \circ R_2$

$$R_1 \circ R_2 = \{(4, l), (4, n), (4, m), (5, l), (5, m), (5, n), (6, l), (6, m), (6, n)\}$$

(ii) The composition relation $R_1 \circ R_1^{-1}$ as shown in fig:

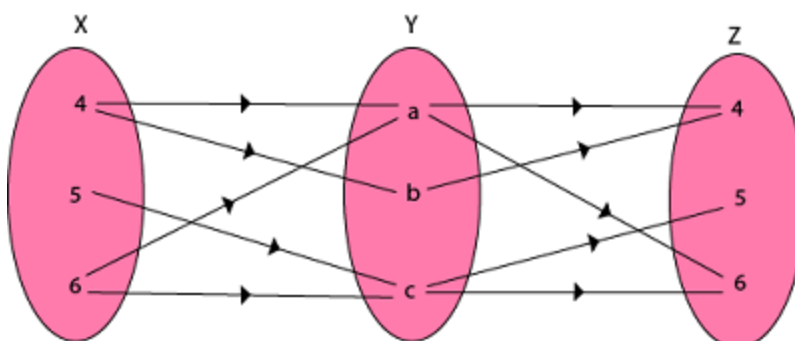


Fig : $R_1 \circ R_1^{-1}$

$$R_1 \circ R_1^{-1} = \{(4, 4), (5, 5), (5, 6), (6, 4), (6, 5), (4, 6), (6, 6)\}$$

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Composition of Relations and Matrices

There is another way of finding $R \circ S$. Let M_R and M_S denote respectively the matrix representations of the relations R and S . Then

Example

1. Let $P = \{2, 3, 4, 5\}$. Consider the relation R and S on P defined by
2. $R = \{(2, 2), (2, 3), (2, 4), (2, 5), (3, 4), (3, 5), (4, 5), (5, 3)\}$
3. $S = \{(2, 3), (2, 5), (3, 4), (3, 5), (4, 2), (4, 3), (4, 5), (5, 2), (5, 5)\}$.
- 4.
5. Find the matrices of the above relations.
6. Use matrices to find the following composition of the relation R and S .
7. (i) $R \circ S$ (ii) $R \circ R$ (iii) $S \circ R$

Solution: The matrices of the relation R and S are shown in fig:

$$M_R = \begin{matrix} & \begin{matrix} 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{matrix} \quad \text{and} \quad M_S = \begin{matrix} & \begin{matrix} 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix} \end{matrix}$$

(i) To obtain the composition of relation R and S . First multiply M_R with M_S to obtain the matrix $M_R \times M_S$ as shown in fig:

The non zero entries in the matrix $M_R \times M_S$ tells the elements related in $R \circ S$. So,

$$M_R \times M_S = \begin{matrix} & \begin{matrix} 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \left\{ \begin{matrix} 2 & 2 & 1 & 4 \\ 2 & 1 & 0 & 2 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{matrix} \right\} \end{matrix}$$

Hence the composition $R \circ S$ of the relation R and S is

1. $R \circ S = \{(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (4, 2), (4, 5), (5, 2), (5, 3), (5, 4), (5, 5)\}$.

(ii) First, multiply the matrix M_R by itself, as shown in fig

$$M_R \times M_R = \begin{matrix} & \begin{matrix} 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \left\{ \begin{matrix} 1 & 2 & 2 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{matrix} \right\} \end{matrix}$$

Hence the composition $R \circ R$ of the relation R and S is

1. $R \circ R = \{(2, 2), (3, 2), (3, 3), (3, 4), (4, 2), (4, 5), (5, 2), (5, 3), (5, 5)\}$

(iii) Multiply the matrix M_S with M_R to obtain the matrix $M_S \times M_R$ as shown in fig:

$$M_S \times M_R = \begin{matrix} & \begin{matrix} 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \left\{ \begin{matrix} 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 \end{matrix} \right\} \end{matrix}$$

The non-zero entries in matrix $M_S \times M_R$ tells the elements related in $S \circ R$.

Hence the composition $S \circ R$ of the relation S and R is

1. $S \circ R = \{(2, 4), (2, 5), (3, 3), (3, 4), (3, 5), (4, 2), (4, 4), (4, 5), (5, 2), (5, 3), (5, 4), (5, 5)\}$.

Types of Relations

1. Reflexive Relation: A relation R on set A is said to be a reflexive if $(a, a) \in R$ for every $a \in A$.

Example: If $A = \{1, 2, 3, 4\}$ then $R = \{(1, 1), (2, 2), (1, 3), (2, 4), (3, 3), (3, 4), (4, 4)\}$. Is a relation reflexive?

Solution: The relation is reflexive as for every $a \in A$, $(a, a) \in R$, i.e. $(1, 1), (2, 2), (3, 3), (4, 4) \in R$.

2. Irreflexive Relation: A relation R on set A is said to be **irreflexive** if $(a, a) \notin R$ for every $a \in A$.

Example: Let $A = \{1, 2, 3\}$ and $R = \{(1, 2), (2, 2), (3, 1), (1, 3)\}$. Is the relation R reflexive or irreflexive?

Solution: The relation R is not reflexive as for every $a \in A$, $(a, a) \notin R$, i.e., $(1, 1)$ and $(3, 3) \notin R$. The relation R is not irreflexive as $(a, a) \in R$, for some $a \in A$, i.e., $(2, 2) \in R$.

3. Symmetric Relation: A relation R on set A is said to be symmetric iff $(a, b) \in R \Leftrightarrow (b, a) \in R$.

Example: Let $A = \{1, 2, 3\}$ and $R = \{(1, 1), (2, 2), (1, 2), (2, 1), (2, 3), (3, 2)\}$. Is a relation R symmetric or not?

Solution: The relation is symmetric as for every $(a, b) \in R$, we have $(b, a) \in R$, i.e., $(1, 2), (2, 1), (2, 3), (3, 2) \in R$ but not reflexive because $(3, 3) \notin R$.

Example of Symmetric Relation:

1. Relation $\perp$ is symmetric since a line a is $\perp$ to b , then b is $\perp$ to a .

2. Also, Parallel is symmetric, since if a line a is $\parallel$ to b then b is also $\parallel$ to a .

Antisymmetric Relation: A relation R on a set A is antisymmetric iff $(a, b) \in R$ and $(b, a) \in R$ then $a = b$.

Example1: Let $A = \{1, 2, 3\}$ and $R = \{(1, 1), (2, 2)\}$. Is the relation R antisymmetric?

Solution: The relation R is antisymmetric as $a = b$ when (a, b) and (b, a) both belong to R .

Example2: Let $A = \{4, 5, 6\}$ and $R = \{(4, 4), (4, 5), (5, 4), (5, 6), (4, 6)\}$. Is the relation R antisymmetric?

Solution: The relation R is not antisymmetric as $4 \neq 5$ but $(4, 5)$ and $(5, 4)$ both belong to R .

5. Asymmetric Relation: A relation R on a set A is called an Asymmetric Relation if for every $(a, b) \in R$ implies that (b, a) does not belong to R .

6. Transitive Relations: A Relation R on set A is said to be transitive iff $(a, b) \in R$ and $(b, c) \in R \Leftrightarrow (a, c) \in R$.

Example1: Let $A = \{1, 2, 3\}$ and $R = \{(1, 2), (2, 1), (1, 1), (2, 2)\}$. Is the relation transitive?

Solution: The relation R is transitive as for every (a, b) (b, c) belong to R , we have $(a, c) \in R$ i.e, $(1, 2)$ $(2, 1) \in R \Rightarrow (1, 1) \in R$.

Note1: The Relation $\leq$, $\subseteq$ and $/$ are transitive, i.e., $a \leq b$, $b \leq c$ then $a \leq c$
(ii) Let $a \subseteq b$, $b \subseteq c$ then $a \subseteq c$
(iii) Let a/b , b/c then a/c .

Note2: $\perp r$ is not transitive since $a \perp r b$, $b \perp r c$ then it is not true that $a \perp r c$. Since no line is $\parallel$ to itself, we can have $a \parallel b$, $b \parallel a$ but $a \nparallel a$. Thus $\parallel$ is not transitive, but it will be transitive in the plane.

7. Identity Relation: Identity relation I on set A is reflexive, transitive and symmetric. So identity relation I is an Equivalence Relation.

Example: $A = \{1, 2, 3\} = \{(1, 1), (2, 2), (3, 3)\}$

8. Void Relation: It is given by $R: A \rightarrow B$ such that $R = \emptyset (\subseteq A \times B)$ is a null relation. Void Relation $R = \emptyset$ is symmetric and transitive but not reflexive.

9. Universal Relation: A relation $R: A \rightarrow B$ such that $R = A \times B (\subseteq A \times B)$ is a universal relation. Universal Relation from $A \rightarrow B$ is reflexive, symmetric and transitive. So this is an equivalence relation.

CLOSURE PROPERTIES

A relation with property P will be called a P -relation. The P -closure of an arbitrary relation R on A , written $P(R)$, is a P -relation such that

$R \subseteq P(R) \subseteq S$ for every P -relation S containing R , For the reflexive, symmetric, and transitive closures of R .

Reflexive and Symmetric Closures

Suppose P is a property such that there is at least one P -relation containing R and that the intersection of any P -relations is again a P -relation.

$P(R) = \cap \{S \mid S \text{ is a } P\text{-relation and } R \subseteq S\}$ $P(R)$ from the “top-down,” that is, as the intersection of relations. $A = \{(a, a) \mid a \in A\}$ is the diagonal or equality relation on A .

Theorem: Let R be a relation on a set A . Then:

- (i) $R \cup A$ is the reflexive closure of R .
- (ii) $R \cup R^{-1}$ is the symmetric closure of R .

In other words, $\text{reflexive}(R)$ is obtained by simply adding to R those elements (a, a) in the diagonal which do not already belong to R , and $\text{symmetric}(R)$ is obtained by adding to R all pairs (b, a) whenever (a, b) belongs to R .

Transitive Closure Let R be a relation on a set A . $R_2 = R \circ R$ and $R_n = R_{n-1} \circ R$.

The following theorem applies:

Theorem: R^* is the transitive closure of R .

Theorem: Let R be a relation on a set A with n elements. Then

$$\text{Transitive}(R) = R \cup R^2 \cup \dots \cup R^n$$

EXAMPLE Consider the relation $R = \{(1, 2), (2, 3), (3, 3)\}$ on $A = \{1, 2, 3\}$. Then: $R^2 = R \circ R = \{(1, 3), (2, 3), (3, 3)\}$ and $R^3 = R^2 \circ R = \{(1, 3), (2, 3), (3, 3)\}$

$$\text{Transitive}(R) = \{(1, 2), (2, 3), (3, 3), (1, 3)\}$$

EQUIVALENCE RELATIONS

Consider a nonempty set S . A relation R on S is an equivalence relation if R is reflexive, symmetric, and transitive. That is, R is an equivalence relation on S if it has the following three properties:

- 1) For every $a \in S$, $a R a$.
- 2) If $a R b$, then $b R a$.
- 3) If $a R b$ and $b R c$, then $a R c$.

S is an equivalence relation; that is:

- (1) $a = a$, for every $a \in S$.
- (2) If $a = b$, then $b = a$.
- (3) If $a = b$, $b = c$, then $a = c$.

Equivalence Relations and Partitions

This subsection explores the relationship between equivalence relations and partitions on a non-empty set S .

Partition P of S is a collection $\{A_i\}$ of nonempty subsets of S with the following two properties:

- (1) Each $a \in S$ belongs to some A_i .
- (2) If $A_i = A_j$ then $A_i \cap A_j = \emptyset$.

Theorem: Let R be an equivalence relation on a set S . Then S/R is a partition of S .

- (i) For each a in S , we have $a \in [a]$.
- (ii) $[a]=[b]$ if and only if $(a, b) \in R$.
- (iii) If $[a] = [b]$, then $[a]$ and $[b]$ are disjoint.

Conversely, given a partition $\{A_i\}$ of the set S , there is an equivalence relation R on S such that the sets A_i are the equivalence classes.

EXAMPLE

Consider the relation $R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}$ on $S = \{1, 2, 3\}$.

One can show that R is reflexive, symmetric, and transitive, that is, that R is an equivalence relation. Also: $[1]=\{1, 2\}$, $[2]=\{1, 2\}$, $[3]=\{3\}$

Observe that $[1] = [2]$ and that $S/R = \{[1], [3]\}$ is a partition of S .

Partial Order Relations

A relation R on a set A is called a partial order relation if it satisfies the following three properties:

1. Relation R is Reflexive, i.e. $aRa \forall a \in A$.
2. Relation R is Antisymmetric, i.e., aRb and $bRa \implies a = b$.
3. Relation R is transitive, i.e., aRb and $bRc \implies aRc$.

Example1: Show whether the relation $(x, y) \in R$, if, $x \geq y$ defined on the set of +ve integers is a partial order relation.

Solution: Consider the set $A = \{1, 2, 3, 4\}$ containing four +ve integers. Find the relation for this set such as $R = \{(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3), (1, 1), (2, 2), (3, 3), (4, 4)\}$.

Reflexive: The relation is reflexive as for every $a \in A$. $(a, a) \in R$, i.e. $(1, 1), (2, 2), (3, 3), (4, 4) \in R$.

Antisymmetric: The relation is antisymmetric as whenever (a, b) and $(b, a) \in R$, we have $a = b$.

Transitive: The relation is transitive as whenever (a, b) and $(b, c) \in R$, we have $(a, c) \in R$.

Example: $(4, 2) \in R$ and $(2, 1) \in R$, implies $(4, 1) \in R$.

As the relation is reflexive, antisymmetric and transitive. Hence, it is a partial order relation.

Example2: Show that the relation 'Divides' defined on N is a partial order relation.

Solution:

Reflexive: We have a divides a , $\forall a \in N$. Therefore, relation 'Divides' is reflexive.

Antisymmetric: Let $a, b, c \in N$, such that a divides b . It implies b divides a iff $a = b$. So, the relation is antisymmetric.

Transitive: Let $a, b, c \in N$, such that a divides b and b divides c .

Then a divides c . Hence the relation is transitive. Thus, the relation being reflexive, antisymmetric and transitive, the relation 'divides' is a partial order relation.

Example3: (a) The relation $\subseteq$ of a set of inclusion is a partial ordering or any collection of sets since set inclusion has three desired properties:

1. $A \subseteq A$ for any set A .
2. If $A \subseteq B$ and $B \subseteq A$ then $B = A$.
3. If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$

(b) The relation $\leq$ on the set R of real no that is Reflexive, Antisymmetric and transitive.

(c) Relation $\leq$ is a Partial Order Relation.

n-Ary Relations

By an n -ary relation, we mean a set of ordered n -tuples. For any set S , a subset of the product set S^n is called an n -ary relation on S . In particular, a subset of S^3 is called a ternary relation on S .

Partial Order Set (POSET):

The set A together with a partial order relation R on the set A and is denoted by (A, R) is called a partial orders set or POSET.

Total Order Relation

Consider the relation R on the set A . If it is also called the case that for all, $a, b \in A$, we have either $(a, b) \in R$ or $(b, a) \in R$ or $a = b$, then the relation R is known total order relation on set A .

Example: Show that the relation ' $<$ ' (less than) defined on N , the set of +ve integers is neither an equivalence relation nor partially ordered relation but is a total order relation.

Solution:

Reflexive: Let $a \in N$, then $a < a$
 $\Rightarrow$ ' $<$ ' is not reflexive.

As, the relation ' $<$ ' (less than) is not reflexive, it is neither an equivalence relation nor the partial order relation.

But, as $\forall a, b \in N$, we have either $a < b$ or $b < a$ or $a = b$. So, the relation is a total order relation.

Equivalence Class

Consider, an equivalence relation R on a set A . The equivalence class of an element $a \in A$, is the set of elements of A to which element a is related. It is denoted by $[a]$.

Example: Let R be an equivalence relations on the set $A = \{4, 5, 6, 7\}$ defined by $R = \{(4, 4), (5, 5), (6, 6), (7, 7), (4, 6), (6, 4)\}$.

Determine its equivalence classes.

Solution: The equivalence classes are as follows:

$$\{4\} = \{6\} = \{4, 6\}$$

$$\{5\} = \{5\}$$

$$\{7\} = \{7\}.$$

Circular Relation

Consider a binary relation R on a set A . Relation R is called circular if $(a, b) \in R$ and $(b, c) \in R$ implies $(c, a) \in R$.

Example: Consider R is an equivalence relation. Show that R is reflexive and circular.

Solution: Reflexive: As, the relation, R is an equivalence relation. So, reflexivity is the property of an equivalence relation. Hence, R is reflexive.

Circular: Let $(a, b) \in R$ and $(b, c) \in R$

$$\Rightarrow (a, c) \in R \quad (\because R \text{ is transitive})$$

$$\Rightarrow (c, a) \in R \quad (\because R \text{ is symmetric})$$

Thus, R is Circular.

Compatible Relation

A binary relation R on a set A that is Reflexive and symmetric is called Compatible Relation.

Every Equivalence Relation is compatible, but every compatible relation need not be an equivalence.

Example: Set of a friend is compatible but may not be an equivalence relation.

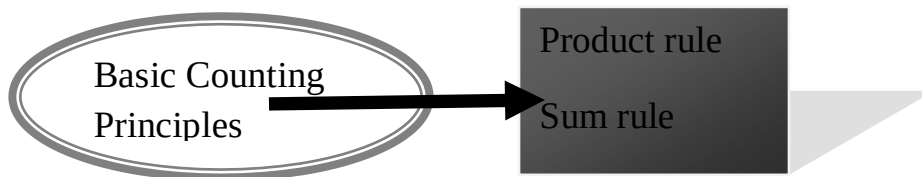
Friend Friend

$a \rightarrow b,$ $b \rightarrow c$ but possible that a and c are not friends.

Counting

Counting problems arise throughout mathematics and computer science. For example, we must count the successful outcomes of experiments and all the possible outcomes of these experiments to determine probabilities of discrete events.

Basic Counting Principles



THE PRODUCT RULE

If the events occur one after the other, then all the events can occur in the order indicated in: $n_1 \cdot n_2 \cdot n_3 \cdot \dots$ ways.

EXAMPLE

How many different license plates can be made if each plate contains a sequence of three uppercase English letters followed by three digits?

Solution: By the product rule there are a total of $26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 17,576,000$ possible license plates.

THE SUM RULE

Suppose some event E can occur in m ways and a second event F can occur in n ways, and suppose both events cannot occur simultaneously. Then E or F can occur in $m + n$ ways.

EXAMPLE Suppose a college has 3 different history courses, 4 different literature courses, and 2 different sociology courses.

Solution: (a) The number m of ways a student can choose one of each kind of courses is: $m = 3(4)(2) = 24$

(b) The number n of ways a student can choose just one of the courses is: $n = 3 + 4 + 2 = 9$

(1) Sum Rule Principle: Suppose A and B are disjoint sets. Then

$$n(A \cup B) = n(A) + n(B)$$

(2) Product Rule Principle: Let $A \times B$ be the Cartesian product of sets A and B . Then $n(A \times B) = n(A) \cdot n(B)$

MATHEMATICAL FUNCTIONS

Two important mathematical functions frequently used are:

Factorial Function

The product of the positive integers from 1 to n inclusive is denoted by $n!$, read “ n factorial.” Namely:

$$n! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot (n-2)(n-1)n = n(n-1)(n-2) \cdot \dots \cdot 3 \cdot 2 \cdot 1$$

Accordingly, $1! = 1$ and $n! = n(n-1)!$. It is also convenient to define $0! = 1$.

EXAMPLE: $3! = 3 \cdot 2 \cdot 1 = 6$, $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$, $5! = 5 \cdot 4! = 5(24) = 120$.

Binomial Coefficients: The symbol $\binom{n}{r}$, read “ n C r ” or “ n Choose r ,” where r and n are positive integers with $r \leq n$, is defined as follows:

$$\binom{n}{r} = \frac{n(n-1)\cdots(n-r+1)}{r(r-1)\cdots 3 \cdot 2 \cdot 1} \text{ or } \frac{n!}{r!(n-r)!}$$

Binomial Coefficients and Pascal's Triangle

The numbers $\binom{n}{r}$ are called binomial coefficients, since they appear as the coefficients in the expansion of $(a + b)^n$.

The Subtraction Rule (Inclusion–Exclusion for Two Sets)

If a task can be done in either n_1 ways or n_2 ways, then the number of ways to do the task is $n_1 + n_2$ minus the number of ways to do the task that are common to the two different ways.

The subtraction rule is also known as the principle of inclusion exclusion, especially when it is used to count the number of elements in the union of two sets.

Formula: $|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$.

THE DIVISION RULE

There are n/d ways to do a task if it can be done using a procedure that can be carried out in n ways, and for every way w , exactly d of the n ways correspond to way w .

Division rule in terms of sets: "If the finite set A is the union of n pair wise disjoint subsets each with d elements, then $n = |A|/d$."

The Pigeonhole Principle

Pigeonhole principle, which states that if there are more pigeons than pigeonholes, then there must be at least one pigeonhole with at least two pigeons in it.

THEOREM 1 THE PIGEONHOLE PRINCIPLE If k is a positive integer and $k + 1$ or more objects are placed into k boxes, then there is at least one box containing two or more of the objects.



(a)



(b)

Figure there are more Pigeons than Pigeonholes

Proof: We prove the pigeonhole principle using a proof by contraposition. Suppose that none of the k boxes contains more than one object. Then the total number of objects would be at most k . This is a contradiction, because there is at least $k + 1$ objects. The pigeonhole principle is also called the Dirichlet drawer principle.

COROLLARY 1 A function f from a set with $k + 1$ or more elements to a set with k elements is not one-to-one.

EXAMPLE Among any group of 367 people, there must be at least two with the same birthday, because there are only 366 possible birthdays.

The Generalized Pigeonhole Principle

THEOREM 2 If N objects are placed into k boxes, then there is at least one box containing at least $\lceil N/k \rceil$ objects.

Proof: Suppose that none of the boxes contains more than $\lceil N/k \rceil - 1$ objects. Then, the total number of objects is at most

$k(\lceil N/k \rceil - 1) < k(\lceil N/k \rceil) = N$, where the inequality $\lceil N/k \rceil < \lceil N/k \rceil + 1$ has been used.

EXAMPLE

What is the minimum number of students required in a discrete mathematics class to be sure that at least six will receive the same grade, if there are five possible grades, A, B, C, D, and F?

Solution: The smallest integer N such that $N/5 = 6$. The smallest such integer is $N = 5 \cdot 5 + 1 = 26$.

Permutations and Combinations

Permutations

A **permutation** of a set of distinct objects is an ordered arrangement of these objects. An ordered arrangement of r elements of a set is called an **r -permutation**.

EXAMPLE Let $S = \{1, 2, 3\}$. The ordered arrangement 3, 1, 2 is a permutation of S . The ordered arrangement 3, 2 is a 2-permutation of S .

The number of **r -permutations** of a set with n elements is denoted by $P(n, r)$. $P(n, r)$ using the product rule.

THEOREM 1 If n is a positive integer and r is an integer with $1 \leq r \leq n$, then there are $P(n, r) = n(n-1)(n-2)\cdots(n-r+1)$ **r -permutations** of a set with n distinct elements.

$n - (r - 1) = n - r + 1$ ways to choose the r^{th} element. By the product rule, there are $n(n-1)(n-2)\cdots(n-r+1)$ **r -permutations** of the set.

COROLLARY If n and r are integers with $0 \leq r \leq n$, then

$$P(n, r) = n! / (n - r)!$$

Proof: When n and r are integers with $1 \leq r \leq n$, by Theorem 1 we have

$$P(n, r) = n(n-1)(n-2)\cdots(n-r+1) = n! / (n-r)!$$

$$n! / (n - 0)! = n! / n! = 1 \text{ whenever } n \text{ is a nonnegative integer}$$

EXAMPLE How many ways are there to select a first-prize winner, a second-prize winner, and a third-prize winner from 100 different people who have entered a contest?

Solution: Number of 3-permutations of a set of 100 elements.

$$P(100, 3) = 100 \cdot 99 \cdot 98 = 970,200.$$

Combinations

An r -combination of elements of a set is an unordered selection of r elements from the set. Thus, an r -combination is simply a subset of the set with r elements.

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EXAMPLE Let S be the set $\{1, 2, 3, 4\}$. Then $\{1, 3, 4\}$ is a 3-combination from S .

The number of r -combinations of a set with n distinct elements is denoted by $C(n, r)$.

THEOREM 2 The number of r -combinations of a set with n elements, where n is a nonnegative integer and r is an integer with $0 \leq r \leq n$, equals $C(n, r) = n! / r! (n - r)!$

EXAMPLE How many poker hands of five cards can be dealt from a standard deck of 52 cards? Also, how many ways are there to select 47 cards from a standard deck of 52 cards?

Solution: $C(52, 5) = 52! / (5! 47!)$

First divide the numerator and denominator by $47!$ to obtain

$$C(52, 5) = (52 \cdot 51 \cdot 50 \cdot 49 \cdot 48) / (5 \cdot 4 \cdot 3 \cdot 2 \cdot 1) = 2,598,960.$$

COROLLARY 2: Let n and r be non-negative integers with $r \leq n$. Then $C(n, r) = C(n, n - r)$.

DEFINITION 1 A combinatorial proof of an identity is a proof that uses counting arguments to prove that both sides of the identity count the same objects but in different ways or a proof that is based on showing that there is a bi-ejection between the sets of objects counted by the two sides of the identity.

EXAMPLE How many ways are there to select five players from a 10-member tennis team to make a trip to a match at another school?

Sol: Number of such combinations is $C(10, 5) = 10! / 5! 5! = 252$.

Combinations and Permutations with and without Repetition.

Type	Repetition Allowed?	Formula
r -permutations	No	$n! / (n - r)!$
r -permutations	No	$n! / r! (n - r)!$

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r-permutations	Yes	n^r
r-combinations	Yes	$(n+r-1)!/r!(n-1)!$

Mathematical Induction

The process to establish the validity of an ordinary result involving natural numbers is the principle of mathematical induction.

Working Rule

Let n_0 be a fixed integer. Suppose $P(n)$ is a statement involving the natural number n and we wish to prove that $P(n)$ is true for all $n \geq n_0$.

1. Basic of Induction: $P(n_0)$ is true i.e. $P(n)$ is true for $n = n_0$.

2. Induction Step: Assume that the $P(k)$ is true for $n = k$.

Then $P(k+1)$ must also be true.

Then $P(n)$ is true for all $n \geq n_0$.

Example 1:

Prove the following by Mathematical Induction:

$$1 + 3 + 5 + \dots + 2n - 1 = n^2.$$

Solution: let us assume that.

$$P(n) = 1 + 3 + 5 + \dots + 2n - 1 = n^2.$$

$$\text{For } n = 1, \quad P(1) = 1 = 1^2 = 1$$

It is true for $n = 1$ (i)

Induction Step: For $n = r$,

$$P(r) = 1 + 3 + 5 + \dots + 2r - 1 = r^2 \text{ is true..... (ii)}$$

Adding $2r + 1$ in both sides

$$\begin{aligned} P(r+1) &= 1 + 3 + 5 + \dots + 2r-1 + 2r+1 \\ &= r^2 + (2r+1) = r^2 + 2r+1 = (r+1)^2 \dots\dots\dots (iii) \end{aligned}$$

As $P(r)$ is true. Hence $P(r+1)$ is also true.

From (i), (ii) and (iii) we conclude that.

$1 + 3 + 5 + \dots + 2n - 1 = n^2$ is true for $n = 1, 2, 3, 4, 5 \dots$ Hence Proved.

Example 2:

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Solution: For $n = 1$,

$$P(1) = 1^2 = \frac{1(1+1)(2(1)+1)}{6} = \frac{6}{6} = 1$$

It is true for $n = 1$.

Induction Step: For $n = r, \dots\dots\dots (i)$

$$P(r) = 1^2 + 2^2 + 3^2 + \dots + r^2 = \frac{r(r+1)(2r+1)}{6} \text{ is true} \dots\dots\dots (ii)$$

Adding $(r+1)^2$ on both sides, we get

$$\begin{aligned} P(r+1) &= 1^2 + 2^2 + 3^2 + \dots + r^2 + (r+1)^2 = \frac{r(r+1)(2r+1)}{6} + (r+1)^2 \\ &= \frac{r(r+1)(2r+1) + 6(r+1)^2}{6} = (r+1) \left[\frac{r(2r+1) + 6(r+1)}{6} \right] \\ &= \frac{(r+1)}{6} [r(2r+1) + 6(r+1)] = \frac{(r+1)}{6} [2r^2 + 7r + 6] \\ &= \frac{(r+1)(r+2)(2r+3)}{6} \dots\dots\dots (iii) \end{aligned}$$

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As $P(r)$ is true, hence $P(r+1)$ is true.

From (i), (ii) and (iii) we conclude that

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6} \text{ is true for } n = 1, 2, 3, 4, 5 \dots \text{ Hence Proved.}$$

Example3: Show that for any integer n

$11^{n+2} + 12^{2n+1}$ is divisible by 133.

Solution:

$$\text{Let } P(n) = 11^{n+2} + 12^{2n+1}$$

For $n = 1$,

$$P(1) = 11^3 + 12^3 = 3059 = 133 \times 23$$

So, 133 divide $P(1)$ (i)

Induction Step: For $n = r$,

$$P(r) = 11^{r+2} + 12^{2r+1} = 133 \times s \dots \dots \dots (ii)$$

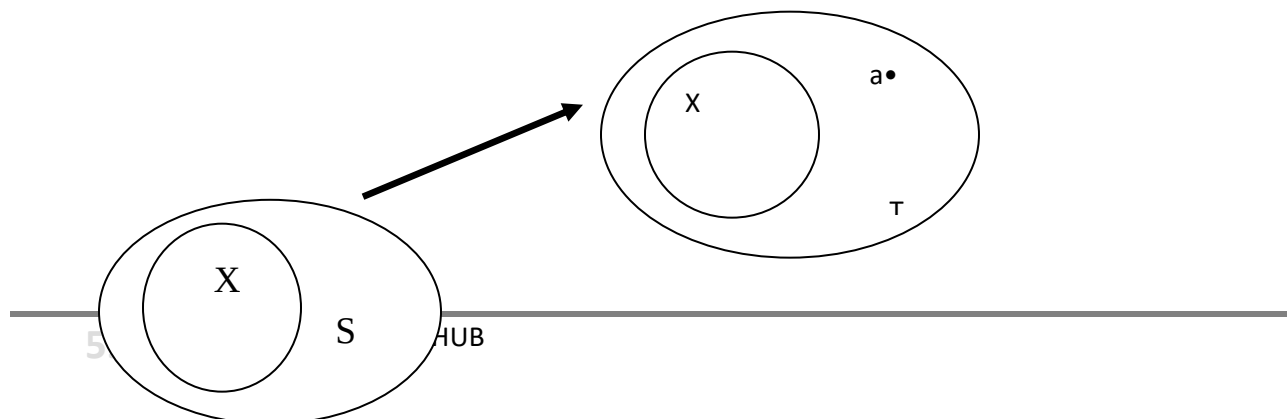
Now, for $n = r + 1$,

$$\begin{aligned} P(r+1) &= 11^{r+2+1} + 12^{2(r)+3} = 11[133s - 12^{2r+1}] + 144 \cdot 12^{2r+1} \\ &= 11 \times 133s + 12^{2r+1} \cdot 133 = 133[11s + 12^{2r+1}] = 133 \times t \dots \dots \dots (iii) \end{aligned}$$

As (i), (ii), and (iii) all are true, hence $P(n)$ is divisible by 133.

PROVING RESULTS ABOUT SETS

Mathematical induction can be used to prove many results about sets.



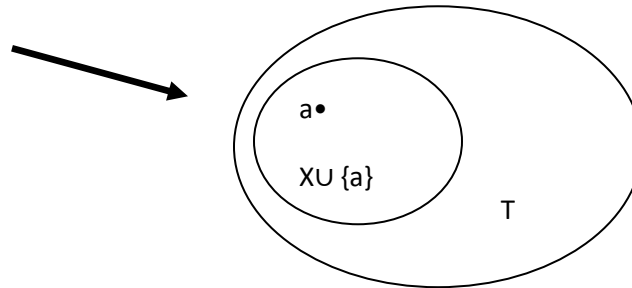


FIGURE:

Generating subsets of a set with $k+1$ elements. Here $T = S \cup \{a\}$.

Strong Induction and Well-Ordering

Strong induction, which can often be used when we cannot easily prove a result using mathematical induction.

To prove that $P(n)$ is true for all positive integers n , where $P(n)$ is a propositional function, we complete two steps:

BASIS STEP: We verify that the proposition $P(1)$ is true.

INDUCTIVE STEP: Conditional statement $[P(1) \wedge P(2) \wedge \cdots \wedge$

$P(k)] \rightarrow P(k+1)$ is true for all positive integers k .

Using Strong Induction in Computational Geometry A polygon is called convex if every line segment connecting two points in the interior of the polygon lies entirely inside the polygon. (A polygon that is not convex is said to be non-convex.)

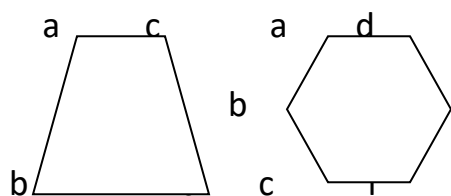


FIGURE Convex and Non-convex Polygons.

THEOREM 1 A simple polygon with n sides, where n is an integer with $n \geq 3$, can be triangulated into $n - 2$ triangles.

LEMMA 1 Every simple polygon with at least four sides has an interior diagonal.

Proof (of Theorem 1): We will prove this result using strong induction. Let $T(n)$ be the statement that every simple polygon with n sides can be triangulated into $n - 2$ triangles.

BASIS STEP: $T(3)$ is true because a simple polygon with three sides is a triangle. Consequently, every simple polygon with $n = 3$ has can be triangulated into $n - 2 = 3 - 2 = 1$ triangle.

INDUCTIVE STEP: For the inductive hypothesis, $T(j)$ is true for all integers j with $3 \leq j \leq k$. That is, we assume that we can triangulate a simple polygon with j sides into $j - 2$ triangles whenever $3 \leq j \leq k$. To complete the inductive step, we must show that when we assume the inductive hypothesis, $P(k + 1)$ is true, that is, that every simple polygon with $k + 1$ sides can be triangulated into $(k + 1) - 2 = k - 1$ triangles.

Proofs Using the Well-Ordering Property

The validity of both the principle of mathematical induction, strong induction follows from a fundamental axiom of the set of integers, the well-ordering property.

EXAMPLE Use the well-ordering property to prove the division algorithm. Recall that the division algorithm states that if a is an integer and d is a positive integer, then there are unique integers q and r with $0 \leq r < d$ and $a = dq + r$.

Solution: Let S be the set of nonnegative integers of the form $a - dq$, where q is an integer. This set is nonempty because $-dq$ can be made as large as desired (taking q to be a negative integer with large absolute value).

By the well-ordering property, S has a least element $r = a - dq_0$.

The integer r is nonnegative. It is also the case that $r < d$.

Recursively Defined Functions

BASIS STEP: Specify the value of the function at zero.

RECURSIVE STEP: Give a rule for finding its value at an integer from its values at smaller integers.

Such a definition is called a recursive or inductive definition.

EXAMPLE Suppose that f is defined recursively by $f(0) = 3$,

$f(n+1) = 2f(n) + 3$. Find $f(1)$, $f(2)$, and $f(4)$.

Solution: $f(1) = 2f(0) + 3 = 2 \cdot 3 + 3 = 9$,

$f(2) = 2f(1) + 3 = 2 \cdot 9 + 3 = 21$, $f(4) = 2f(3) + 3 = 2 \cdot 45 + 3 = 93$.

Probability

The word 'Probability' means the chance of occurring of a particular event. It is generally possible to predict the future of an event quantitatively with a certain probability of being correct. The probability is used in such cases where the outcome of the trial is uncertain.

Probability Definition:

The probability of happening of an event A , denoted by $P(A)$, is defined as

$$P(A) = \frac{\text{number of cases favourable to } A}{\text{number of possible outcomes}}$$

Thus, if an event can happen in m ways and fails to occur in n ways and $m+n$ ways is equally likely to occur then the probability of happening of the event A is given by

$$P(A) = \frac{m}{m+n}$$

And the probability of non-happening of A is

$$P(\overline{A}) = \frac{n}{m+n}$$

Note:

1. The probability of an event which is certain to occur is one.
2. The probability of an event which is impossible is zero.
3. If the probability of happening of an event $P(A)$ and that of not happening is $P(\overline{A})$, then
$$P(A) + P(\overline{A}) = 1, 0 \leq P(A) \leq 1, 0 \leq P(\overline{A}) \leq 1.$$

Important Terms related to Probability:

1. Trial and Event: The performance of an experiment is called a trial, and the set of its outcomes is termed an event.

Example: Tossing a coin and getting head is a trial. Then the event is {HT, TH, HH}

2. Random Experiment: It is an experiment in which all the possible outcomes of the experiment are known in advance. But the exact outcomes of any specific performance are not known in advance.

Example:

1. Tossing a Coin
2. Rolling a die
3. Drawing a card from a pack of 52 cards.
4. Drawing a ball from a bag.

3. Outcome: The result of a random experiment is called an Outcome.

Example: 1. Tossing a coin is an experiment and getting head is called an outcome.

2. Rolling a die and getting 6 is an outcome.

4. Sample Space: The set of all possible outcomes of an experiment is called sample space and is denoted by S .

Example: When a die is thrown, sample space is $S = \{1, 2, 3, 4, 5, 6\}$

It consists of six outcomes 1, 2, 3, 4, 5, 6

Note1: If a die is rolled n times the total number of outcomes will be 6^n .

Note2: If 1 die rolled n times then n die rolled 1 time.

5. Complement of Event: The set of all outcomes which are in sample space but not an event is called the complement of an event.

6. Impossible Events: An event which will never be happened.

Example1: Tossing double-headed coins and getting tails in an impossible event.

Example2: Rolling a die and getting number > 10 in an impossible outcome.

$P(\text{impossible outcome}) = 0$

7. Sure Outcome/Certain Outcome: An Outcome which will definitely be happen

Example1: Tossing double-headed coins and getting heads only.

Example2: Rolling a die and getting number < 6

$P(\text{sure outcome}) = 1$

$\{1, 2, 3, 4, 5, 6\}$ is called sure event

$P(\text{sure outcome}) = 1$

8. Possible Outcome: An outcome which is possible to occur is called Possible Outcome.

Example1: Tossing a fair coin and getting a head on it.

Example2: Rolling a die and getting an odd number.

9. Equally Likely Events: Events are said to be equally likely if one of them cannot be expected to occur in preference to others. In other words, it means each outcome is as likely to occur as any other outcome.

Example: When a die is thrown, all the six faces, i.e., 1, 2, 3, 4, 5 and 6 are equally likely to occur.

10. Mutually Exclusive or Disjoint Events: Events are called mutually exclusive if they cannot occur simultaneously.

Example: Suppose a card is drawn from a pack of cards, then the events getting a jack and getting a king are mutually exclusive because they cannot occur simultaneously.

11. Exhaustive Events: The total number of all possible outcomes of an experiment is called exhaustive events.

Example: In the tossing of a coin, either head or tail may turn up. Therefore, there are two possible outcomes. Hence, there are two exhaustive events in tossing a coin.

12. Independent Events: Events A and B are said to be independent if the occurrence of any one event does not affect the occurrence of any other event.

$$P(A \cap B) = P(A) P(B).$$

Example: A coin is tossed thrice, and all 8 outcomes are equally likely

A: "The first throw results in heads."

B: "The last throw results in Tails."

Prove that event A and B are independent.

Solution:

Discrete Structures and Optimization Unit - 1

Sample Space: [HHH, HHT, HTH, THH, TTT, TTH, THT, HTT]

A: [HHH, HHT, HTH, HTT]

B: [HHT, TTT, THT, HTT]

$A \cap B$: [HHT, HTT]

$$P(A) = \frac{4}{8} = \frac{1}{2}$$

$$P(B) = \frac{4}{8} = \frac{1}{2}$$

$$P(A \cap B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

13. Dependent Event: Events are said to be dependent if occurrence of one affect the occurrence of other events.

Addition Theorem

Theorem1: If A and B are two mutually exclusive events, then

$$P(A \cup B) = P(A) + P(B)$$

Proof: Let the n = total number of exhaustive cases

n_1 = number of cases favorable to A.

n_2 = number of cases favorable to B.

Now, we have A and B two mutually exclusive events. Therefore, $n_1 + n_2$ is the number of cases favorable to A or B.

$$P(A \cup B) = \frac{\text{favorable cases}}{\text{Total number of exhaustive cases}} = \frac{n_1 + n_2}{n} = \frac{n_1}{n} + \frac{n_2}{n}$$

But we have, $P(A) = \frac{n_1}{n}$ and $P(B) = \frac{n_2}{n}$

Hence, $P(A \cup B) = P(A) + P(B)$.

Example: Two dice are tossed once. Find the probability of getting an even number on first dice or a total of 8.

Solution: An even number can be got on a die in 3 ways because any one of 2, 4, 6, can come. The other die can have any number. This can happen in 6 ways.

$$\therefore P(\text{an even number on 1st die}) = \frac{3 \times 6}{36} = \frac{18}{36} = \frac{1}{2}$$

A total of 8 can be obtained in the following cases:

$$\{(2,6),(3,5),(4,4),(5,3),(6,2)\}$$

$$\therefore P(\text{a total of 8}) = \frac{5}{36}$$

$$\therefore \text{Total Probability} = \frac{1}{2} + \frac{5}{36} = \frac{23}{36}$$

Theorem2: If A and B are two events that are not mutually exclusive, then
 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Proof: Let n = total number of exhaustive cases

n_1 = number of cases favorable to A

n_2 = number of cases favorable to B

n_3 = number of cases favorable to both A and B

But A and B are not mutually exclusive. Therefore, A and B can occur simultaneously. So, $n_1 + n_2 - n_3$ is the number of cases favorable to A or B.

$$\text{Therefore, } P(A \cup B) = \frac{n_1 + n_2 - n_3}{n} = \frac{n_1}{n} + \frac{n_2}{n} - \frac{n_3}{n}$$

$$\text{But we have, } P(A) = \frac{n_1}{n}, P(B) = \frac{n_2}{n} \text{ and } P(A \cap B) = \frac{n_3}{n}$$

$$\text{Hence, } P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Example1: Two dice are tossed once. Find the probability of getting an even number on first die or a total of 8.

Solution: $P(\text{even number on 1st die or a total of 8}) = P(\text{even number on 1st die}) + P(\text{total of 8}) - P(\text{even number on 1st die and a total of 8})$

$$\therefore \text{Now, } P(\text{even number on 1st die}) = \frac{18}{36} = \frac{1}{2}$$

Discrete Structures and Optimization Unit - 1

Ordered Pairs showing a total of 8 = {(6, 2), (5, 3), (4, 4), (3, 5), (2, 6)} = 5

$$\therefore \text{Probability; } P(\text{total of 8}) = \frac{5}{36}$$

$$P(\text{even number on 1st die and total of 8}) = \frac{3}{36}$$

$$\therefore \text{Required Probability} = \frac{18}{36} + \frac{5}{36} - \frac{3}{36} = \frac{20}{36} = \frac{5}{9}$$

Example2: Two dice are thrown. The events A, B, C, D, E, F

A = getting even number on first die.

B = getting an odd number on the first die.

C = getting a sum of the number on dice ≤ 5

D = getting a sum of the number on dice > 5 but less than 10.

E = getting sum of the number on dice ≥ 10 .

F = getting odd number on one of the dice.

Show the following:

1. A, B are a mutually exclusive event and Exhaustive Event.
2. A, C are not mutually exclusive.
3. C, D are a mutually exclusive event but not Exhaustive Event.
4. C, D, E are a mutually exclusive and exhaustive event.
5. A' $\cap$ B' are a mutually exclusive and exhaustive event.
6. A, B, F are not a mutually exclusive event.

Solution:

$$\text{Sample Space} = \begin{bmatrix} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{bmatrix}$$

Discrete Structures and Optimization Unit - 1

A: (2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)

B: (1,1), (1,2),(1,3),(1,4),(1,5),(1,6)
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)

C: (1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(3,1),(3,2),(4,1)

D: (1,5),(1,6),(2,4),(2,5),(2,6)
(3,3),(3,4),(3,5),(3,6)
(4,2),(4,3),(4,4),(4,5)
(5,1),(5,2),(5,3),(5,4)
(6,1),(6,2),(6,3)

E: (4,6),(5,5),(5,6),(6,5),(6,6),(6,4)

F: (1,2),(1,4),(1,6)
(2,1),(2,3),(2,5)
(3,2),(3,4),(3,6)
(4,1),(4,3),(4,5)
(5,2),(5,4),(5,6)
(6,1),(6,3),(6,5)

1. $(A \cap B) = \emptyset$ and $(A \cup B) = S$

A, B are a mutually exclusive and exhaustive event.

2. $(A \cap C)$ are not mutually exclusive

$(2,1),(2,3),(4,1) \neq \emptyset$

3. $C \cap D$ are a mutually exclusive but not exhaustive event.

$C \cap D = \emptyset$ $C \cup D \neq S$

4. $C \cap D = \emptyset, D \cap E = \emptyset, C \cap E = \emptyset$ are mutually exclusive and exhaustive event.

5. $A' \cap B' = (A \cup B)'$ are a mutually exclusive and exhaustive event.

6. $(A \cap B) = \emptyset$ are a mutually exclusive

A, B, F are not mutually exclusive events.

Multiplication Theorem

Theorem: If A and B are two independent events, then the probability that both will occur is equal to the product of their individual probabilities.

$$P(A \cap B) = P(A) \times P(B)$$

Proof: Let event

A can happen in n_1 ways of which p are successful

B can happen in n_2 ways of which q are successful

Now, combine the successful event of A with successful event of B.

Thus, the total number of successful cases = $p \times q$

We have, total number of cases = $n_1 \times n_2$.

Therefore, from definition of probability

$$P(A \text{ and } B) = P(A \cap B) = \frac{p \times q}{n_1 \times n_2} = \frac{p}{n_1} \times \frac{q}{n_2}$$

$$\text{We have } P(A) = \frac{p}{n_1}, P(B) = \frac{q}{n_2}$$

So, $P(A \cap B) = P(A) \times P(B)$

If, there are three independent events A, B and C, then

$$\begin{aligned} P(A \cap B \cap C) &= P((A \cap B) \cap C) = P(A \cap B) \times P(C) \\ &= P(A) \times P(B) \times P(C). \end{aligned}$$

In general, if there are n independent events, then

$$P(\cap_{i=1}^n A_i) = \prod_{i=1}^n P(A_i).$$

Example: A bag contains 5 green and 7 red balls. Two balls are drawn. Find the probability that one is green and the other is red.

Solution: $P(A) = P(\text{a green ball}) = \frac{1}{5}$

$$P(B) = P(\text{a red ball}) = \frac{1}{7}$$

By Multiplication Theorem

$$P(A) \text{ and } P(B) = P(A) \times P(B) = \frac{1}{5} \times \frac{1}{7} = \frac{1}{35}$$

CONDITIONAL PROBABILITY

Suppose E is an event in a sample space S with $P(E) > 0$. The probability that an event A occurs once E has occurred or, specifically, the conditional probability of A given E , written $P(A|E)$, is defined as follows: $P(A|E) = P(A \cap E) / P(E)$

Theorem: Suppose S is an equi-probable space and A and E are events. Then

$$P(A|E) = \text{number of elements in } A \cap E / \text{number of elements in } E = n(A \cap E) / n(E)$$

Example: A couple has two children; the sample space is $S = \{bb, bg, gb, gg\}$ with probability $1/4$ for each point. Find the probability p that both children are boys if it is known that: (i) at least one of the children is a boy; (ii) the older child is a boy.

Solution: Here the reduced space consists of three elements, $\{bb, bg, gb\}$; hence $p = 1/3$

Here the reduced space consists of only two elements $\{bb, bg\}$; hence $p = 1/2$

Multiplication Theorem for Conditional Probability

Theorem: $P(A \cap B) = P(A) P(B|A)$

The multiplication theorem gives us a formula for the probability that events A and B both occur. It can easily be extended to three or more events $A_1, A_2, \dots, A_m$; that is,

$$P(A_1 \cap A_2 \cap \dots \cap A_m) = P(A_1) \cdot P(A_2|A_1) \cdots P(A_m|A_1 \cap A_2 \cap \dots \cap A_{m-1})$$

INDEPENDENT EVENTS

Definition 7.2: Events A and B are independent if $P(A \cap B) = P(A)P(B)$; otherwise they are dependent.

$P(A \cap B) = P(A) P(B)$ implies both $P(B|A) = P(B)$ and $P(A|B) = P(A)$

Discrete Structures and Optimization Unit - 1

EXAMPLE A fair coin is tossed three times yielding the equi-probable space.

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

Solution Consider the events: $A = \{\text{first toss is heads}\} = \{HHH, HHT, HTH, HTT\}$

$$B = \{\text{second toss is heads}\} = \{HHH, HHT, THH, THT\}$$

$$C = \{\text{exactly two heads in a row}\} = \{HHT, THH\}$$

$$P(A) = 4/8 = 1/2, P(B) = 4/8 = 1/2, P(C) = 2/8 = 1/4$$

$$P(A \cap B) = P(\{HHH, HHT\}) = 1/4, P(A \cap C) = P(\{HHT\}) = 1/8, P(B \cap C) = P(\{HHT, THH\}) = 1/4$$

INDEPENDENT REPEATED TRIALS, BINOMIAL DISTRIBUTION

Definition: Let S be a finite probability space. By the space of n independent repeated trials, we mean the probability space S^n consisting of ordered n -tuples of elements of S , with the probability of an n -tuples defined to be the product of the probabilities of its components:

$$P((S_1, S_2, \dots, S_n)) = P(S_1)P(S_2) \dots P(S_n)$$

RANDOM VARIABLES

Definition: A random variable X is a rule that assigns a numerical value to each outcome in a sample space S .

EXAMPLE A pair of fair dice is tossed. The sample space S consists of the 36 ordered pairs (a, b) where a and b can be any of the integers from 1 to 6.

Solution: Let x assign to each point in S the sum of the numbers; then x is a random variable with range space

$$R_x = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

Let y assign to each point the maximum of the two numbers; then y is a random variable with range space

$$R_y = \{1, 2, 3, 4, 5, 6\}$$

Probability Distribution of a Random Variable

Theorem: Let S be an equiv.-probable space, and let f be the distribution of a random variable X on S with the range space $R_x = \{x_1, x_2, \dots, x_t\}$.

$$P_i = f(x_i) = \text{No. of points in } S \text{ whose image is } x_i / \text{No. of points in } S$$

EXAMPLE Let X be the random variable in Example 7.13 which assigns the sum to the toss of a pair of dice. Note $n(S) = 36$, and $R_x = \{2, 3, \dots, 12\}$.

Solution: Using Theorem, we obtain the distribution f of X as follows:

$f(2) = 1/36$, since there is one outcome $(1, 1)$ whose sum is 2.

$f(3) = 2/36$, since there are two outcomes, $(1, 2)$ and $(2, 1)$, whose sum is 3.

$f(4) = 3/36$, since there are three outcomes, $(1, 3)$, $(2, 2)$ and $(3, 1)$, whose sum is 4.

Similarly, $f(5) = 4/36$, $f(6) = 5/36$... $f(12) = 1/36$. Thus the distribution of X follows:

x	2	3	4	5	6	7	8	9	10
$F(x)$	1/36	2/36	3/36	4/36	5/36	6/36	7/36	8/36	7/36

Binomial Distribution

Theorem: Consider the binomial distribution $B(n, p)$. Then:

(i) Expected value $E(X) = \mu = n p$.

(ii) Variance $V a r(X) = \sigma^2 = n p q$.

(iii) Standard deviation $\sigma = \sqrt{n p q}$.

EXAMPLE

The probability that a man hits a target is $p = 1/5$. He fires 100 times. Find the expected number μ of times

Solution: Man will hit the target and the standard deviation σ .

Here $p = 1/5$ and so $q = 4/5$. Hence

$$\mu = n p = 100 * 1/5 = 20 \text{ and } \sigma = \sqrt{n p q} = \sqrt{100 * 1/5 * 4/5} = 4$$

Bayes' theorem in Artificial intelligence

Bayes' theorem:

Bayes' theorem is also known as **Bayes' rule**, **Bayes' law**, or **Bayesian reasoning**, which determines the probability of an event with uncertain knowledge.

In probability theory, it relates the conditional probability and marginal probabilities of two random events.

Bayes' theorem was named after the British mathematician **Thomas Bayes**.

The **Bayesian inference** is an application of Bayes' theorem, which is fundamental to Bayesian statistics.

It is a way to calculate the value of $P(B|A)$ with the knowledge of $P(A|B)$.

Bayes' theorem allows updating the probability prediction of an event by observing new information of the real world.

Example: If cancer corresponds to one's age then by using Bayes' theorem, we can determine the probability of cancer more accurately with the help of age.

Bayes' theorem can be derived using product rule and conditional probability of event A with known event B:

As from product rule we can write:

1. $P(A \wedge B) = P(A|B) P(B)$ or

Similarly, the probability of event B with known event A:

$$1. P(A \cap B) = P(B|A) P(A)$$

Equating right hand side of both the equations, we will get:

$$P(A|B) = \frac{P(B|A) P(A)}{P(B)} \quad \dots(a)$$

The above equation (a) is called as **Bayes' rule** or **Bayes' theorem**. This equation is basic of most modern AI systems for **probabilistic inference**.

It shows the simple relationship between joint and conditional probabilities. Here,

$P(A|B)$ is known as **posterior**, which we need to calculate, and it will be read as Probability of hypothesis A when we have occurred an evidence B.

$P(B|A)$ is called the likelihood, in which we consider that hypothesis is true, then we calculate the probability of evidence.

$P(A)$ is called the **prior probability**, probability of hypothesis before considering the evidence

$P(B)$ is called **marginal probability**, pure probability of an evidence.

In the equation (a), in general, we can write $P(B) = \sum_{i=1}^k P(A_i) * P(B|A_i)$, hence the Bayes' rule can be written as:

$$P(A_i|B) = \frac{P(A_i) * P(B|A_i)}{\sum_{i=1}^k P(A_i) * P(B|A_i)}$$

Where $A_1, A_2, A_3, \dots, A_n$ is a set of mutually exclusive and exhaustive events.

Applying Bayes' rule:

Bayes' rule allows us to compute the single term $P(B|A)$ in terms of $P(A|B)$, $P(B)$, and $P(A)$. This is very useful in cases where we have a good probability of these three terms and want to determine the fourth one. Suppose we want to perceive the effect of some unknown cause, and want to compute that cause, then the Bayes' rule becomes:

$$P(\text{cause} | \text{effect}) = \frac{P(\text{effect} | \text{cause}) P(\text{cause})}{P(\text{effect})}$$

Example-1:

Question: what is the probability that a patient has diseases meningitis with a stiff neck?

Given Data:

A doctor is aware that disease meningitis causes a patient to have a stiff neck, and it occurs 80% of the time. He is also aware of some more facts, which are given as follows:

- The Known probability that a patient has meningitis disease is 1/30,000.
- The Known probability that a patient has a stiff neck is 2%.

Let a be the proposition that patient has stiff neck and b be the proposition that patient has meningitis. , so we can calculate the following as:

$$P(a | b) = 0.8$$

$$P(b) = 1/30000$$

$$P(a) = .02$$

$$P(b | a) = \frac{P(a | b) P(b)}{P(a)} = \frac{0.8 * (\frac{1}{30000})}{0.02} = 0.001333333.$$

Hence, we can assume that 1 patient out of 750 patients has meningitis disease with a stiff neck.

Example-2:

Question: From a standard deck of playing cards, a single card is drawn. The probability that the card is king is 4/52, then calculate posterior probability $P(\text{King} | \text{Face})$, which means the drawn face card is a king card.

Solution:

$$P(\text{king} | \text{face}) = \frac{P(\text{Face} | \text{king}) \cdot P(\text{King})}{P(\text{Face})} \dots\dots(i)$$

P(king): probability that the card is King= $\frac{4}{52} = \frac{1}{13}$

P(face): probability that a card is a face card= $\frac{3}{13}$

P(Face | King): probability of face card when we assume it is a king = 1

Putting all values in equation (i) we will get:

$$P(\text{king} | \text{face}) = \frac{1 * (\frac{1}{13})}{(\frac{3}{13})} = \frac{1}{3}, \text{ it is a probability that a face card is a king card.}$$

Application of Bayes' theorem in Artificial intelligence:

Following are some applications of Bayes' theorem:

- It is used to calculate the next step of the robot when the already executed step is given.
- Bayes' theorem is helpful in weather forecasting.
- It can solve the Monty Hall problem.

Group Theory

Group

Let G be a non-void set with a binary operation * that assigns to each ordered pair (a, b) of elements of G an element of G denoted by a * b. We say that G is a group under the binary operation * if the following three properties are satisfied:

1) Associativity: The binary operation * is associative i.e. $a * (b * c) = (a * b) * c$, $\forall a, b, c \in G$

2) Identity: There is an element e , called the identity, in G , such that $a * e = e * a = a$, $\forall a \in G$

3) Inverse: For each element a in G , there is an element b in G , called an inverse of a such that $a * b = b * a = e$, $\forall a, b \in G$

Note: If a group has the property that $a * b = b * a$ i.e., commutative law holds then the group is called an abelian.

Properties of Groups:

The following theorems can understand the elementary features of Groups:

Theorem1:-

1. Statement: - In a Group G , there is only one identity element (uniqueness of identity) Proof: - let e and e' are two identities in G and let $a \in G$

$$\therefore ae = a \rightarrow (i)$$

$$\therefore ae' = a \rightarrow (ii)$$

R.H.S of (i) and (ii) are equal $\Rightarrow ae = ae'$

Thus by the left cancellation law, we obtain $e = e'$

There is only one identity element in G for any $a \in G$. Hence the theorem is proved.

2. Statement: - For each element a in a group G , there is a unique element b in G such that $ab = ba = e$ (uniqueness of inverses)

Proof: - let b and c are both inverses of $a \in G$

Then $ab = e$ and $ac = e$

$$\therefore c = ce \text{ \{existence of identity element\}}$$

$$\Rightarrow c = c(ab) \text{ \{ \because } ab = e \}}$$

$$\Rightarrow c = (ca)b$$

$$\Rightarrow c = (ac)b \text{ \{ \because } ac = ca \}}$$

$$\Rightarrow c = eb$$

$$\Rightarrow c = b \{ \because b = eb \}$$

Hence inverse of a G is unique.

Theorem 2:-

1. Statement: - In a Group $G, (a^{-1})^{-1} = a, \forall a \in G$

Proof: We have $a a^{-1} = a^{-1} a = e$

Where e is the identity element of G

Thus a is inverse of $a^{-1} \in G$

$$\text{i.e., } (a^{-1})^{-1} = a, \forall a \in G$$

2. Statement: In a Group $G, (a b^{-1}) = b^{-1} a^{-1}, \forall a, b \in G$

Proof: - By associativity we have

$$(b^{-1} a^{-1})ab = b^{-1} (a^{-1} a)b$$

$$\Rightarrow (b^{-1} a^{-1})ab = b^{-1} (e)b \quad \{ \because a^{-1} a = e \}$$

$$\Rightarrow (b^{-1} a^{-1})ab = b^{-1} b \quad \{ \because eb = b \}$$

$$\Rightarrow (b^{-1} a^{-1})ab = e, \quad \{ \because b^{-1} b = e \}$$

Similarly

$$(ab) (b^{-1} a^{-1}) = a(b b^{-1}) a^{-1}$$

$$\Rightarrow (ab) (b^{-1} a^{-1}) = a (e) a^{-1}$$

$$\Rightarrow (ab) (b^{-1} a^{-1}) = a a^{-1}$$

$$\Rightarrow (ab) (b^{-1} a^{-1}) = e \quad \{ \because aa^{-1} = e \}$$

$$\text{Thus } (b^{-1} a^{-1})ab = (ab)(b^{-1} a^{-1}) = e$$

$\therefore b^{-1} a^{-1}$ is the inverse of ab

$$\text{i.e., } b^{-1} a^{-1} = a b^{-1}$$

Hence the theorem is proved.

Theorem 3:-

In a group G , the left and right cancellation laws hold i.e.

(i) $ab = ac$ implies $b = c$

(ii) $ba = ca$ implies $b = c$

Proof

(i) Let $ab = ac$

Premultiplying a^{-1} on both sides we get

$$\begin{aligned} a^{-1}(ab) &= a^{-1}(ac) \\ \Rightarrow (a^{-1}a)b &= (a^{-1}a)c \\ \Rightarrow eb &= ec \\ \Rightarrow b &= c \end{aligned}$$

Hence Proved.

(ii) Let $ba = ca$

Post-multiplying a^{-1} on both sides

$$\begin{aligned} \Rightarrow (ba)a^{-1} &= (ca)a^{-1} \\ \Rightarrow b(aa^{-1}) &= c(aa^{-1}) \\ \Rightarrow be &= ce \\ \Rightarrow b &= c \end{aligned}$$

Hence the theorem is proved.

Finite and Infinite Group:

A group $(G, *)$ is called a finite group if G is a finite set.

A group $(G, *)$ is called a infinite group if G is an infinite set.

Example1: The group $(\mathbb{I}, +)$ is an infinite group as the set $\mathbb{I}$ of integers is an infinite set.

Example2: The group $G = \{1, 2, 3, 4, 5, 6, 7\}$ under multiplication modulo 8 is a finite group as the set G is a finite set.

Order of Group:

Discrete Structures and Optimization Unit - 1

The order of the group G is the number of elements in the group G . It is denoted by $|G|$. A group of order 1 has only the identity element, i.e., $(\{e\}, *)$.

A group of order 2 has two elements, i.e., one identity element and one some other element.

Example1: Let $(\{e, x\}, *)$ be a group of order 2. The table of operation is shown in fig:

*	e	x
e	e	x
x	x	e

The group of order 3 has three elements i.e., one identity element and two other elements.

Symmetric Group S_n

A one-to-one mapping σ of the set $\{1, 2, \dots, n\}$ onto itself is called a permutation. $\sigma =$

$\begin{matrix} 1 & 2 & 3 & \dots & n \end{matrix}$

$\begin{matrix} j_1 & j_2 & j_3 & \dots & j_n \end{matrix}$

The set of all such permutations is denoted by S_n , and there are $n! = n(n-1) \cdot \dots \cdot 2 \cdot 1$ of them.

Subgroup

If a non-void subset H of a group G is itself a group under the operation of G , we say H is a subgroup of G .

Theorem: - A subset H of a group G is a subgroup of G if:

- the identity element $a \in H$.
- H is closed under the operation of G i.e. if $a, b \in H$, then $a \cdot b \in H$ and

- H is closed under inverses, that is if $a \in H$ then $a^{-1} \in H$.

Cyclic Subgroup:-

A Subgroup K of a group G is said to be cyclic subgroup if there exists an element $x \in G$ such that every element of K can be written in the form x^n for some $n \in \mathbb{Z}$.

The element x is called generator of K and we write $K = \langle x \rangle$

Cyclic Group:-

In the case when $G = K$, we say G is cyclic and x is a generator of G. That is, a group G is said to be cyclic if there is an element $x \in G$ such that every element of G can be written in the form x^n for the some $n \in \mathbb{Z}$.

Example: The group $G = \{1, -1, i, -i\}$ under usual multiplication is a finite cyclic group with i as generator, since $i^1 = i, i^2 = -1, i^3 = -i$ and $i^4 = 1$

Abelian Group:

Let us consider an algebraic system $(G, *)$, where $*$ is a binary operation on G. Then the system $(G, *)$ is said to be an abelian group if it satisfies all the properties of the group plus a additional following property:

- (1) The operation $*$ is commutative i.e.,
 $a * b = b * a \quad \forall a, b \in G$

Example: Consider an algebraic system $(G, *)$, where G is the set of all non-zero real numbers and $*$ is a binary operation defined by

$$a * b = \frac{ab}{4}.$$

Show that $(G, *)$ is an abelian group.

Solution:

Closure Property: The set G is closed under the operation $*$, since $a * b = \frac{ab}{4}$ is a real number. Hence, it belongs to G.

Discrete Structures and Optimization Unit - 1

Associative Property: The operation $*$ is associative. Let $a, b, c \in G$, then we have

$$(a * b) * c = \left(\frac{ab}{4}\right) * c = \frac{(ab)c}{16} = \frac{abc}{16}$$

Similarly,
$$a * (b * c) = a * \left(\frac{bc}{4}\right) = \frac{a(bc)}{16} = \frac{abc}{16}$$

Identity: To find the identity element, let us assume that e is a +ve real number. Then $e * a = a$, where $a \in G$.

$$\frac{Ea}{4} = a \quad \text{or} \quad e = 4$$

Similarly,
$$a * e = a$$

$$\frac{Ae}{4} = a \quad \text{or} \quad e = 4.$$

Thus, the identity element in G is 4.

Inverse: let us assume that $a \in G$. If $a^{-1} \in G$, is an inverse of a , then $a * a^{-1} = 4$

Therefore,
$$\frac{aa^{-1}}{4} = 4 \quad \text{or} \quad a^{-1} = \frac{16}{a}$$

Similarly,
$$a^{-1} * a = 4$$

Therefore,
$$\frac{a^{-1}a}{4} = 4 \quad \text{or} \quad a^{-1} = \frac{16}{a}$$

Thus, the inverse of element a in G is $\frac{16}{a}$

Commutative: The operation $*$ on G is commutative.

Since,
$$a * b = \frac{ab}{4} = b * a$$

Thus, the algebraic system $(G, *)$ is closed, associative, identity element, inverse and commutative. Hence, the system $(G, *)$ is an abelian group.

Product of Groups:

Theorem: Prove that if $(G_1, *_1)$ and $(G_2, *_2)$ are groups, then $G = G_1 \times G_2$ i.e., $(G, *)$ is a group with operation defined by $(a_1, b_1) * (a_2, b_2) = (a_1 *_1 a_2, b_1 *_2 b_2)$.

Proof: To prove that $G_1 \times G_2$ is a group, we have to show that $G_1 \times G_2$ has the associativity operator, has an identity and also exists inverse of every element.

Associativity. Let $a, b, c \in G_1 \times G_2$, then

$$\begin{aligned}\text{So, } a * (b * c) &= (a_1, a_2) * ((b_1, b_2) * (c_1, c_2)) \\ &= (a_1, a_2) * (b_1 *_1 c_1, b_2 *_2 c_2) \\ &= (a_1 *_1 (b_1 *_1 c_1), a_2 *_2 (b_2 *_2 c_2)) \\ &= ((a_1 *_1 b_1) *_1 c_1, (a_2 *_2 b_2) *_2 c_2) \\ &= (a_1 *_1 b_1, a_2 *_2 b_2) * (c_1, c_2) \\ &= ((a_1, a_2) * (b_1, b_2)) * (c_1, c_2) \\ &= (a * b) * c.\end{aligned}$$

Identity: Let e_1 and e_2 are identities for G_1 and G_2 respectively. Then, the identity for $G_1 \times G_2$ is $e = (e_1, e_2)$. Assume same $a \in G_1 \times G_2$

$$\begin{aligned}\text{Then, } a * e &= (a_1, a_2) * (e_1, e_2) \\ &= (a_1 *_1 e_1, a_2 *_2 e_2) \\ &= (a_1, a_2) = a\end{aligned}$$

Similarly, we have $e * a = a$.

Inverse: To determine the inverse of an element in $G_1 \times G_2$, we will determine it component wise i.e.,

$$a^{-1} = (a_1, a_2)^{-1} = (a_1^{-1}, a_2^{-1})$$

Now to verify that this is the exact inverse, we will compute $a * a^{-1}$ and $a^{-1} * a$.

$$\begin{aligned}\text{Now, } a * a^{-1} &= (a_1, a_2) * (a_1^{-1}, a_2^{-1}) \\ &= (a_1 *_1 a_1^{-1}, a_2 *_2 a_2^{-1}) = (e_1, e_2) = e\end{aligned}$$

Similarly, we have $a^{-1} * a = e$.

Thus, $(G_1 \times G_2, *)$ is a group.

In general, if $G_1, G_2, \dots, G_n$ are groups, then $G = G_1 \times G_2 \times \dots \times G_n$ is also a group.

Cosets:

Let H be a subgroup of a group G . A left coset of H in G is a subset of G whose elements may be expressed as $xH = \{xh \mid h \in H\}$ for any $x \in G$. The element x is called a representation of the coset. Similarly, a right coset of H in G is a subset that may be expressed as $Hx = \{hx \mid h \in H\}$, for any $x \in G$. Thus complexes xH and Hx are called respectively a left coset and a right coset.

If the group operation is additive (+) then a left coset is denoted as $x + H = \{x+h \mid h \in H\}$ and a right coset is denoted by $H + x = \{h+x \mid h \in H\}$

Normal SubGroup

Let G be a group. A subgroup H of G is said to be a normal subgroup of G if for all $h \in H$ and $x \in G$, $x h x^{-1} \in H$

If $x H x^{-1} = \{x h x^{-1} \mid h \in H\}$ then H is normal in G if and only if $x H x^{-1} \subseteq H, \forall x \in G$

Statement: If G is an abelian group, then every subgroup H of G is normal in G .

Proof: Let any $h \in H, x \in G$, then

$$x h x^{-1} = x (h x^{-1})$$

$$x h x^{-1} = (x x^{-1}) h$$

$$x h x^{-1} = e h$$

$$x h x^{-1} = h \in H$$

Hence H is normal subgroup of G .

Group Homomorphism:

A homomorphism is a mapping $f: G \rightarrow G'$ such that $f(xy) = f(x)f(y), \forall x, y \in G$. The mapping f preserves the group operation although the binary operations of the group G and G' are different. Above condition is called the homomorphism condition.

Discrete Structures and Optimization Unit - 1

Kernel of Homomorphism: - The Kernel of a homomorphism f from a group G to a group G' with identity e' is the set $\{x \in G \mid f(x) = e'\}$

The kernel of f is denoted by $\text{Ker } f$.

If $f: G \rightarrow G'$ is a homomorphism of G into G' , then the image set of f is the range, denoted by $f(G)$, of the map f . Thus

$$\text{Im}(f) = f(G) = \{f(x) \in G' \mid x \in G\}$$

If $f(G) = G'$, then G' is called a homomorphic image of G .

Note: - A group homomorphism

Isomorphism:

Let $(G_1, *)$ and $(G_2, \circ)$ be two algebraic system, where $*$ and $\circ$ both are binary operations. The systems $(G_1, *)$ and $(G_2, \circ)$ are said to be isomorphic if there exists an isomorphic mapping $f: G_1 \rightarrow G_2$

When two algebraic systems are isomorphic, the systems are structurally equivalent and one can be obtained from another by simply renaming the elements and operation.

Example: Let $(A_1, *)$ and $(A_2, \square)$ be the two algebraic systems as shown in fig. Determine whether the two algebraic systems are isomorphic.

x	a	b	c
a	a	b	c
b	b	c	a
c	c	a	b

$\square$	1	w	w^2
1	1	w	w^2
w	w	w^2	1
w^2	w^2	1	w

Solution: The two algebraic system $(A_1, *)$ and $(A_2, \square)$ are isomorphic and $(A_2, \square)$ is an isomorphic image of A_1 , such that

$$f(a) = 1$$

$$f(b) = w$$

$$f(c) = w^2$$

Automorphism:

Let $(G_1, *)$ and $(G_2, 0)$ be two algebraic system, where $*$ and 0 both are binary operations on G_1 and G_2 respectively. Then an isomorphism from $(G_1, *)$ to $(G_2, 0)$ is called an automorphism if $G_1 = G_2$

Rings:

An algebraic system $(R, +, \cdot)$ where R is a set with two arbitrary binary operations $+$ and $\cdot$, is called a ring if it satisfies the following conditions

1. $(R, +)$ is an abelian group.
2. $(R, \cdot)$ is a semigroup.
3. The multiplication operation, is distributive over the addition operation
i.e.,
$$a(b+c) = ab + ac \text{ and } (b+c)a = ba + ca \text{ for all } a, b, c \in R.$$

Example1: Consider M be the set of all matrices of the type $\begin{bmatrix} c & d \\ 0 & 0 \end{bmatrix}$ over integers under matrix addition and matrix multiplication. Thus M form a ring.

Example2: The set $Z_9 = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$ under the operation addition and multiplication modulo 9 forms a ring.

Types of Rings:

1. Commutative Rings: A ring $(R, +, \cdot)$ is called a commutative ring if it holds the commutative law under the operation of multiplication i.e., $a \cdot b = b \cdot a$, for every $a, b \in R$

Example1: Consider a set E of all even integers under the operation of addition and multiplication. The set E forms a commutative ring.

2. Ring with Unity: A ring $(R, +, \cdot)$ is called a ring with unity, if it has a multiplicative identity i.e,

Example: Consider a set M of all 2×2 matrices over integers under matrix multiplication and matrix addition. The set M forms a ring with unity $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

3. Ring with Zero Divisions: If $a \cdot b = 0$, where a and b are any two non-zero elements of R in the ring $(R, +)$ then a and b are called divisions of zero and the ring $(R, +)$ is called ring with zero division.

4. Rings without Zero Division: An algebraic system $(R, +)$ where R is a set with two arbitrary binary operation $+$ and is called a ring without divisors of zero if for every $a, b \in R$, we have $a \cdot b \neq 0 \Rightarrow a \neq 0$ and $b \neq 0$

SubRings:

A subset A of a ring $(R, +)$ is called a subring of R , if it satisfies following conditions:

$(A, +)$ is a subgroup of the group $(R, +)$

A is closed under the multiplication operation i.e., $a \cdot b \in A$, for every $a, b \in A$.

Example: The ring $(\mathbb{I}, +)$ of integers is a subring of ring $(\mathbb{R}, +)$ of real numbers.

Note:

1. If R is any ring then $\{0\}$ and R are subrings of R .
2. Sum of two subrings may not be a subring.
3. Intersection of subring is a subring.

SemiGroup

Let us consider, an algebraic system $(A, *)$, where $*$ is a binary operation on A . Then, the system $(A, *)$ is said to be semi-group if it satisfies the following properties:

1. The operation $*$ is a closed operation on set A .
2. The operation $*$ is an associative operation.

Example: Consider an algebraic system $(A, *)$, where $A = \{1, 3, 5, 7, 9, \dots\}$, the set of positive odd integers and $*$ is a binary operation means multiplication. Determine whether $(A, *)$ is a semi-group.

Solution: Closure Property: The operation $*$ is a closed operation because multiplication of two +ve odd integers is a +ve odd number.

Associative Property: The operation $*$ is an associative operation on set A . Since every $a, b, c \in A$, we have

$$(a * b) * c = a * (b * c)$$

Hence, the algebraic system $(A, *)$, is a semigroup.

Subsemigroup:

Consider a semigroup $(A, *)$ and let $B \subseteq A$. Then the system $(B, *)$ is called a subsemigroup if the set B is closed under the operation $*$.

Example: Consider a semigroup $(N, +)$, where N is the set of all natural numbers and $+$ is an addition operation. The algebraic system $(E, +)$ is a subsemigroup of $(N, +)$, where E is a set of +ve even integers.

Free Semigroup:

Consider a non empty set $A = \{a_1, a_2, \dots, a_n\}$.

Now, A^* is the set of all finite sequences of elements of A , i.e., A^* consist of all words that can be formed from the alphabet of A .

If α, β , and γ are any elements of A^* , then $\alpha(\beta \cdot \gamma) = (\alpha \cdot \beta) \cdot \gamma$.

Here $\circ$ is a concatenation operation, which is an associative operation as shown above.

Thus $(A^*, \circ)$ is a semigroup. This semigroup $(A^*, \circ)$ is called the free semigroup generated by set A .

Product of Semigroup:

Theorem: If $(S_1, *)$ and $(S_2, *)$ are semigroups, then $(S_1 \times S_2, *)$ is a semigroup, where $*$ defined by $(s_1', s_2') * (s_1'', s_2'') = (s_1' * s_1'', s_2' * s_2'')$.

Proof: The semigroup $S_1 \times S_2$ is closed under the operation $*$.

Associativity of $*$. Let $a, b, c \in S_1 \times S_2$

$$\begin{aligned}\text{So, } a * (b * c) &= (a_1, a_2) * ((b_1, b_2) * (c_1, c_2)) \\ &= (a_1, a_2) * (b_1 * c_1, b_2 * c_2) \\ &= (a_1 * (b_1 * c_1), a_2 * (b_2 * c_2)) \\ &= ((a_1 * b_1) * c_1, (a_2 * b_2) * c_2) \\ &= (a_1 * b_1, a_2 * b_2) * (c_1, c_2) \\ &= ((a_1, a_2) * (b_1, b_2)) * (c_1, c_2) \\ &= (a * b) * c.\end{aligned}$$

Since $*$ is closed and associative. Hence, $S_1 \times S_2$ is a semigroup.

Monoid:

Let us consider an algebraic system (A, o) , where o is a binary operation on A . Then the system (A, o) is said to be a monoid if it satisfies the following properties:

1. The operation o is a closed operation on set A .
2. The operation o is an associative operation.
3. There exists an identity element, i.e., the operation o .

Example: Consider an algebraic system $(N, +)$, where the set $N = \{0, 1, 2, 3, 4, \dots\}$. The set of natural numbers and $+$ is an addition operation. Determine whether $(N, +)$ is a monoid.

Solution: (a) Closure Property: The operation $+$ is closed since the sum of two natural numbers.

(b) Associative Property: The operation $+$ is an associative property since we have $(a+b)+c=a+(b+c) \forall a, b, c \in N$.

(c)Identity: There exists an identity element in set N the operation $+$. The element 0 is an identity element, i.e., the operation $+$. Since the operation $+$ is a closed, associative and there exists an identity. Hence, the algebraic system $(N, +)$ is a monoid.

SubMonoid:

Let us consider a monoid (M, o) , also let $S \subseteq M$. Then (S, o) is called a submonoid of (M, o) , if and only if it satisfies the following properties:

1. S is closed under the operation o .
2. There exists an identity element $e \in T$.

Example: Let us consider, a monoid $(M, *)$, where $*$ is a binary operation and M is a set of all integers. Then $(M_1, *)$ is a submonoid of $(M, *)$ where M_1 is defined as $M_1 = \{a^i \mid i \text{ is from } 0 \text{ to } n, a \text{ a positive integer, and } a \in M\}$.

Homomorphism

A mapping f from a group G into G' is called a homomorphism if, for every $a, b \in G$, $f(ab) = f(a)f(b)$

If f is one to one and onto, then f is called an isomorphism; and G and G' are said to be isomorphic, $G \sim G'$.

Theorem: Suppose $f: G \rightarrow G'$ is a homomorphism with kernel K . Then K is a normal subgroup of G , and the quotient group G/K is isomorphic to $f(G)$.

Example: Let G be the group of real numbers under addition, and let G' be the group of positive real numbers under multiplication. The mapping $f: G \rightarrow G'$ defined by $f(a) = 2^a$ is a homomorphism

$$f(a + b) = 2^{a+b} = 2^a 2^b = f(a)f(b)$$

f is one to one and onto; hence G and G' are isomorphic.

Rings, Integral Domains, and Fields

Let R be a non-empty set with two binary operations, an operation of addition (denoted by $+$) and an operation of multiplication (denoted by juxtaposition). Then R is called a ring if the following axioms are satisfied:

[R1] For any $a, b, c \in R$, we have $(a + b) + c = a + (b + c)$.

[R2] There exists an element $0 \in R$, called the zero element, such that for every $a \in R$, $a + 0 = 0 + a = a$.

[R3] For each $a \in R$ there exists an element $-a \in R$.

[R4] For any $a, b \in R$.

[R5] For any $a, b, c \in R$.

[R6] For any $a, b, c \in R$. $a + (b + c) = a + b + c$, $(b + c) a = b a + c a$.

Special Kinds of Rings: Integral Domain and fields

Definition: A commutative ring R is an integral domain if R has no zero divisors, that is, if $a b = 0$ implies $a = 0$ or $b = 0$.

Definition: A commutative ring R with an identity element 1 (not equal to 0) is a field if every non-zero $a \in R$ is a unit, that is, has a multiplicative inverse. A field is necessarily an integral domain; for if

$a b = 0$ and $a \neq 0$.

Example: The rational numbers Q and the real numbers R each form a field with respect to the usual operations of addition and multiplication.

Ideals

A subset J of a ring R is called an ideal in R if the following three properties hold:

(i) $0 \in J$.

(ii) For any $a, b \in J$, we have $a - b \in J$.

(iii) For any $r \in R$ and $a \in J$, we have $r a, a r \in J$.

Theorem: Let J be an ideal in a ring R . Then the cosets $\{a + J \mid a \in R\}$ form a ring under the cosets operations

$$(a + J) + (b + J) = a + b + J \text{ and } (a + J)(b + J) = ab + J$$

This ring is denoted by R/J and is called the quotient ring.

Example: Let R be any ring. Then $\{0\}$ and R are ideals. In particular, if R is a field, then $\{0\}$ and R are the only ideals.

Ring Homeomorphisms

A mapping f from a ring R into a ring R is called a ring homomorphism or, simply, homomorphism if, for every $a, b \in R$,

$f(a + b) = f(a) + f(b)$, $f(ab) = f(a)f(b)$ In addition, if f is one-to-one and onto, then f is called an isomorphism; and R and R are said to be isomorphic, written $R \cong R$

Theorem: Let $f: R \rightarrow R$ be a ring homomorphism with kernel K .

Then K is an ideal in R , and the quotient ring R/K is isomorphic to $f(R)$.

Divisibility in Integral Domains

An element $u \in D$ is called a unit if u divides 1, if u has a multiplicative inverse. An element $b \in D$ is called an associate of

$a \in D$ if $b = ua$ for some unit $u \in D$.

An integral domain D is called a unique factorization domain (UFD), if every non-unit $a \in D$.

Example

The ring Z of integers is the classical example of a unique factorization domain. The units of Z are 1 and -1 .

The only associates of $n \in Z$ are n and $-n$. The irreducible elements of Z are the prime numbers.

POLYNOMIALS OVER A FIELD

This section investigates polynomials whose coefficients come from some integral domain or field K .

Theorem: Let K be an integral domain. Then $K[t]$ under the operations of addition and multiplication of polynomials is a commutative ring with an identity element 1.

Proposition: Let K be an integral domain and let f and g be polynomials over K .

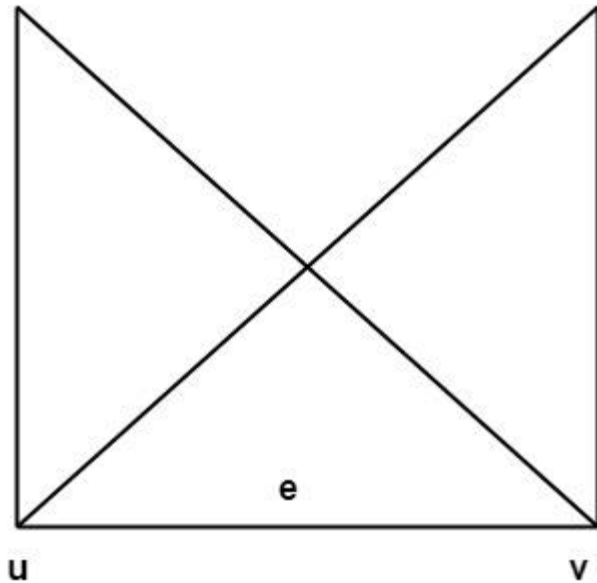
- (i) $K[t]$ is an integral domain.
- (ii) The units of $K[t]$ are the units in K .
- (iii) If g divides f , then $\deg(g) \leq \deg(f)$ or $f \equiv 0$.
- (iv) If g divides f and f divides g , then $f(t) = k g(t)$ where k is a unit in K .
- (v) If d and d are monic polynomials such that d divides d and d divides d , then $d = d$

Graph Theory

Graph:

Graph G consists of two things:

1. A set $V=V(G)$ whose elements are called vertices, points or nodes of G .
2. A set $E = E(G)$ of an unordered pair of distinct vertices called edges of G .
3. We denote such a graph by $G(V, E)$ vertices u and v are said to be adjacent if there is an edge $e = \{u, v\}$.

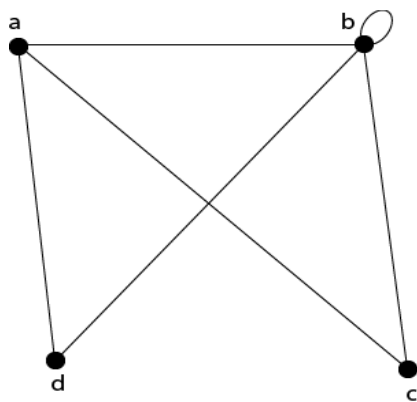


4. In such a case u and v are called the endpoints of $e=\{u, v\}$ and e are said to connect u and v .

Degree of a Vertex:

The degree of a vertex is the number of edges incident on a vertex v . The self-loop is counted twice. The degree of a vertex is denoted by $d(v)$.

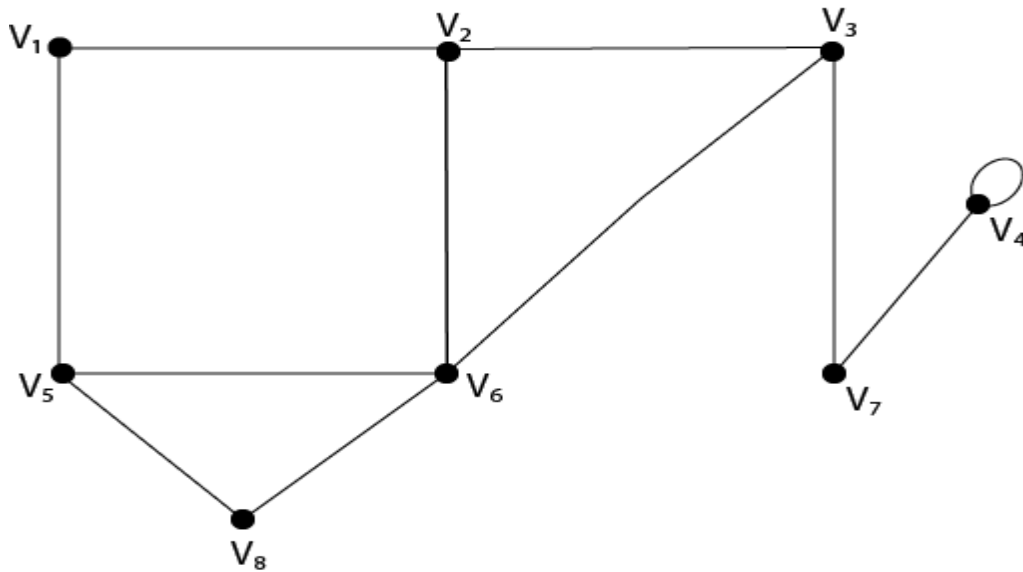
Example1: Consider the graph G shown in fig. Determine the degree of each vertex.



Solution: The degree of each vertex is as follows:

$$d(a)=3; \quad d(b)=5; \quad d(c) = 2; \quad d(d)=2.$$

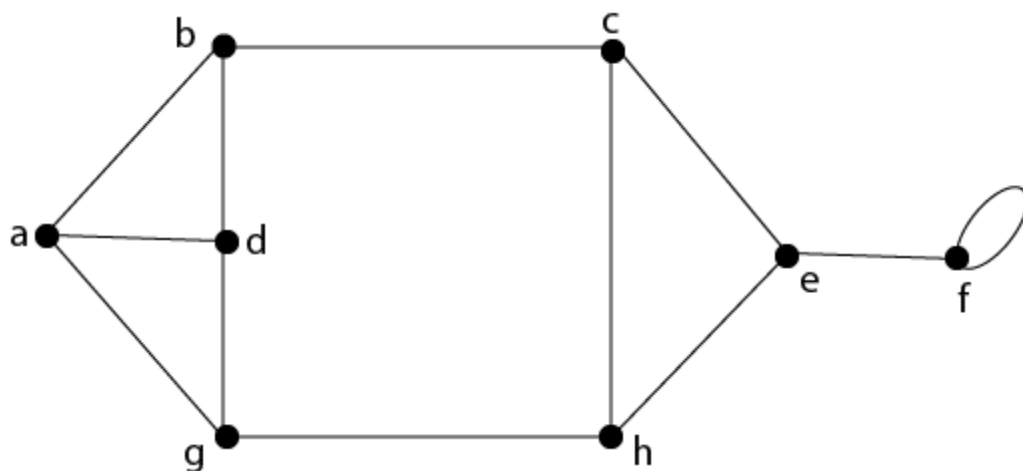
Example2: Verify that the sum of the degrees of all the vertices is even for the graph shown in fig:



Solution: The sum of the degrees of all the vertices is:

$$\begin{aligned} &= d(v_1) + d(v_2) + d(v_3) + d(v_4) + d(v_5) + d(v_6) + d(v_7) + d(v_8) \\ &= 2 + 3 + 3 + 3 + 3 + 4 + 2 + 2 = 22, \text{ which is even.} \end{aligned}$$

Example3: Verify that there are even numbers of vertices of odd degrees in the graph shown in fig:



Solution: The number of vertices of degree odd is 8, and each has a degree three in the above graph. Hence, we have even number of vertices of odd degrees.

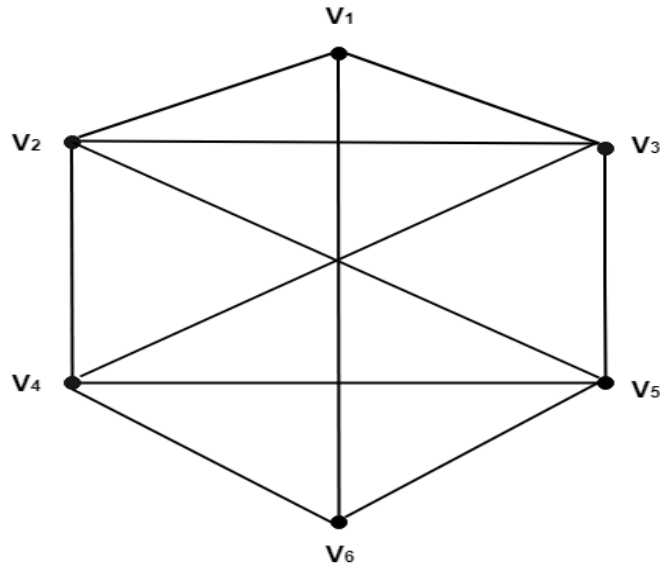
Path:

A path of length n is a sequence of $n+1$ vertices of a graph in which each pair of vertices is an edge of the graph.

1. **A Simple Path:** The path is called simple one if no edge is repeated in the path, i.e., all the vertices are distinct except that first vertex equal to the last vertex.
2. **An Elementary Path:** The path is called elementary one if no vertex is repeated in the path, i.e., all the vertices are distinct.
3. **Circuit or Closed Path:** The circuit or closed path is a path in which starts and ends at the same vertex, i.e., $v_0=v_n$.
4. **Simple Circuit Path:** The simple circuit is a simple path which is a circuit.

Example: Consider the graph shown in fig: Give an example of the following:

1. A simple path from V_1 to V_6 .
2. An elementary path from V_1 to V_6 .
3. A simple path which is not elementary from V_1 to V_6 .
4. A path which is not simple and starting from V_2 .
5. A simple circuit starting from V_1
6. A circuit which is not simple and starting from V_2 .



Solution:

1. A simple path from V_1 to V_6 .
 $V_1, V_2, V_3, V_4, V_5, V_6$.
2. An elementary path from V_1 to V_6 .
 $V_1, V_2, V_3, V_5, V_4, V_6$.
3. A simple path which is not elementary from V_1 to V_6 .
 $V_1, V_2, V_3, V_5, V_2, V_4, V_6$.
4. A path which is not simple and starting from V_2 .
 $V_2, V_3, V_4, V_5, V_3, V_4, V_6$.
5. A simple circuit starting from V_1 .
 $V_1, V_2, V_4, V_6, V_5, V_3, V_1$
6. A circuit which is not simple and starting from V_2 .
 $V_2, V_3, V_1, V_2, V_5, V_4, V_2$.

Pendant Vertex: A vertex with degree one is called a Pendant Vertex.

Pendant Edge: The only edge which is an incident with a pendant vertex is called the Pendant Edge.

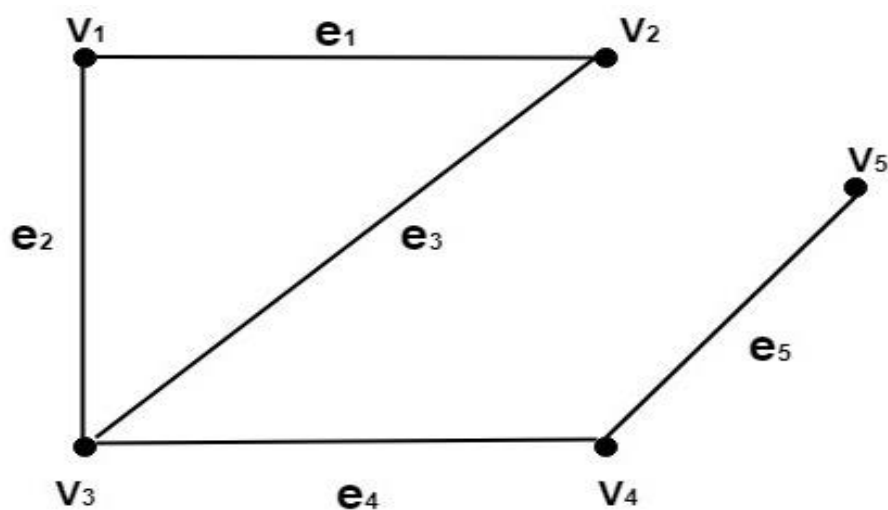
Odd Vertex: A vertex having degree odd is called an odd vertex.

Even Vertex: A vertex having a degree even is called an even vertex.

Incident Edge: An edge is called incident with the vertices it connects.

Adjacent Vertices: Two vertices are called adjacent if an edge links them. If there is an edge (u, v) , then we can say vertex u is adjacent to vertex v , and vertex v is adjacent to vertex u .

Example: Consider the graph as shown in fig:



Determine the following:

1. Pendant Vertices
2. Pendant Edges
3. Odd vertices
4. Even Vertices
5. Incident Edges
6. Adjacent Vertices

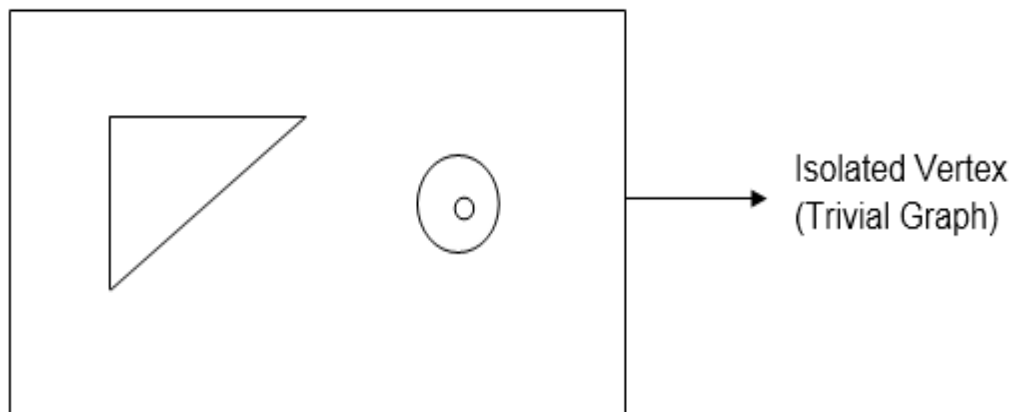
Solution:

1. The vertex V_5 is the pendant vertex.
2. The edge (V_4, V_5) or e_5 is the pendant edge.
3. The vertices V_3 and V_5 are odd vertices.
4. The vertices V_1, V_2 , and V_4 are even vertices.
5. The edge e_1 is an incident on V_1 , and V_2 .
The edge e_2 is an incident on V_1 and V_3 .
The edge e_3 is an incident on V_2 and V_3 .
The edge e_4 is an incident on V_3 and V_4 .
The edge e_5 is an incident on V_4 and V_5 .
6. The vertex V_1 is adjacent to V_2 and V_3 .
The vertex V_2 is adjacent to V_1 and V_3 .
The vertex V_3 is adjacent to V_1 and V_4 .
The vertex V_4 is adjacent to V_3 and V_5 .
The vertex V_5 is adjacent to V_4 .

Self-Loop: A self-loop is an edge e if it has the same endpoint.

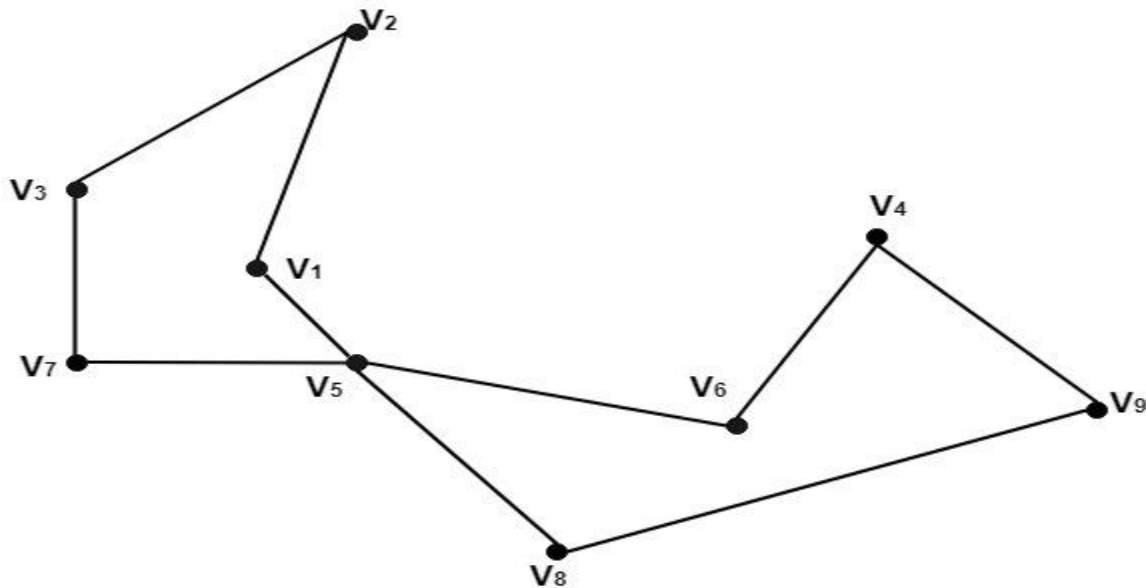
The graph shown in fig contains the self-loop at vertex b , i.e., $e=(b, b)$.

Isolated Vertex: A vertex with degree 0 is called Isolated Vertex.



Cut Set: Consider a graph $G=(V, E)$. A cut set for G is the smallest set of edges such that the removal of the set, disconnected the graph whereas the removal of any proper subset of this set left a connected subgraph.

Example: Consider the graph shown in fig. Determine the cut set for this group.



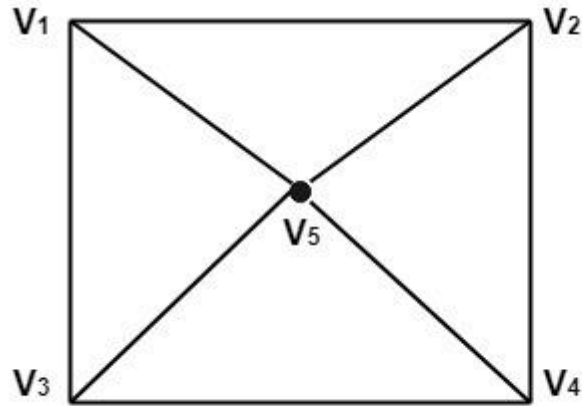
Solution: For this graph, the edge set $\{(V_1, V_5), (V_7, V_5)\}$ is a cut set. After the removal of the set, we have left with a disconnected subgraph. While after the removal of any of its proper subset, we have left with a connected subgraph.

Cut Points or Cut Vertices: Consider a graph $G=(V, E)$. A cut point for a graph G is a vertex v such that $G-v$ has more connected components than G or disconnected.

The subgraph $G-v$ is obtained by deleting the vertex v from graph G and also deleting the entire edges incident on v .

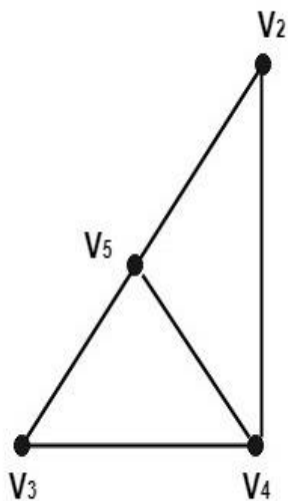
Example: Consider the graph shown in fig. Determine the subgraphs

- (i) $G-v_1$ (ii) $G-v_3$ (iii) $G-v_5$

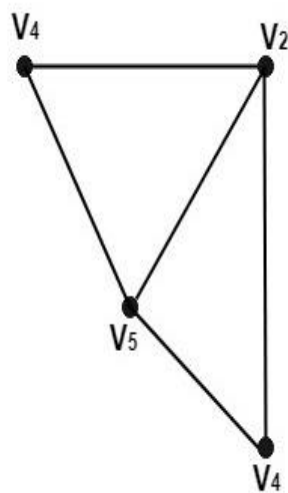


Solution:

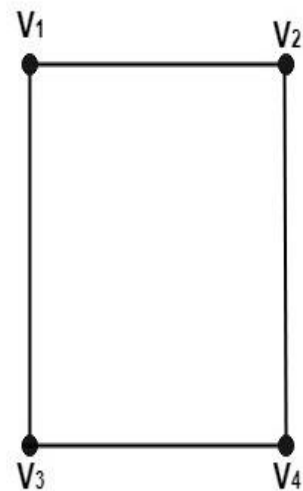
1. The subgraph $G-v_1$ is shown in fig
2. The subgraph $G-v_3$ is shown in fig
3. The subgraph $G-v_5$ is shown in fig



$G-v_1$



$G-v_3$

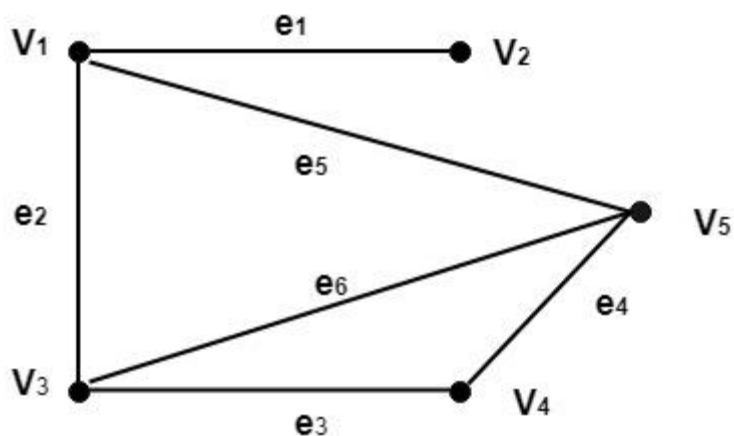


$G-v_5$

Bridge (Cut Edges): Consider a graph $G=(V, E)$. A bridge for a graph G , is an edge e such that $G-e$ has more connected components than G or disconnected.

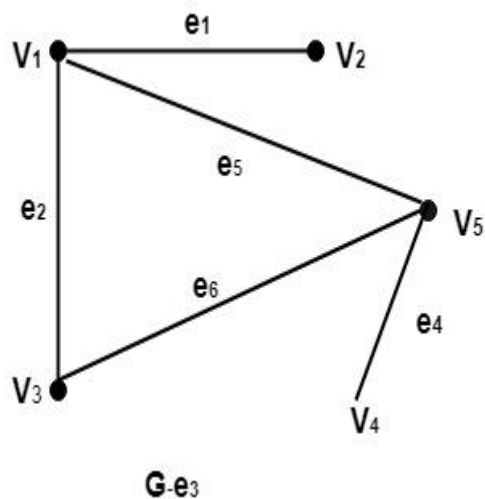
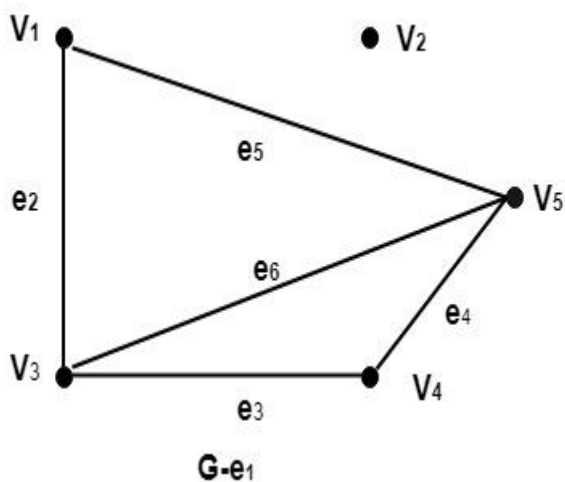
Example: Consider the graph shown in fig. Determine the subgraphs

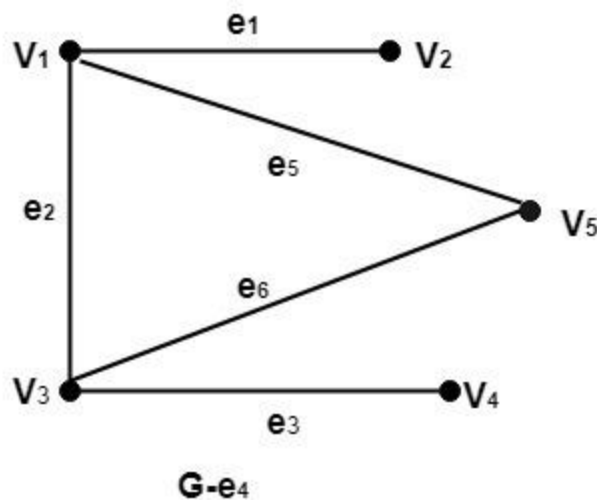
(i) $G-e_1$ (ii) $G-e_3$ (iii) $G-e_4$



Solution:

1. The subgraph $G-e_1$ is shown in fig
2. The subgraph $G-e_3$ is shown in fig
3. The subgraph $G-e_4$ is shown in fig





Types of Graphs:

1. Null Graph: A null graph is defined as a graph which consists only the isolated vertices.

Example: The graph shown in fig is a null graph, and the vertices are isolated vertices.

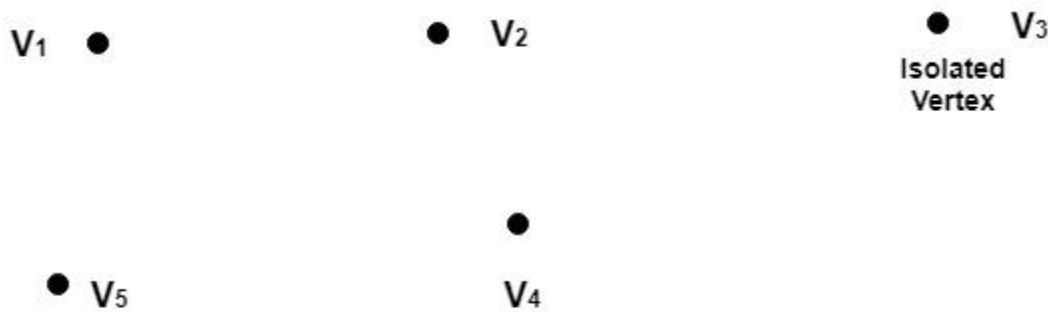


Fig:Null Graph

2. Undirected Graphs: An Undirected graph G consists of a set of vertices, V and a set of edge E . The edge set contains the unordered pair of vertices. If $(u, v) \in E$ then we say u and v are connected by an edge where u and v are vertices in the set V .

Example: Let $V = \{1, 2, 3, 4\}$ and $E = \{(1, 2), (1, 4), (3, 4), (2, 3)\}$. Draw the graph.

Solution: The graph can be drawn in several ways.

Two of which are as follows:

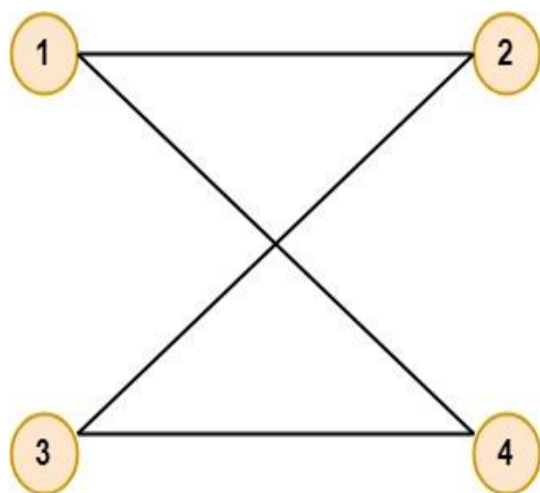


Fig:1

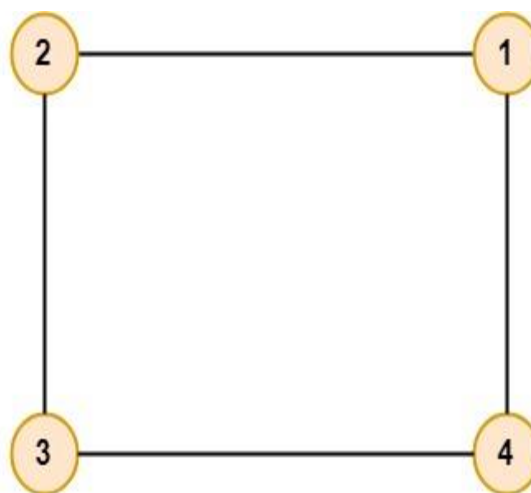


Fig:2

3. Multigraph: If in a graph multiple edges between the same set of vertices are allowed, it is known as Multigraph. In other words, it is a graph having at least one loop or multiple edges.

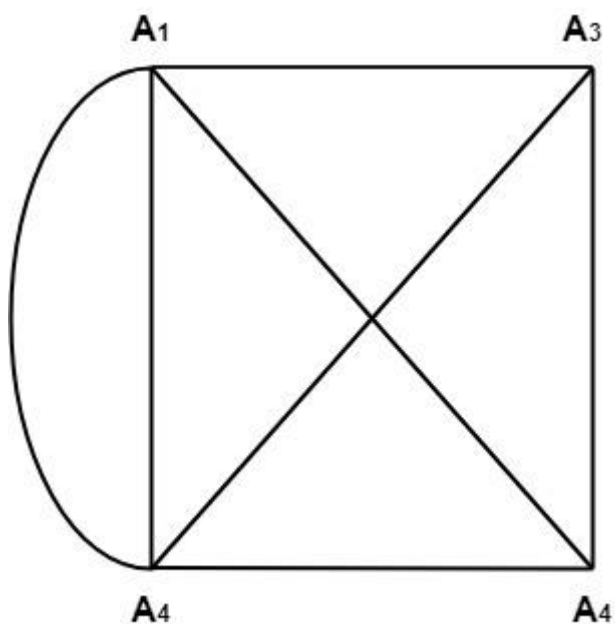
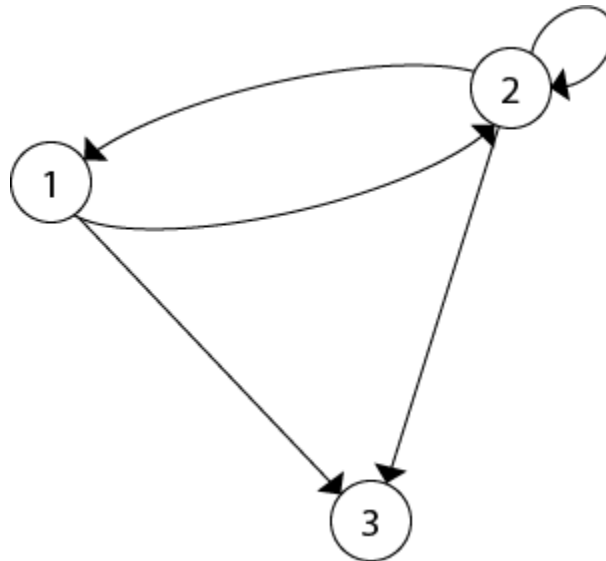


Fig:Multigraph

4. Directed Graphs: A directed graph or digraph G is defined as an unordered pair (V, E) , where V is the set of points called vertices and E is the set of edges. Each edge in the graph G is assigned a direction and is identified with an ordered pair (u, v) , where u is the initial vertex, and v is the end vertex.

Example: Consider the graph $G = (V, E)$ as shown in fig. Determine the vertex set and edge set of graph G .



Solution: The vertex and edge set of graph $G = (V, E)$ is as follow

$$G = \{\{1, 2, 3\}, \{(1, 2), (2, 1), (2, 2), (2, 3), (1, 3)\}\}.$$

5. Undirected Complete Graph: An undirected complete graph $G = (V, E)$ of n vertices is a graph in which each vertex is connected to every other vertex i.e., and edge exist between every pair of distinct vertices. It is denoted by K_n . A

complete graph with n vertices will have $\frac{n(n-1)}{2}$ edges.

Example: Draw Undirected Complete Graphs k_4 and k_6 .

Solution: The undirected complete graph of k_4 is shown in fig1 and that of k_6 is shown in fig2.

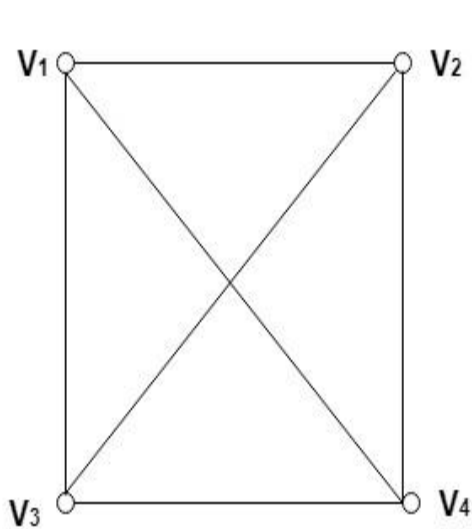


Fig: K_4

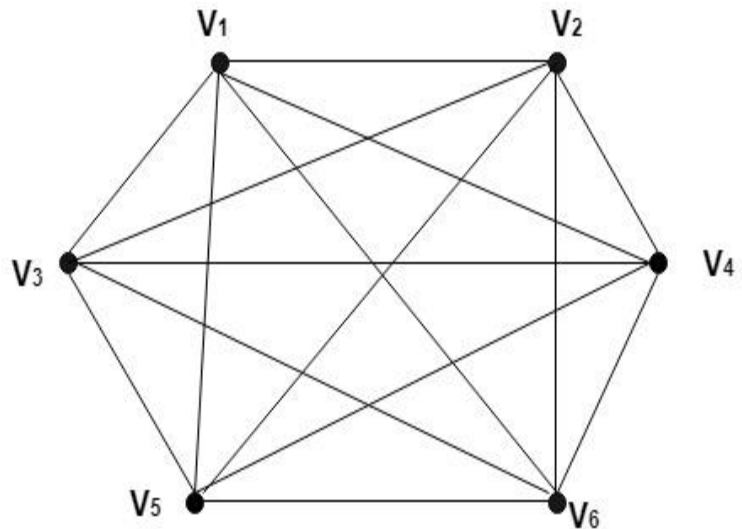


Fig: K_6

6. Connected and Disconnected Graph:

Connected Graph: A graph is called connected if there is a path from any vertex u to v or vice-versa.

Disconnected Graph: A graph is called disconnected if there is no path between any two of its vertices.

Example: Consider the graph shown in fig. Determine whether the graphs are

(a) Disconnected Graph

(b) Connected Graph.

Also, write their connected components.

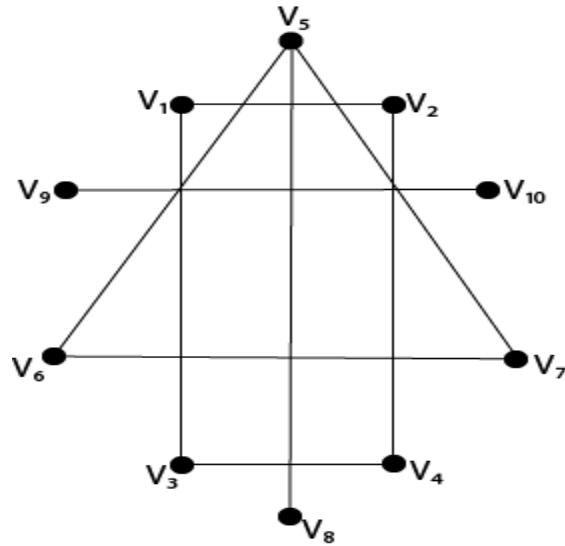


Fig:Disconnected Graph

Solution:

(i) The graph is shown in fig is a Disconnected Graph, and its connected components are

$\{V_1, V_2, V_3, V_4\}, \{V_5, V_6, V_7, V_8\}$ and $\{V_9, V_{10}\}$.

(ii) The graph shown in fig is a Disconnected Graph and its connected components are

$\{V_1, V_2\}, \{V_3, V_4\}, \{V_5, V_6\}, \{V_7, V_8\}, \{V_9, V_{10}\}$ and $\{V_{11}, V_{12}\}$.

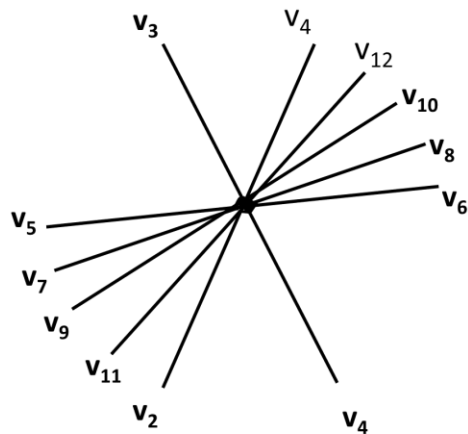


Fig: Disconnected graph

(iii) The graph shown in fig is a connected graph.

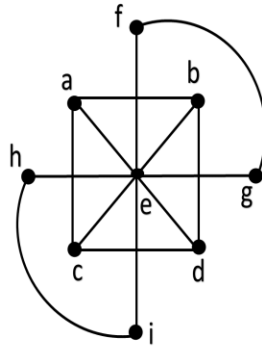
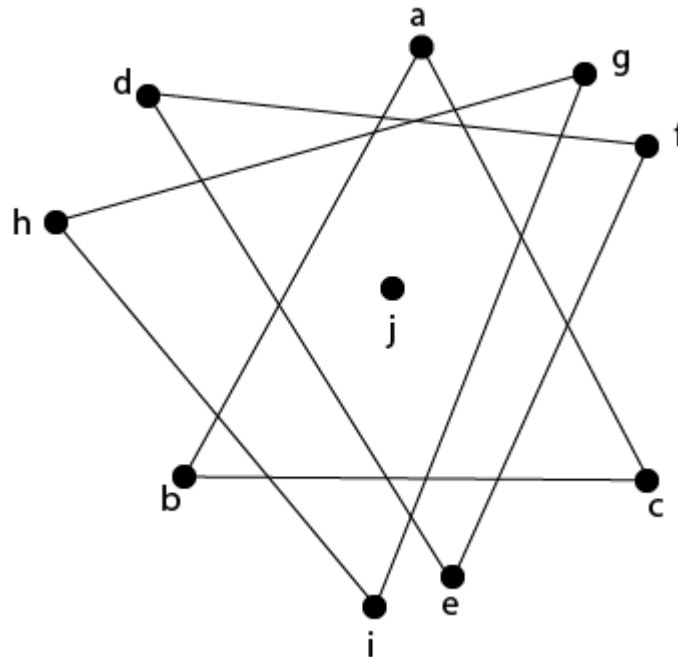


Fig: Connected graph

7. Connected Component: A subgraph of graph G is called the connected component of G , if it is not contained in any bigger subgraph of G , which is connected. It is defined by listing its vertices.

Example: Consider the graph shown in fig. Determine its connected components.



Solution: The connected components of this graph is $\{a, b, c\}$, $\{d, e, f\}$, $\{g, h, i\}$ and $\{j\}$.

8. Directed Complete Graph: A directed complete graph $G = (V, E)$ on n vertices is a graph in which each vertex is connected to every other vertex by an arrow. It is denoted by K_n .

Example: Draw directed complete graphs K_3 and K_5 .

Solution: Place the number of vertices at the appropriate place and then draw an arrow from each vertex to every other vertex as shown in fig:

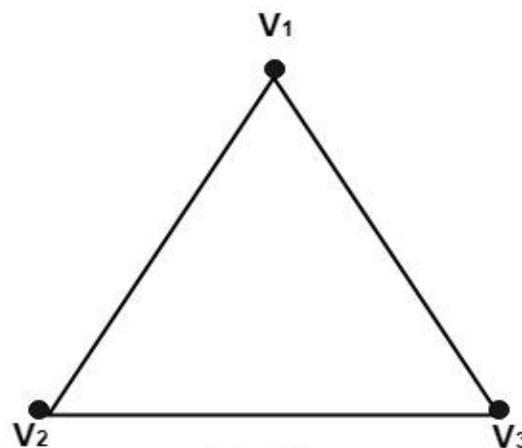


Fig: K_3

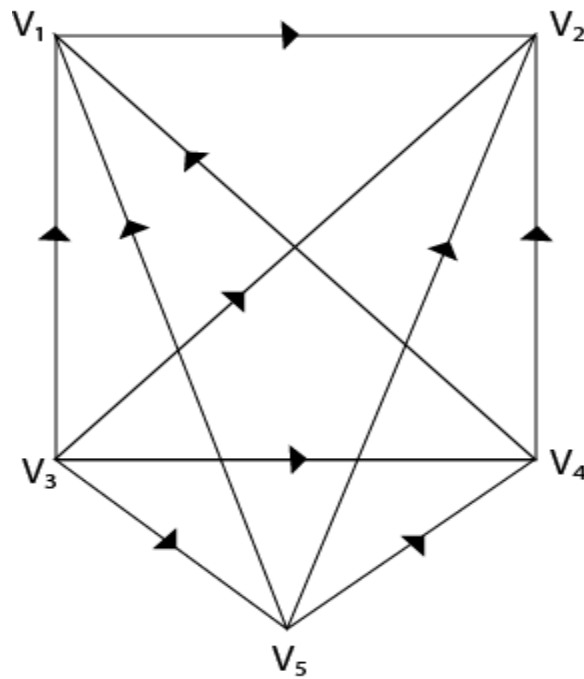
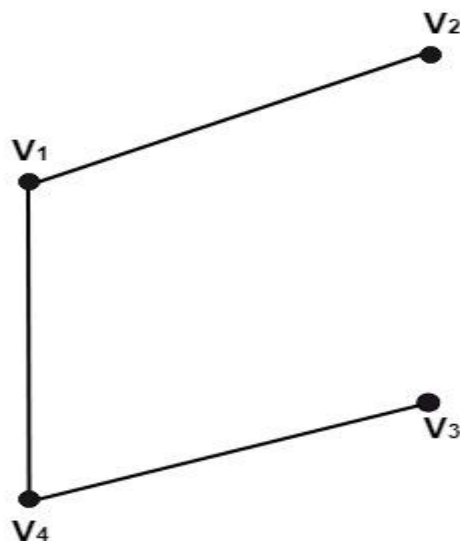


Fig: K_5

9. Complementary Graph: The complement of a graph G is defined to be a graph which has the same number of vertices as in graph G and has two vertices connected if and only if they are not related in the graph.

Example: Consider the graph G shown in fig. Find the complement of this graph.



Solution: The complement of the above graph is shown in Fig:

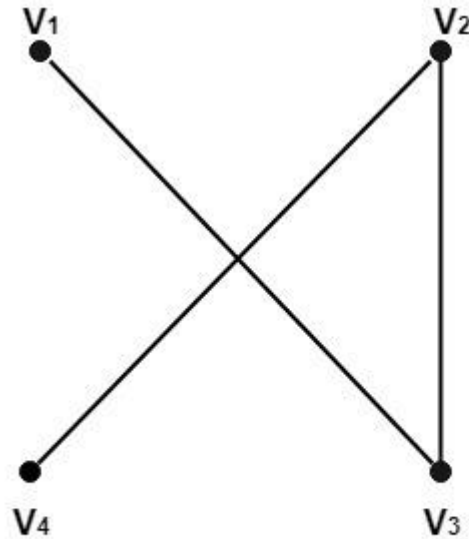


Fig: Complement of Graph G

10. Labeled Graphs: A graph $G=(V, E)$ is called a labeled graph if its edges are labeled with some name or data. So, we can write these labels in place of an ordered pair in its edges set.

Example: The graph shown in fig is labeled graphs.

$$G= \{\{a, b, c, d\}, \{e_1, e_2, e_3, e_4\}\}$$

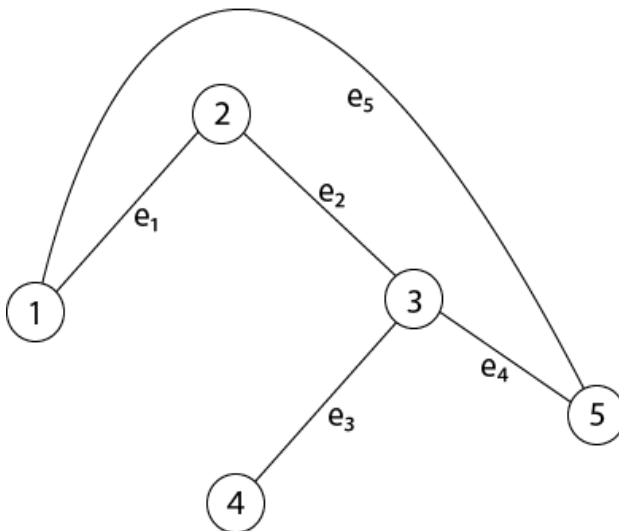


Fig: Undirected Labeled Graph

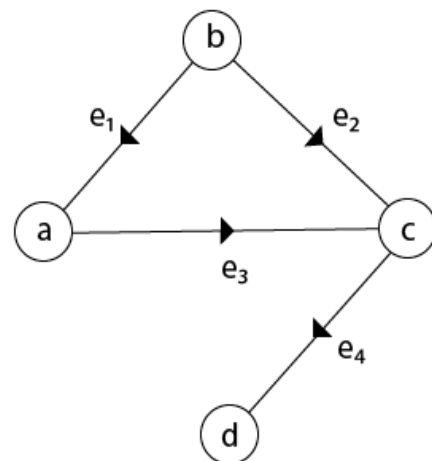


Fig: Directed Labeled Graph

11. Weighted Graphs: A graph $G=(V, E)$ is called a weighted graph if each edge of graph G is assigned a positive number w called the weight of the edge e .

Example: The graph shown in fig is a Weighted Graph.

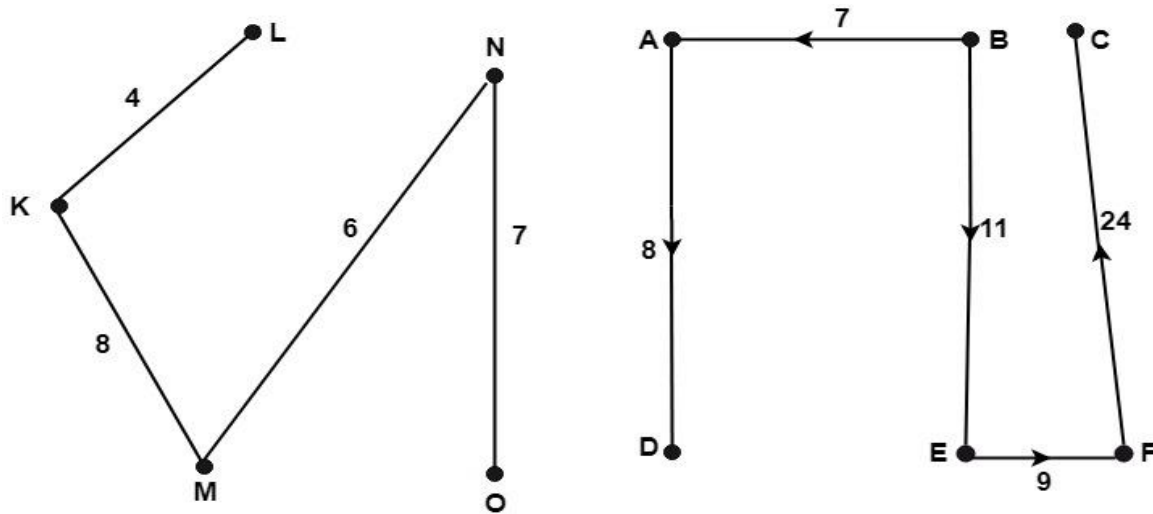


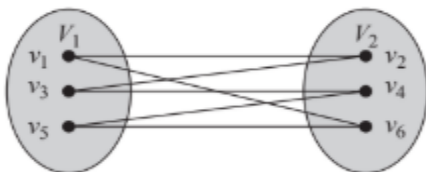
Fig:Weighted Graph

Degree of a Vertex

The degree of a vertex v in a graph G , written $\deg(v)$, is equal to the number of edges in G which contain v , that is, which are incident on v .

Bipartite Graphs

DEFINITION: A simple graph G is called bipartite if its vertex set V can be partitioned into two disjoint sets V_1 and V_2 such that every edge in the graph connects a vertex in V_1 and a vertex in V_2 .



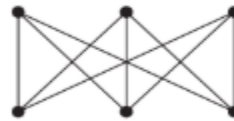
- C_6 is bipartite, as shown in Figure, because its vertex set can be partitioned into the two sets V_1 and V_2 .

- $V_1 = \{v_1, v_3, v_5\}$ and $V_2 = \{v_2, v_4, v_6\}$, and every edge of C_6 connects a vertex in V_1 and a vertex in V_2 .

Complete Bipartite Graphs: The complete bipartite graphs $K_{2,3}$, $K_{3,3}$, and are displayed in Figure



$K_{2,3}$



$K_{3,3}$

New Graphs from Old

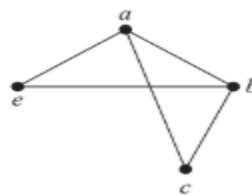
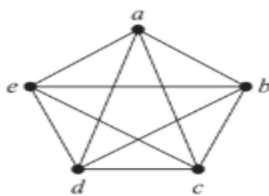
DEFINITION: A sub-graph of a graph $G = (V, E)$ is a graph.

$H = (W, F)$, where $W \subseteq V$ and $F \subseteq E$. A sub-graph H of G is a proper sub-graph of G if $H \neq G$.

DEFINITION: Let $G = (V, E)$ be a simple graph.

The sub-graph induced by a subset W of the vertex set V is the graph (W, F) , where the edge set F contains an edge in E if and only if both endpoints of this edge are in W .

The graph G is a sub-graph of K_5 . If we add the edge connecting c and e to G , $W = \{a, b, c, e\}$.



A Sub-graph of K_5

DEFINITION

The union of two simple graphs $G_1 = (V_1, E_1)$ $G_2 = (V_2, E_2)$ is the simple graph with vertex set $V_1 \cup V_2$ and edge set $E_1 \cup E_2$. The union of G_1 and G_2 is denoted by $G_1 \cup G_2$.

EXAMPLE: Find the union of the graphs G_1 and G_2 .

Solution: The vertex set of the union $G_1 \cup G_2$ is the union of the two vertex sets as: $\{a, b, c, d, e, f\}$.

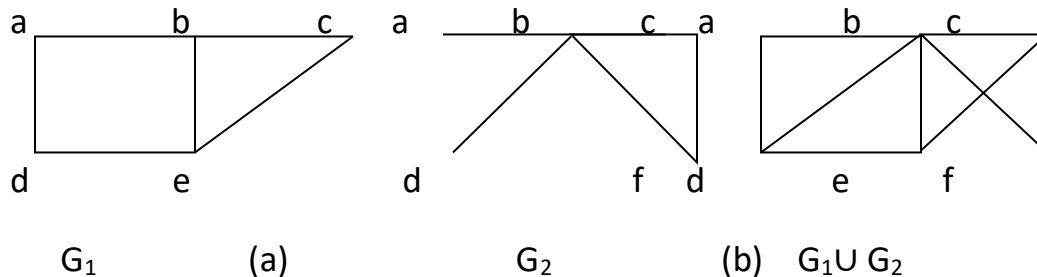


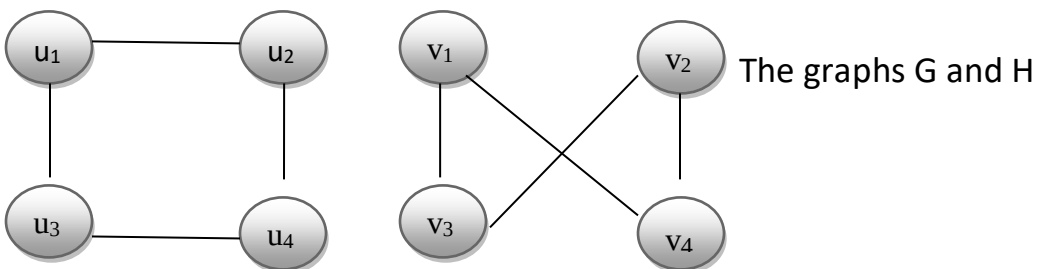
FIGURE (a) The Simple Graphs G_1 and G_2 ; (b) Their Union $G_1 \cup G_2$.

Isomorphism of Graphs

The simple graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are isomorphic.

- If there exists a one-to-one and onto function f from V_1 to V_2 with the property that a, b are adjacent in G_1 .
- If and only if $f(a)$ and $f(b)$ are adjacent in G_2 , for all a, b in V_1 . Such a function f is called an isomorphism.
- Two simple graphs that are not isomorphic are called non-isomorphic.

Example: Show that the graphs $G = (V, E)$ and $H = (W, F)$, displayed in figure, are isomorphic.



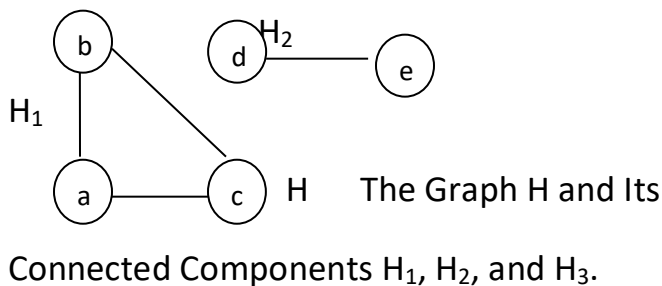
Paths

A path is a sequence of edges that begins at a vertex of a graph and travels from vertex to vertex along edges of the graph.

- Let n be a nonnegative integer and G an undirected graph.
- A path of length n from u to v in G is a sequence of n edges $e_1, \dots, e_n$ of G .
- The path is a circuit if it begins and ends at the same vertex, that is, if $u = v$, and has length greater than zero.

Connectedness in Undirected Graphs

- An undirected graph is called connected if there is a path between every pair of distinct vertices of the graph.
- An undirected graph that is not connected is called disconnected.

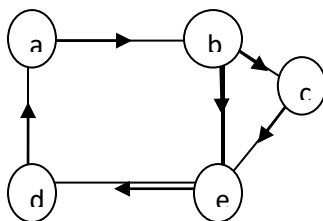


Connectedness in Directed Graphs

A directed graph is strongly connected if there is a path from a to b and from b to a whenever a and b are vertices in the graph.

A directed graph is weakly connected if there is a path between every two vertices in the underlying undirected graph.

FIGURE: The directed graphs G and H .



Paths and Isomorphism

A path in a multi-graph G consists of an alternating sequence of vertices and edges of the form

$v_0, e_1, v_1, e_2, v_2 \dots e_{n-1}, v_{n-1}, e_n, v_n$, where each edge e_i contains the vertices v_{i-1} and v_i (which appear on the sides of e_i in the sequence). The number n of edges is called the length of the path. The path is said to be closed if $v_0 = v_n$. Otherwise, we say the path is from v_0 to v_n or between v_0 and v_n , or connects v_0 to v_n .

Isomorphic Graphs

Graphs $G(V, E)$ and $G(V^*, E^*)$ are said to be isomorphic if there exists a one-to-one correspondence $f: V \rightarrow V^*$ such that $\{u, v\}$ is an edge of G if and only if $\{f(u), f(v)\}$ is an edge of G^* .

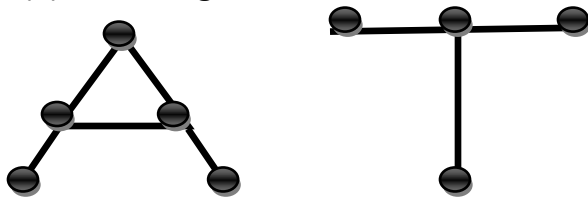
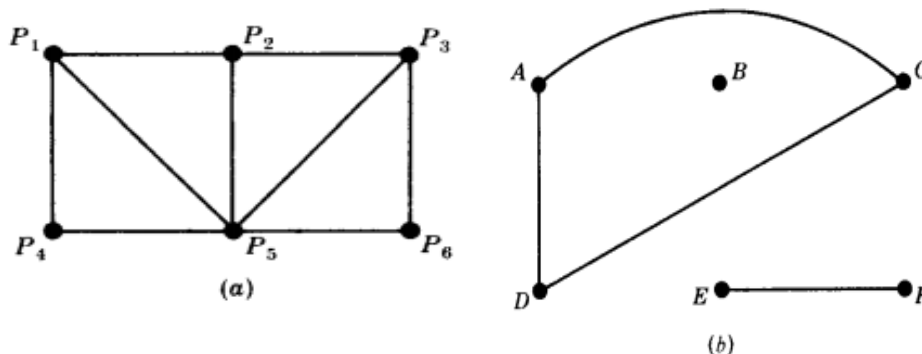


Figure: A and T are isomorphic graphs

Theorem: There is a path from a vertex u to a vertex v if and only if there exists a simple path from u to v .

Connectivity, Connected Components

A graph G is connected if there is a path between any two of its vertices.



The graph in Figure (a) is connected, but the graph in Figure (b) is not connected since, for example, there is no path between vertices D and E .

- The graph G in Fig. 8-8(b) has three connected components, the subgraphs induced by the vertex sets $\{A, C, D\}$, $\{E, F\}$, and $\{B\}$.

- The vertex B in Figure (b) is called an isolated vertex since B does not belong to any edge or, in other words, $\deg(B) = 0$. Therefore, as noted, B itself forms a connected component of the graph.

Euler and Hamilton Paths

DEFINITION 1 An Euler circuit in a graph G is a simple circuit containing every edge of G . An Euler path in G is a simple path containing every edge of G .

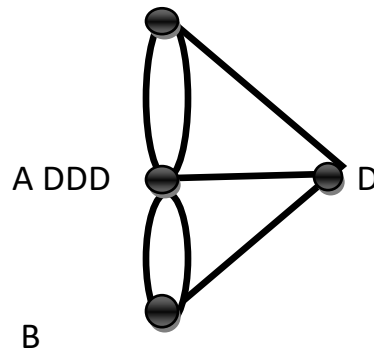
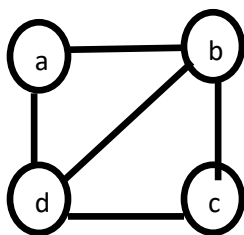


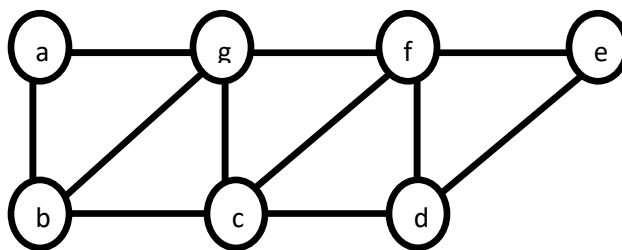
Figure: Multi-graph Model of the Town of Königsberg

THEOREM 1: A connected **multi-graph** with at least two vertices has an Euler circuit if and only if each of its vertices has even degree.

THEOREM 2: A connected **multi-graph** has an **Euler path** but not an Euler circuit if and only if it has exactly two vertices of odd degree.

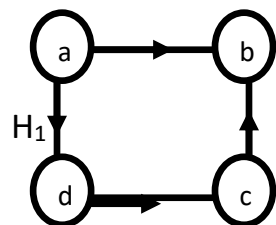


G_1

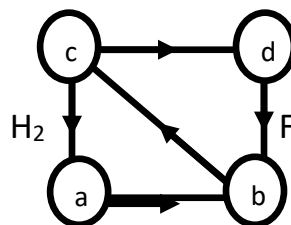


G_2

Figure: Two Undirected Graph



H_1



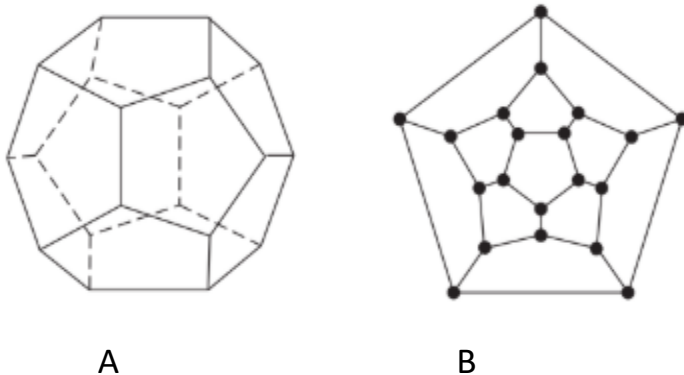
H_2

Figure: Two directed graph

Hamilton Paths and Circuits

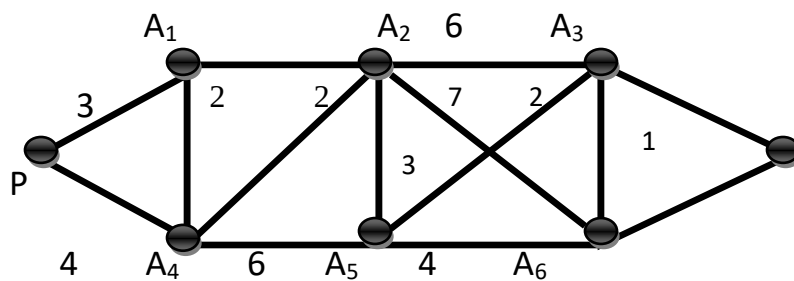
DEFINITION: A simple path in a graph G that passes through every vertex exactly once is called a **Hamilton path**, and a simple circuit in a graph G that passes through every vertex exactly once is called a **Hamilton circuit**.

That is, the simple path $x_0, x_1, \dots, x_{n-1}, x_n$ in the graph $G = (V, E)$ is a **Hamilton path** if $V = \{x_0, x_1, \dots, x_{n-1}, x_n\}$ and $x_i \neq x_j$ for $0 \leq i < j \leq n$, and the simple circuit $x_0, x_1, \dots, x_{n-1}, x_n, x_0$ (with $n > 0$) is a **Hamilton circuit** if $x_0, x_1, \dots, x_{n-1}, x_n$ is a **Hamilton path**.



Labeled and Weighted graphs

A graph G is called a **labeled graph** if its edges and/or vertices are assigned data of one kind or another. In particular, G is called a **weighted graph** if each edge e of G is assigned a non negative number $w(e)$ called the weight or length of e .



The weight (or length) of a path in such a weighted graph G is defined to be the sum of the weights of the edges in the path. One important problem in graph theory is to find a shortest path, that is, a path of minimum weight (length),

between any two given vertices. The length of a shortest path between P and Q in figure is 14; one such path is

$(P, A_1, A_2, A_5, A_3, A_6, Q)$

Complete, Regular, and Bipartite graphs

There are many different types of graphs. This section considers three of them: complete, regular, and bipartite graphs.

Complete Graphs

A graph G is said to be complete if every vertex in G is connected to every other vertex in G . Thus a complete graph G must be connected. The complete graph with n vertices is denoted by K_n . Figure shows the graphs K_1 through K_4 .

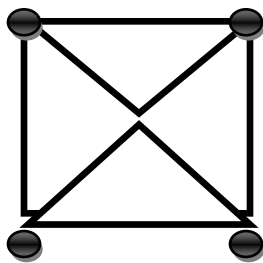
Regular Graphs

A graph G is regular of degree k or k -regular if every vertex has degree k . In other words, a graph is regular if every vertex has the same degree.

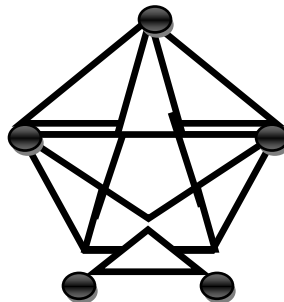
K_1 = Isolated vertex



K_2 = Line Segment



K_3



K_4



0-Regular



1-Regular

Bipartite Graphs

A graph G is said to be bipartite if its vertices V can be partitioned into two subsets M and N such that each edge of G connects a vertex of M to a vertex of N .

By a complete bipartite graph, we mean that each vertex of M is connected to each vertex of N ; this graph is denoted by $K_{m,n}$ where m is the number of vertices in M and n is the number of vertices in N , and, for standardization, we will assume $m \leq n$.

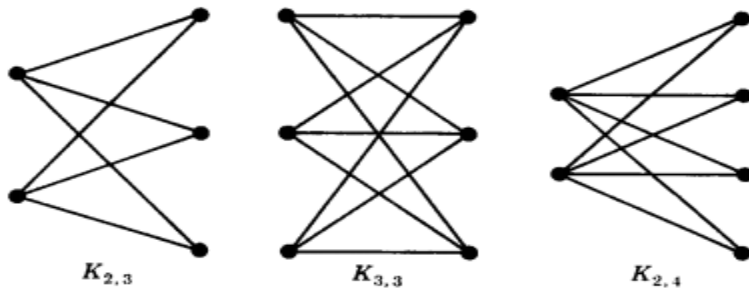


Figure shows the graphs $K_{2,3}$, $K_{3,3}$, and $K_{2,4}$, clearly the graph $K_{m,n}$ has $m n$ edges.

Tree Graphs

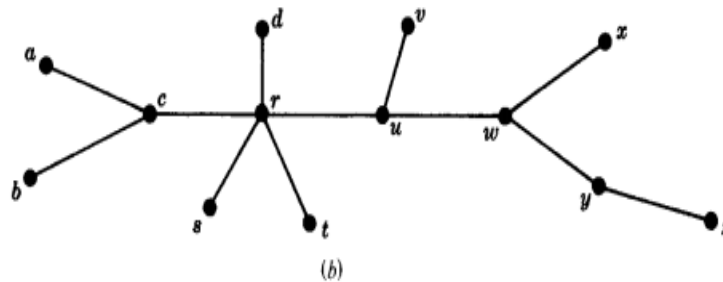
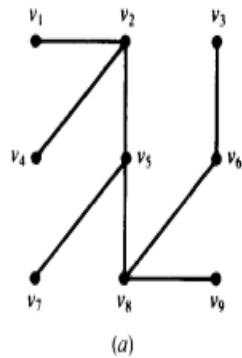
- A graph T is called a tree if T is connected and T has no cycles. Examples of trees are shown in Figure.
- A forest G is a graph with no cycles; hence the connected components of a forest G are trees.
- A graph without cycles is said to be cycle-free. The tree consisting of a single vertex with no edges is called the degenerate tree.

Theorem: Let G be a graph with $n > 1$ vertices. Then the following are equivalent:

- (i) G is a tree.
- (ii) G is a cycle-free and has $n - 1$ edges.
- (iii) G is connected and has $n - 1$ edges.

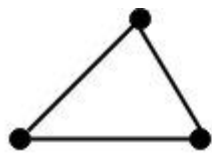
This theorem also tells us that a finite tree T with n vertices must have $n - 1$ edges. For example, the tree in

Fig. 8-17(a) has 9 vertices and 8 edges, and the tree in Figure has 13 vertices and 12 edges.

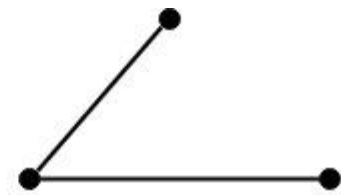


Spanning Tree

A subgraph T of a connected graph G is called spanning tree of G if T is a tree and T include all vertices of G .



Cycle Graph



Subgraph/Spanning Tree

Minimum Spanning Tree:

Suppose G is a connected weight graph i.e., each edge of G is assigned a non-negative number called the weight of edge, then any spanning tree T of G is assigned a total weight obtained by adding the weight of the edge in T .

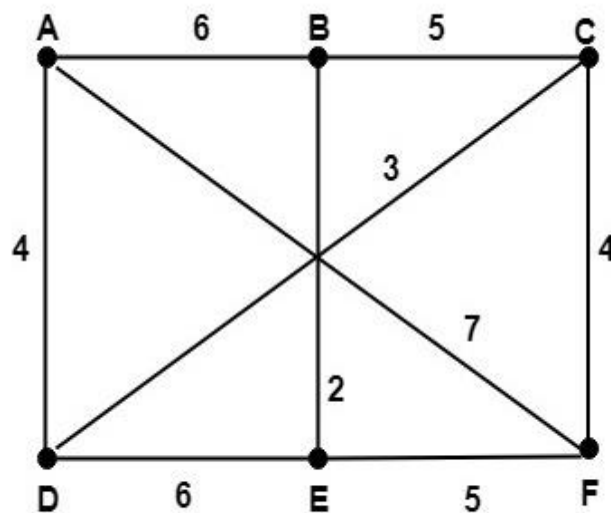
A minimum spanning tree of G is a tree whose total weight is as small as possible.

Kruskal's Algorithm to find a minimum spanning tree: This algorithm finds the minimum spanning tree T of the given connected weighted graph G .

1. Input the given connected weighted graph G with n vertices whose minimum spanning tree T , we want to find.
2. Order all the edges of the graph G according to increasing weights.
3. Initialize T with all vertices but do include an edge.

4. Add each of the graphs G in T which does not form a cycle until $n-1$ edges are added.

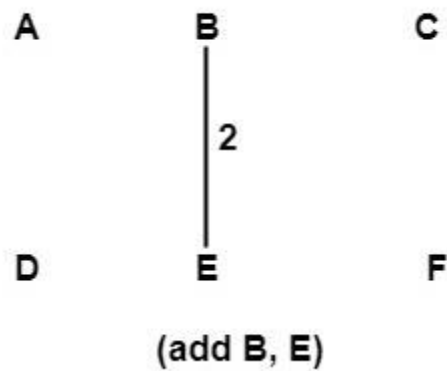
Example1: Determine the minimum spanning tree of the weighted graph shown in fig:



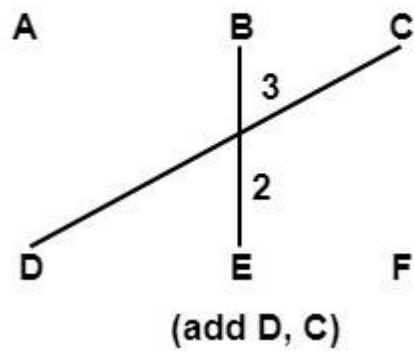
Solution: Using kruskal's algorithm arrange all the edges of the weighted graph in increasing order and initialize spanning tree T with all the six vertices of G . Now start adding the edges of G in T which do not form a cycle and having minimum weights until five edges are not added as there are six vertices.

Edges	Weights	Added or Not
(B, E)	2	Added
(C, D)	3	Added
(A, D)	4	Added
(C, F)	4	Added
(B, C)	5	Added
(E, F)	5	Not added
(A, B)	6	Not added
(D, E)	6	Not added
(A, F)	7	Not added

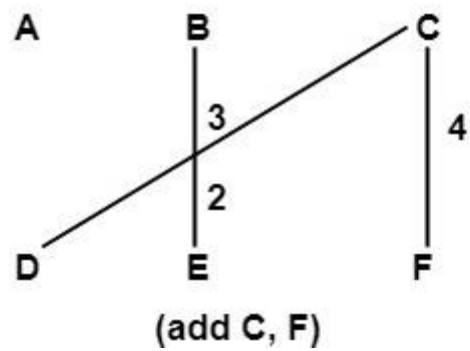
Step1:



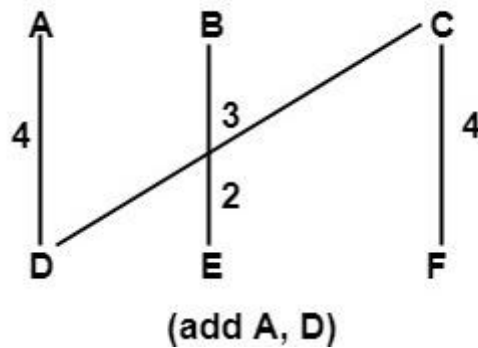
Step2:



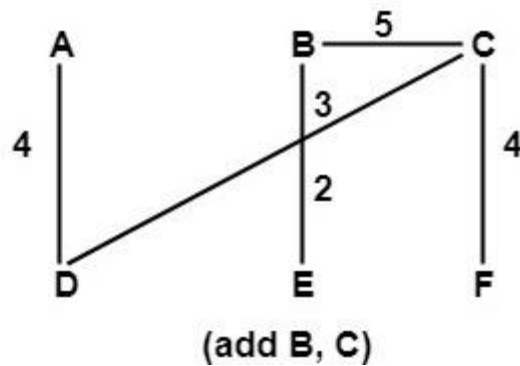
Step3:



Step4:

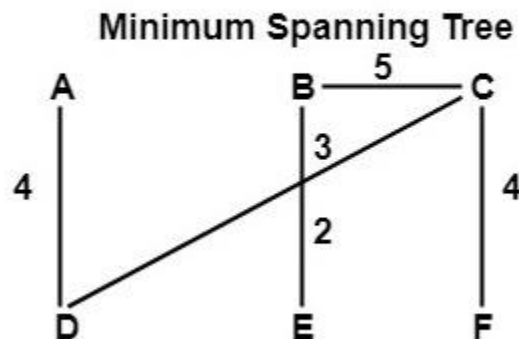


Step5:

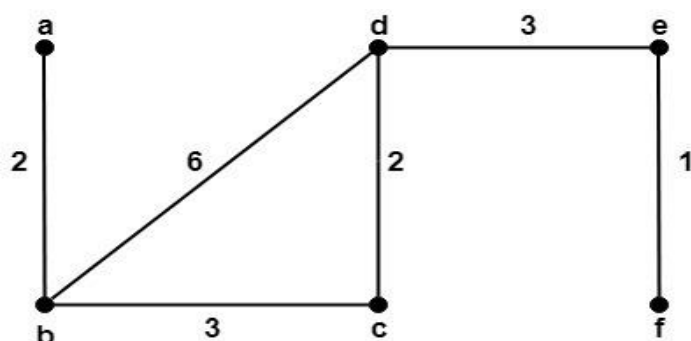


Step6: Edge (A, B), (D, E) and (E, F) are discarded because they will form the cycle in a graph.

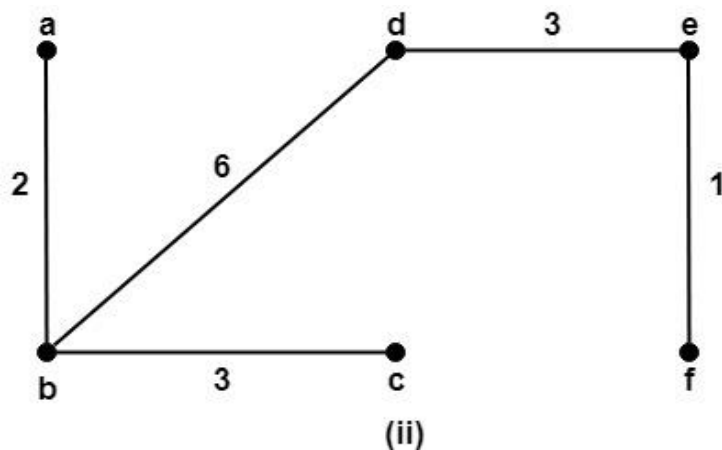
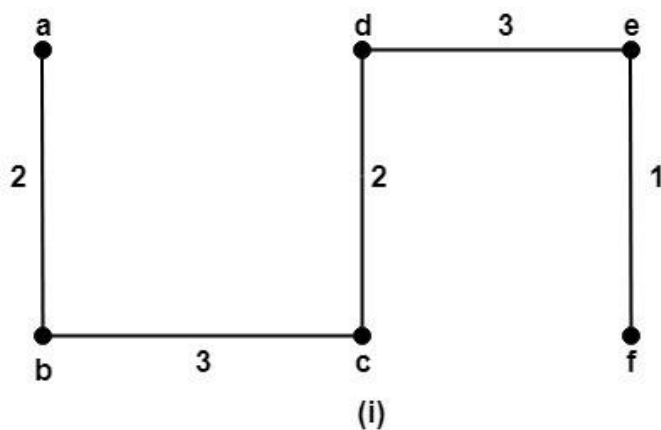
So, the minimum spanning tree form in step 5 is output, and the total cost is 18.

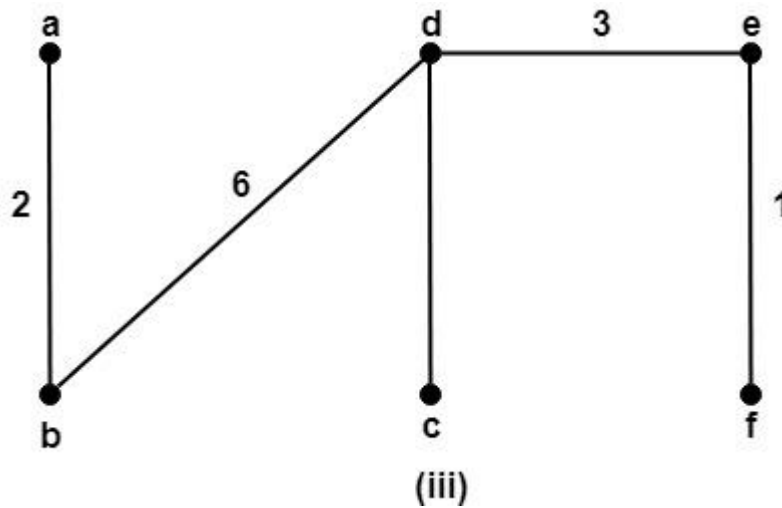


Example2: Find all the spanning tree of graph G and find which is the minimal spanning tree of G shown in fig:



Solution: There are total three spanning trees of the graph G which are shown in fig:

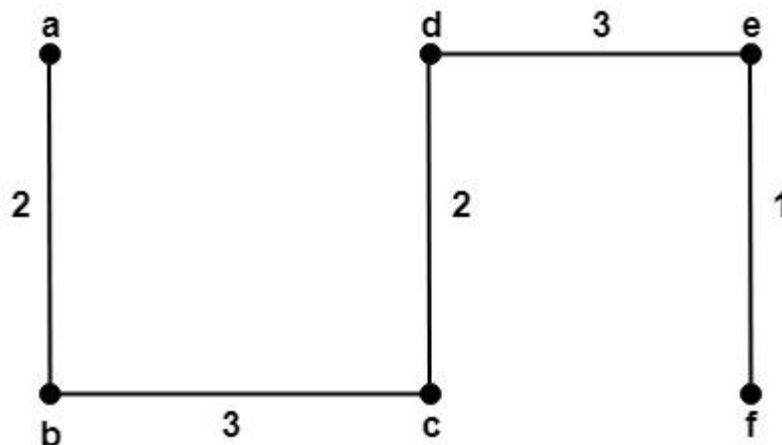




To find the minimum spanning tree, use the KRUSKAL'S ALGORITHM. The minimal spanning tree is shown in fig:

Edges	Weights	Added or Not
(E, F)	1	Added
(A, B)	2	Added
(C, D)	2	Added
(B, C)	3	Added
(D, E)	3	Added
(B, D)	6	Not Added

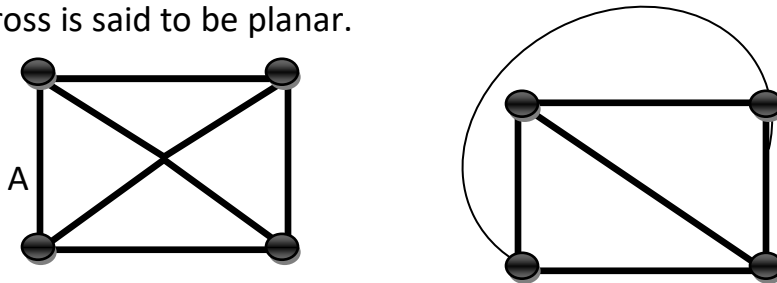
Minimum Spanning Tree



The first one is the minimum spanning having the minimum weight = 11.

Planner Graphs

A graph or multi-graph which can be drawn in the plane so that its edges do not cross is said to be planar.



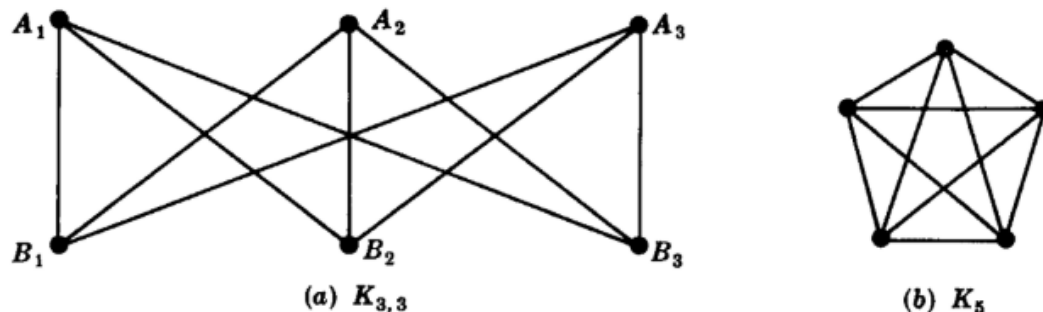
The complete graph with four vertices K_4 is usually pictured with crossing edges as in Figure A, it can also be drawn with non-crossing edges as in Figure B; hence K_4 is planar. Tree graphs form an important class of planar graphs.

Maps, Regions

A particular planar representation of a finite planar multi-graph is called a map. The map is connected if the underlying multi-graph is connected. A given map divides the plane into various regions.

Non-planar Graphs, Kuratowski's Theorem

1. Consider first the utility graph; that is, three houses A_1, A_2 and A_3 are to be connected to outlets for water, gas and electricity, B_1, B_2 and B_3 , as in Figure.
2. Observe that this is the graph $K_{3,3}$ and it has $p = 6$ vertices and $q = 9$ edges.



- Suppose the graph is planar. By Euler's formula a planar representation has $r = 5$ regions. Observe that no three vertices are

connected to each other; hence the degree of each region must be 4 or more and so the sum of the degrees of the regions must be 20 or more.

- Consider next the star graph in Figure. This is the complete graph K_5 on $p = 5$ vertices and has $q = 10$ edges. $10 = q \leq 3p - 6 = 15 - 6 = 9$ which is impossible. Thus K_5 is non-planar.

Graph Coloring

A coloring of a simple graph is the assignment of a color to each vertex of the graph so that no two adjacent vertices are assigned the same color.

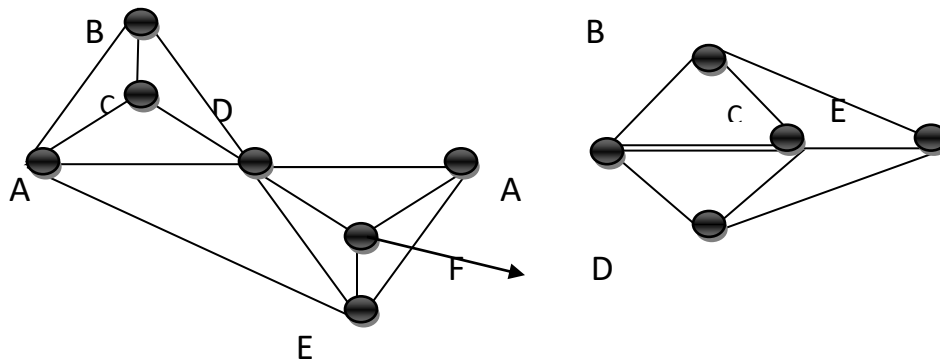
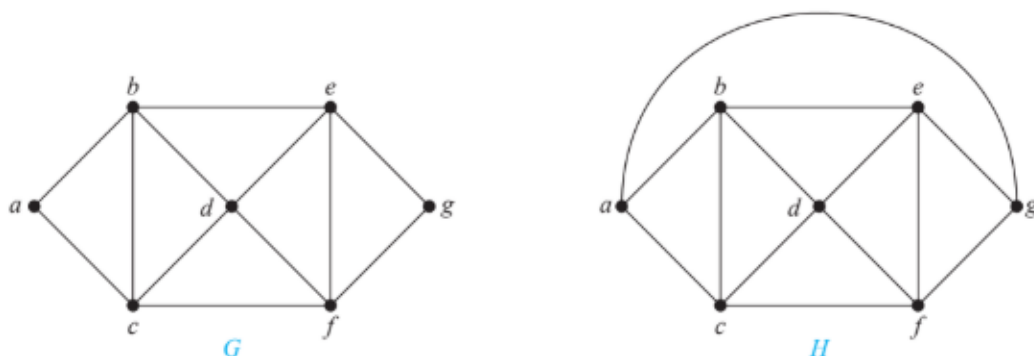


FIGURE 2 Dual Graphs of the Maps in Figure 1

DEFINITION: The chromatic number of a graph is the least number of colors needed for a coloring of this graph. The chromatic number of a graph G is denoted by $\chi(G)$.

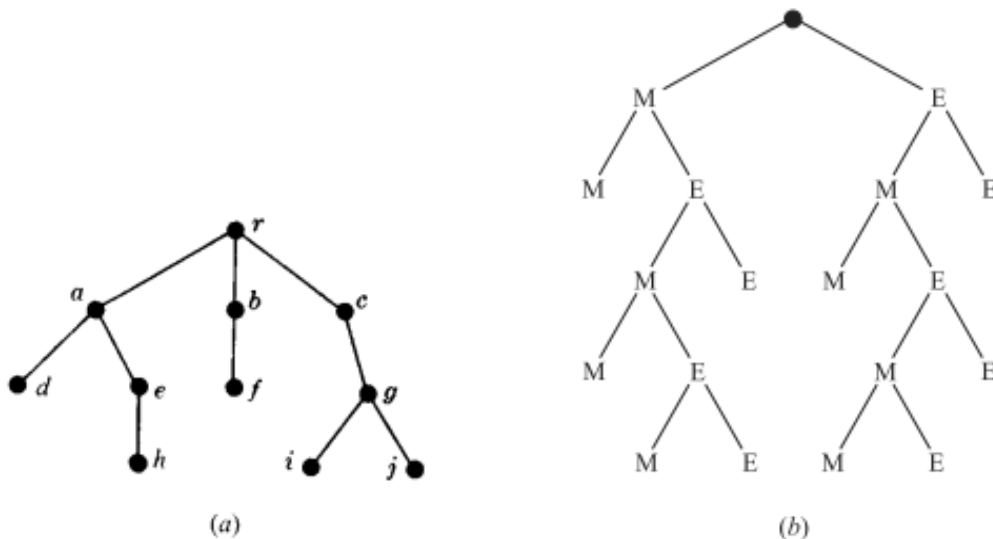
THEOREM 1 THE FOUR COLOR THEOREM The chromatic number of a planar graph is no greater than four.



Rooted trees

- A rooted tree T is a tree graph with a designated vertex r called the root of the tree.
- Consider a rooted tree T with root r . The length of the path from the root r to any vertex v is called the level (or depth) of v , and the maximum vertex level is called the depth of the tree.
- Those vertices with degree 1, other than the root r , are called the leaves of T , and a directed path from a vertex to a leaf is called a branch.

The tree has five leaves, d , f , h , i , and j . Observe that: level (a) = 1, level (f) = 2, level (j) = 3. Furthermore, the depth of the tree is 3.



- The fact that a rooted tree T gives a direction to the edges means that we can give a precedence relationship between the vertices.
- Specifically, we will say that a vertex u precedes a vertex v or that v follows u if there is a (directed) path from v to u .
- In particular, we say that v immediately follows u if (u, v) is an edge, that is, if v follows u and v is adjacent to u .

EXAMPLE

Suppose Marc and Erik are playing a tennis tournament such that the first person to win two games in a row or who wins a total of three games wins the tournament. Find the number of ways the tournament can proceed.

Solution: The rooted tree in Figure (b) shows the various ways that the tournament could proceed. There are 10 leaves which correspond to the 10 ways that the tournament can occur:

MM, MEMM, MEMEM, MEMEE, MEE, EMM, EMMEM, EMEME, EMEE, EE

Specifically, the path from the root to the leaf describes who won which games in the particular tournament.

Ordered Rooted Trees

Consider a rooted tree T in which the edges leaving each vertex are ordered. Then we have the concept of an ordered rooted tree.

Boolean algebra

Boolean Functions

Boolean algebra provides the operations and the rules for working with the set $\{0, 1\}$. The complement of an element, denoted with a bar, is defined by $\bar{0} = 1$ and $\bar{1} = 0$.

The Boolean sum, denoted by $+$ or by OR, has the following values:

$$1 + 1 = 1, 1 + 0 = 1, 0 + 1 = 1, 0 + 0 = 0$$

The Boolean product, denoted by $\cdot$ or by AND, has the following values:

$$1 \cdot 1 = 1, 1 \cdot 0 = 0, 0 \cdot 1 = 0, 0 \cdot 0 = 0$$

EXAMPLE 1 Find the value of $1 \cdot 0 + (0 + 1)$.

Solution: $1 \cdot 0 + (0 + 1) = 0 + 1 = 0 + 0 = 0$.

EXAMPLE 2 Translate $1 \cdot 0 + (0 + 1) = 0$, the equality into a logical equivalence.

Solution:

- Translate each 1 into a T, 0 into an F,
- Boolean sum into a disjunction, each Boolean product into a conjunction, and each complementation into a negation.

$$(T \wedge F) \vee \neg (T \vee F) \equiv F$$

Boolean Expressions and Boolean Functions

- Let $B = \{0, 1\}$. Then $B_n = \{(x_1, x_2, \dots, x_n) \mid x_i \in B \text{ for } 1 \leq i \leq n\}$ is the set of all possible n -tuples of 0s and 1s.
- The variable x is called a Boolean variable if it assumes values only from B , that is, if its only possible values are 0 and 1.
- A function from B_n to B is called a Boolean function of degree n .

EXAMPLE: The function $F(x, y) = x \cdot y$ from the set of ordered pairs of Boolean variables to the set $\{0, 1\}$ is a Boolean function of degree 2 with $F(1, 1) = 0$, $F(1, 0) = 1$, $F(0, 1) = 0$, and $F(0, 0) = 0$.

We display these values of F in Table.

x	y	F(x, y)
1	1	0
1	0	1
0	1	0
0	0	0

The Boolean expressions in the variables $x_1, x_2, \dots, x_n$ are defined recursively as 0, 1, $x_1, x_2, \dots, x_n$ are Boolean expressions;

If E_1 and E_2 are Boolean expressions, then $\neg E_1$, $(E_1 E_2)$, and $(E_1 + E_2)$ are Boolean expressions.

Identities of Boolean algebra

Law of the double complement: $x'' = x$

Discrete Structures and Optimization Unit - 1

Idempotent laws: $x + x = x$, $x \cdot x = x$

Identity laws: $x + 0 = x$, $x \cdot 1 = x$

Domination laws: $x + 1 = 1$, $x \cdot 0 = 0$

Commutative laws: $x + y = y + x$, $x \cdot y = y \cdot x$

Associative laws: $x + (y + z) = (x + y) + z$, $x \cdot (y \cdot z) = (x \cdot y) \cdot z$

Distributive laws: $x + y \cdot z = (x + y)(x + z)$, $x \cdot (y + z) = x \cdot y + x \cdot z$

De Morgan's laws: $\overline{(x \cdot y)} = \overline{x} + \overline{y}$, $\overline{(x + y)} = \overline{x} \cdot \overline{y}$

Absorption laws: $x + x \cdot y = x$, $x \cdot (x + y) = x$

Unit property: $x + \overline{x} = 1$

Zero property $x \cdot \overline{x} = 0$

EXAMPLE: Prove the absorption law $x(x + y) = x$ using the other identities of Boolean algebra.

Solution: $x(x + y) = (x + 0)(x + y)$ Identity law for the Boolean sum

$= x + 0 \cdot y$ Distributive law

$= x + y \cdot 0$ Commutative law

$= x + 0$ Domination law

$= x$ Identity law

Duality

The dual of a Boolean expression is obtained by interchanging Boolean sums and Boolean products and interchanging 0s and 1s.

EXAMPLE: Find the duals of $x(y + 0)$ and $x \cdot 1 + (y + z)$.

Solution: Interchanging $\cdot$ signs and $+$ signs and interchanging 0s and 1s. The duals are $x + (y \cdot 1)$ and $(x + 0)(y \cdot z)$, respectively.

The Abstract Definition of a Boolean Algebra

DEFINITION: A Boolean Algebra is a set B with two binary operations $\vee$ and $\wedge$, elements 0 and 1 , and a unary operation such that these properties hold for all x , y , and z in B :

Identity laws: $x \vee 0 = x$, $x \wedge 1 = x$

Complement laws: $x \vee x = 1$, $x \wedge x = 0$

Associative laws: $(x \vee y) \vee z = x \vee (y \vee z)$, $(x \wedge y) \wedge z = x \wedge (y \wedge z)$

Commutative laws: $x \vee y = y \vee x$, $x \wedge y = y \wedge x$

Distributive laws: $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$, $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$

Sum-of-Products Expansions

EXAMPLE: Find Boolean expressions that represent the functions $F(x, y, z)$ and $G(x, y, z)$.

x	y	z	F	G
1	1	1	0	0
1	1	0	0	1
1	0	1	1	0
1	0	0	0	0
0	1	1	0	0
0	1	0	0	1
0	0	1	0	0
0	0	0	0	0

Solution: An expression that has the value 1 when $x = z = 1$ and $y = 0$, and the value 0 otherwise, is needed to represent F .

To represent G , we need an expression that equals 1 when $x = y = 1$ and $z = 0$, or $x = z = 0$ and $y = 1$.

DEFINITION: A literal is a Boolean variable or its complement.

Discrete Structures and Optimization Unit - 1

A Min-term of the Boolean variables $x_1, x_2, \dots, x_n$ is a Boolean product $y_1 y_2 \cdots y_n$, where $y_i = x_i$ or $y_i = \bar{x}_i$. Hence, a min-term is a product of n literals, with one literal for each variable.

EXAMPLE: Find a min-term that equals 1 if $x_1 = x_3 = 0$ and $x_2 = x_4 = x_5 = 1$, and equals 0 otherwise.

Solution: The midterm $\bar{x}_1 x_2 x_3 \bar{x}_4 x_5$ has the correct set of values.

Functional Completeness

- Each min-term is the Boolean product of Boolean variables or their complements.
- Shows that every Boolean function can be represented using the Boolean operators $\cdot$, $+$, and $-$. Because every Boolean function can be represented using these operators we say that the set $\{\cdot, +, -\}$ is **functionally complete**.

We can eliminate all Boolean sums using the identity $x + y = x \bar{y} \bar{\bar{y}}$

Optimization

Linear Programming

- Problems which seek to maximize (or, minimize) profit (or, cost) form a general class of problems called optimization problems.
- A special but a very important class of optimization is linear programming problem.

A linear programming (LP) problem is an optimization problem that can be written

Maximize: $c x$

Subject to: $Ax \leq b$

Where A is a given $q \times n$ matrix, c is a given row vector of length n , and b is a given column vector of length q .

Linear programming problem

A linear programming problem is one that is concerned with finding the optimal value (maximum or minimum) of a linear function of several variables (called **objective function**) subject to the conditions that:

1. The variables are non-negative and satisfy a set of linear inequalities (called **linear constraints**).
2. Variables are sometimes called decision variables and are **non-negative**.

Objective function: Linear function $Z = ax + by$, where a, b are constants, which has to be maximized or minimized is called a linear objective function.

Constraints: The linear inequalities or equations or restrictions on the variables of a linear programming problem are called constraints.

Optimization problem: A problem which seeks to maximize or minimize a linear function subject to certain constraints as determined by a set of linear inequalities is called an Optimization problem.

The common region determined by all the constraints including the non-negative constraints $x \geq 0, y \geq 0$ of a linear programming problem is called **feasible region** (or **solution region**) for the problem.

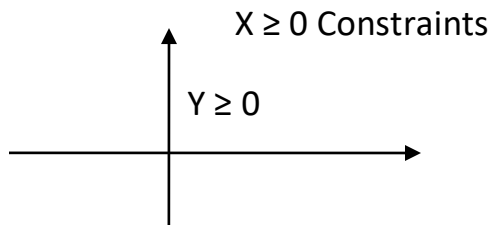
1. Points within and on the boundary of the feasible region represent **feasible solutions** of the constraints.
2. Any point outside the **feasible region** is an **infeasible solution**.

Optimal Solution

Any point in the **feasible region** that gives the optimal value (maximum or minimum) of the objective function is called an **optimal solution**.

Theorem 1 Let R be the feasible region (Convex Polygon) for a linear programming problem and let $Z = ax + by$ be the Objective function. When Z has an optimal value (maximum or minimum), where the variables x and y are subject to constraints described by linear inequalities, this optimal value must occur at a corner point (vertex) of the feasible region.

Theorem 2 Let R be the feasible region for a linear programming problem and let $Z = ax + by$ be the Objective function. If R is bounded, then the objective function Z has both a maximum and a minimum value on R and each of these occurs at a corner point (vertex) of R . A corner point of a feasible region is a point in the region which is the intersection of two boundary lines.



Bounded and Unbounded

A feasible region of a system of linear inequalities is said to be bounded.

- If it can be enclosed within a circle. Otherwise, it is called unbounded.
- Unbounded means that the feasible region does not extend indefinitely in any direction.

Corner Point Method (Steps)

1. Find the feasible region of the linear programming problem and determine its corner point either by inspection or by solving the two equations of the lines intersecting at that point.
2. Evaluate the objective function $Z = ax + by$ at each corner point. Let M and m , respectively denote the largest and smallest values of these points.
3. When the feasible region is **bounded**, M and m are the maximum and minimum value of Z .
4. In case feasible region is unbounded, we have:
 - a) M is the maximum value of Z , if the open half plane determined by $ax + by > M$ has no point in common with the feasible region. Otherwise, Z has no maximum value.

- b) Similarly, m is the minimum value of Z , if the open half plane determined by $ax + by > M$ has no point in common with the feasible region. Otherwise, Z has no minimum value.

Corner Point Solution

The optimal solution to LPP, if it exists, occurs at the corners of the feasible region.

The method includes the following steps:

- **Step 1:** Find the feasible region of the LPP.
- **Step 2:** Find the co-ordinates of each vertex of the feasible region. These co-ordinates can be obtained from the graph or by solving the equation of the lines.
- **Step 3:** At each vertex (corner point) compute the value of the objective function.
- **Step 4:** Identify the corner point at which the value of the objective function maximum (or minimum depending on the LP).
- The co-ordinates of this vertex are the optimal solution and the value of Z is the optimal value.

Different types of linear programming

Example: Maximize y

$$-x + y \leq 1$$

$$3x + 2y \leq 12$$

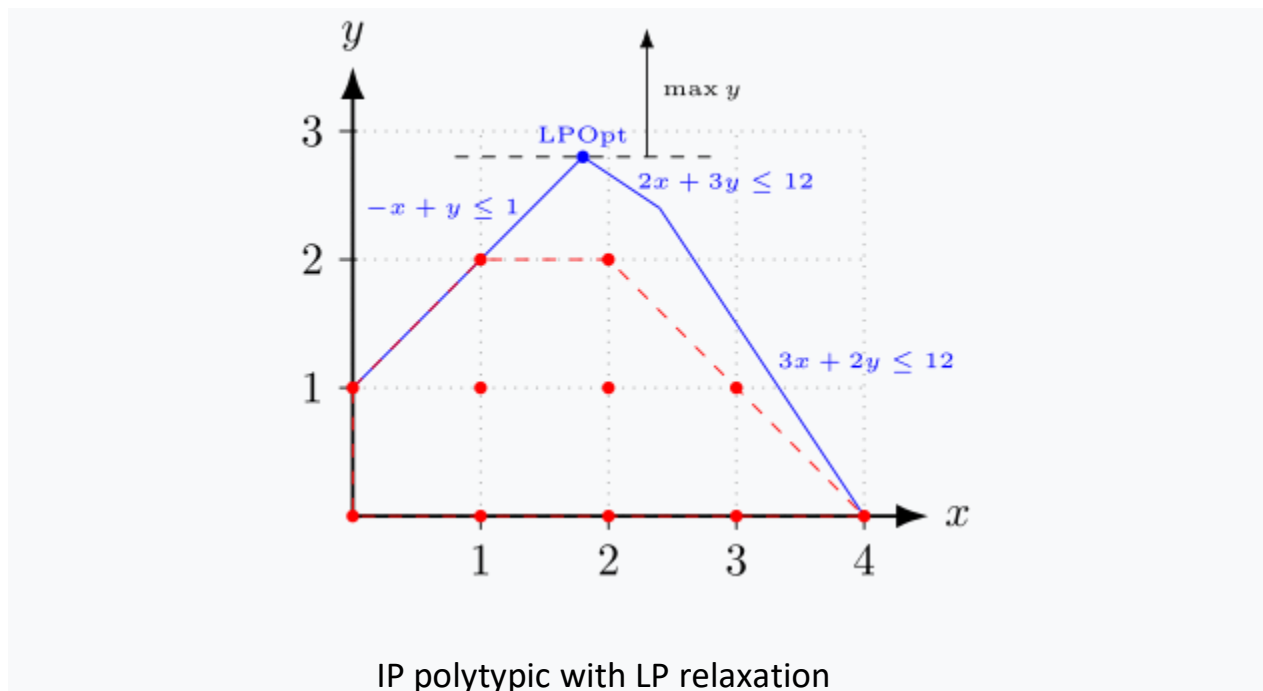
$$2x + 3y \leq 12$$

$$x, y \geq 0$$

- The feasible integer points are shown in red, and the red dashed lines indicate their convex hull, which is the smallest convex polyhedron that contains all of these points.

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- The blue lines together with the coordinate axes define the polyhedron of the LP relaxation, which is given by the inequalities without the integrality constraint.
- The goal of the optimization is to move the black dotted line as far upward while still touching the polyhedron. The optimal solutions of the integer problem are the points and which both have an objective value of 2.
- The unique optimum of the relaxation is with objective value of 2.8. If the solution of the relaxation is rounded to the nearest integers, it is not feasible for the ILP.



LINEAR PROGRAMMING – SIMPLEX METHOD

- If the linear programming problem has larger number of variables, the suitable method for solving is Simplex Method.

The **Simplex method** also helps the decision maker/manager to identify the following:

- Redundant Constraints

- Multiple Solutions
- Unbounded Solution
- Infeasible Problem

Basics of Simplex Method

The basic of simplex method is explained with the following linear programming problem.

Example:

Maximize

$$60x_1 + 70x_2$$

Subject to:

$$2x_1 + x_2 \leq 300$$

$$3x_1 + 4x_2 \leq 509$$

$$4x_1 + 7x_2 \leq 812$$

$$x_1, x_2 \geq 0$$

Solution: First we introduce the variables $s_3, s_4, s_5 \geq 0$

So that the constraints becomes equations, thus

$$2x_1 + x_2 + s_3 = 300$$

$$3x_1 + 4x_2 + s_4 = 509$$

$$4x_1 + 7x_2 + s_5 = 812$$

Corresponding to the three constraints, the variables s_3, s_4, s_5 are called as slack variables.

There are two types of solutions they are basic and basic feasible, which are discussed as follows:

Basic Solution

- We may equate any two variables to zero in the above system of equations, and then the system will have three variables.
- Thus, if this system of three equations with three variables is solvable such a solution is called as basic solution.
- The variables s_3 , s_4 , and s_5 are known as basic variables where as the variables x_1 , x_2 are known as non-basic variables.

Basic Feasible Solution

- A basic solution of a linear programming problem is called as basic feasible solutions if it is feasible it means all the variables are non-negative.
- The solution $s_3=300$, $s_4=509$ and $s_5=812$ is a basic feasible solution.

Simplex Method Computation

Consider the following linear programming problem

Maximize

$$60x_1 + 70x_2$$

Subject to:

$$2x_1 + x_2 + s_3 = 300$$

$$3x_1 + 9x_2 + s_4 = 509$$

$$4x_1 + 7x_2 + s_5 = 812$$

$$x_1, x_2, s_3, s_4, s_5 \geq 0$$

The profit $Z = 60x_1 + 70x_2$ i.e. Maximize $60x_1 + 70x_2$

The standard form can be summarized in a compact table form as

C_B	Basic	C_j	60	70	0	0	0
	Variables	X_B	x_1	x_2	s_3	s_4	s_5

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0	s_3	300	2	1	1	0	0
0	s_4	509	3	4	0	1	0
0	s_5	812	4	7	0	0	1
	Z	-60	-70	0	0	0	0

Consider the first equation $2x_1 + x_2 + s_3 = 300$

$$2x_1 + s_3 = 300 - x_2 \quad (x_1 = 0, s_3 \geq 0)$$

$$300 - x_2 \geq 0 \text{ i.e. } x_2 \leq 300$$

Second equation: $3x_1 + 9x_2 + s_4 = 509$

$$3x_1 + s_4 = 509 - 9x_2 \quad (x_1 = 0, s_4 \geq 0)$$

$$509 - 9x_2 \geq 0$$

$$\text{So, } x_2 \geq 509/9$$

Third equation: $4x_1 + 7x_2 + s_5 = 812$

$$4x_1 + s_5 = 812 - 7x_2 \quad (x_1 = 0, s_5 \geq 0)$$

$$812 - 7x_2 \geq 0$$

$$\text{So, } x_2 \geq 812/7$$

Therefore the three equations: $x_2 \leq 300, x_2 \geq 509/9, x_2 \geq 812/7$

Thus $x_2 = \min(x_2 \leq 300, x_2 \geq 509/9, x_2 \geq 812/7)$

$$\text{So, } x_2 = \min(x_2 \leq 300/1, x_2 \geq 509/9, x_2 \geq 812/7) = 116$$

Therefore $x_2 = 116$, if $x_2 = 116$, you may be note from the third equation

$$7x_2 + s_5 = 812 \text{ i.e. } s_5 = 0 \text{ So, } x_2 = 812/7 = 116$$

Thus, the variable s_5 becomes non-basic in the next iteration.

The other two basic variables are

Discrete Structures and Optimization Unit - 1

$$S_3 = 300 - x_2 = 184$$

$$S_4 = 509 - 4 \cdot 116 = 45$$

Using the following rules the Table 2 is computed from the Table 1.

- i. The revised basic variables are s_3 , s_4 and x_2 . Accordingly, we make $CB_1=0$, $CB_2=0$ and $CB_3=70$.
- ii. As x_2 is the incoming basic variable we make the coefficient of x_2 one by dividing each element of row-3 by 7. Thus the numerical value of the element corresponding to x_1 is $4/7$, corresponding to s_5 is $1/7$ in Table 2.
- iii. The incoming basic variable should appear only in the third row. So we multiply the third-row of Table 2 by 1 and subtract it from the first-row of Table 1 element by element. Thus the element corresponding to x_2 in the first-row of Table 2 is 0.

Therefore the element corresponding to x_1 is $2 - 1 \cdot 4/7 = 10/7$ and the element corresponding to s_5 is $0 - 1 \cdot 1/7 = -1/7$

In this way we obtain the elements of the first and the second row in Table 2. In Table 2 the numerical values can also be calculated in a similar way.

C_B	Basic	C_j	60	70	0	0	0
	Variables	XB	x_1	x_2	s_3	s_4	s_5
0	s_3	184	$10/7$	1	1	0	$-1/7$
0	s_4	45	$5/7$	4	0	1	$-4/7$
0	s_5	116	$4/7$	7	0	0	$1/7$
	$Z_j - C_j$		$-140/7$	0	0	0	$70/7$

$$z_1 - c_1 = 10/7 \cdot 0 + 5/7 \cdot 0 + 70 \cdot 4/7 - 60 = -140/7$$

$$z_5 - c_5 = -1/7 \cdot 0 - 4/7 \cdot 0 + 1/7 \cdot 70 - 0 = 70/7$$

- i. Now we apply rule (1) to Table 2. Here the only negative $z_j - c_j$ is $z_1 - c_1 = -140/7$

Hence x_1 should become a basic variable at the next iteration.

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- ii. This minimum occurs corresponding to s_4 , it becomes a non basic variable in next iteration.

C_B	Basic	C_j	60	70	0	0	0
	Variables	XB	x_1	x_2	s_3	s_4	s_5
0	s_3	94	10/7	1	1	-2	1
0	s_4	63	5/7	4	0	7/5	-4/5
0	s_5	80	4/7	7	0	-4/5	3/5
	$Z_j - C_j$		0	0	0	28	-6

- Now we apply rule (1) to Table 2. Here the only negative $Z_j - C_j$ is $z_1 - c_1 = -140/7$
- We compute the minimum of the ratio

$$\text{Min } (180/10/7, 45/5/7, 116/4/7) = \text{Min } (644/5, 63, 203) = 63$$

This minimum occurs corresponding to s_4 ; it becomes a non basic variable in next iteration.

- $z_5 - c_5 < 0$ should be made a basic variable in the next iteration.
- Now compute the minimum ratios

$$\text{Min } (94/1, 80/3/5) = 94$$

Since $y_{25} = -4/5 < 0$, the corresponding ratio is not taken for comparison. The variable s_3 becomes non basic in the next iteration.

C_B	Basic	C_j	60	70	0	0	0
	Variables	XB	x_1	x_2	s_3	s_4	s_5
0	s_3	94	10/7	1	1	2	1
0	s_4	691/5	5/7	4	4/5	-1/5	0
0	s_5	118/5	4/7	7	-3/5	2/5	0
	$Z_j - C_j$		0	0	6	16	0

Thus, the objective function is maximized for $x_1 = 691/5$ and $x_2 = 118/5$ and the maximum value of the objective function is 9944.

Key Terms

Basic Variable: Variable of a basic feasible solution has n non-negative value.

Non Basic Variable: Variable of a feasible solution has a value equal to zero.

Artificial Variable: A non-negative variable introduced to provide basic feasible solution and initiate the simplex procedures.

Slack Variable: A variable corresponding to a $\leq$ type constraint is a non-negative variable introduced to convert the inequalities into equations.

Surplus Variable: A variable corresponding to a $\geq$ type constraint is a non-negative variable introduced to convert the constraint into equations.

Basic Solution: System of m -equation and n -variables i.e. $m < n$ is a solution where at least $n-m$ variables are zero.

Basic Feasible Solution: System of m -equation and n -variables i.e. $m < n$ is a solution where m variables are non-negative and $n-m$ variables are zero.

Optimum Solution: A solution where the objective function is minimized or maximized.

DUAL LINEAR PROGRAMMING PROBLEMS

For every linear programming problem there is a corresponding linear programming problem called the **dual**.

Dual Problem Formulation

If the original problem is in the standard form then the dual problem can be formulated using the following rules:

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- The number of constraints in the original problem is equal to the number of dual variables. The number of constraints in the dual problem is equal to the number of variables in the original problem.
- The original problem profit coefficients appear on the right hand side of the dual problem constraints.
- If the original problem is a maximization problem then the dual problem is a minimization problem. Similarly, if the original problem is a minimization problem then the dual problem is a maximization problem.
- The original problem has less than or equal to ($\leq$) type of constraints while the dual problem has greater than or equal to ($\geq$) type constraints.
- The coefficients of the constraints of the original problem which appear from left to right are placed from top to bottom in the constraints of the dual problem and vice versa.

Simple Way of Solving Dual Problem

Minimize: $P = x_1 + 2x_2$

Subject to: $x_1 + x_2 \geq 8$

$$2x_1 + x_2 \geq 12$$

$$x_1 \geq 1$$

Solution: Step 1: Set up the P-matrix and its transpose

1	1	8	1	2	1	1
2	1	12	1	1	0	2
1	0	1	8	12	1	0
1	2	0				

Step 2: Form the objective function and constraints for the dual

$$w_1 + 2w_2 + w_3 \leq 1$$

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$$w_1 + w_2 + w_3 \leq 2$$

$$Z = 8w_1 = 12w_2 + 2$$

Step 3: Construct the initial simplex tableau for the dual

w_1	w_2	w_3	s_1	s_2	g	z
1	2	1	1	0	0	1
1	1	0	0	1	0	2
-8	-12	-1	0	0	1	0

The most negative number in the bottom row to the left of the last column is -12. This establishes the pivot column. The smallest non-negative ratio is $\frac{1}{2}$.

Step 4: Pivoting

Pivoting about the 2 we get:

w_1	w_2	w_3	s_1	s_2	g	z
1	2	1	1	0	0	1
$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	1	0	$\frac{3}{2}$
-2	0	5	6	0	1	6

w_1	w_2	w_3	s_1	s_2	g	z
$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{2}$
1	1	0	0	1	0	2
-8	-12	-1	0	0	1	0

The most negative entry in the bottom row to the left of the last column is -2

Pivoting about the $\frac{1}{2}$

w_1	w_2	w_3	s_1	s_2	g	z
$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{2}$
$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	1	0	$\frac{3}{2}$
-2	0	5	6	0	1	6

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Therefore, the optimum solution providing by $x_1 = 8$ and $x_2 = 0$

w_1	w_2	w_3	s_1	s_2	g	z
1	2	1	1	0	0	1
0	-1	-1	-1	1	0	1
0	4	7	8	0	1	8

Therefore, the optimum solution providing by $x_1 = 8$ and $x_2 = 0$. The minimum value is 8.

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Discrete Structures and Optimization Unit – 1 MCQs

As per updated syllabus

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THE LEARN WITH EXPERTIES

Discrete Structures and Optimization Unit – 1 MCQs

1. Which of the following statement is a proposition?
- a) Get me a glass of milkshake.
 - b) God bless you!
 - c) What is the time now?
 - d) The only odd prime number is 2

Answer: d

Explanation: Only this statement has got the truth value which is false.

2. The truth value of '4+3=7 or 5 is not prime'.
- a) False
 - b) True

Answer: b

Explanation: Compound statement with 'or' is true when either of the statement is true. Here the first part of the statement is true, hence the whole is true.

3. Which of the following option is true?
- a) If the Sun is a planet, elephants will fly
 - b) $3 + 2 = 8$ if $5 - 2 = 7$

- c) $1 > 3$ and 3 is a positive integer
- d) $-2 > 3$ or 3 is a negative integer

Answer: a

Explanation: Hypothesis is false, thus the whole statement is true.

4. What is the value of x after this statement, assuming the initial value of x are 5?

'If x equals to one then $x=x+2$ else $x=0$ '.

- a) 1
- b) 3
- c) 0
- d) 2

Answer: c

Explanation: If condition is false so value decided according to else condition.

5. Let P: I am in Bangalore; Q: I love cricket; then $q \rightarrow p$ (q implies p) is?

- a) If I love cricket then I am in Bangalore
- b) If I am in Bangalore then I love cricket
- c) I am not in Bangalore

Discrete Structures and Optimization Unit – 1 MCQs

d) I love cricket

Answer: a

Explanation: Q is hypothesis and P is conclusion. So the compound statement will be if hypothesis then conclusion.

6. Which of the following bits is the negation of the bits "010110"?

- a) 111001
- b) 001001
- c) 101001
- d) 111111

Answer: c

Explanation: Flip each of the bits to get the negation of the required string.

7. Which of the following option is suitable, if A is "10110110", B is "11100000" and C is "10100000"?

- a) $C=A \text{ or } B$
- b) $C=\sim A$
- c) $C=\sim B$
- d) $C=A \text{ and } B$

Answer: d

Explanation: Output of and is 1 when both other inputs are one.

8. How many bits string of length 4 are possible such that they contain 2 ones and 2 zeroes?

- a) 4
- b) 2
- c) 5
- d) 6

Answer: d

Explanation: The strings are {0011, 0110, 1001, 1100, 1010 and 0101}.

9. If a bit string contains {0, 1} only, having length 5 has no more than 2 ones in it. Then how many such bit strings are possible?

- a) 14
- b) 12
- c) 15
- d) 16

Answer: d

Explanation: The total strings are 1(having no one in it) +5(having 1 one in it) +10(having 2 ones in it) = 16.

10. If A is "001100" and B is "010101" then what is the value of A (Ex-or) B?

Discrete Structures and Optimization Unit – 1 MCQs

- a) 000000
- b) 111111
- c) 001101
- d) 011001

Answer: d

Explanation: In Ex-or if both the inputs are same then output is 0 otherwise 1.

11. Let P and Q be statements, then $P \leftrightarrow Q$ is logically equivalent to
-

- a) $P \leftrightarrow \sim Q$
- b) $\sim P \leftrightarrow Q$
- c) $\sim P \leftrightarrow \sim Q$
- d) None of the mentioned

Answer: c

Explanation: Both of them have same truth table, hence they are equal.

12. What is the negation of the statement $A \rightarrow (B \vee (\text{or}) C)$?
- a) $A \wedge \sim B \wedge \sim C$
 - b) $A \rightarrow B \rightarrow C$
 - c) $\sim A \wedge B \vee C$
 - d) None of the mentioned

Answer: a

Explanation: $A \rightarrow P$ is logically equivalent to $\sim A \vee P$.

13. The compound statement $A \rightarrow (A \rightarrow B)$ is false, then the truth values of A, B are respectively
-

- a) T, T
- b) F, T
- c) T, F
- d) F, F

Answer: c

Explanation: For implications to be false hypothesis should be true and conclusion should be false

14. The statement which is logically equivalent to $A \wedge (\text{and}) B$ is?

- a) $A \rightarrow B$
- b) $\sim A \wedge \sim B$
- c) $A \wedge \sim B$
- d) $\sim(A \rightarrow \sim B)$

Answer: d

Explanation: The truth table of both statements is same.

15. Let P, Q, R be true, false true, respectively, which of the following is true?

- a) $P \wedge Q \wedge R$
- b) $P \wedge \sim Q \wedge \sim R$
- c) $Q \rightarrow (P \wedge R)$
- d) $P \rightarrow (Q \wedge R)$

Discrete Structures and Optimization Unit – 1 MCQs

Answer: c

Explanation: Hypothesis is false, hence statement is true.

16. If there are 'M' switches in series numbered from 1, 2, ..., M. For circuit to be complete and bulb to glow which of the following is necessary?
- a) $1 \wedge 2 \wedge 3 \wedge \dots \wedge M$ should be on.
 - b) $1 \wedge 2 \wedge 3 \wedge \dots \wedge M$ should be off.
 - c) $1 \vee 2 \vee 3 \vee \dots \vee M$ should be on.
 - d) None of the mentioned.

Answer: a

Explanation: All should be on in-order to complete the circuit.

17. Which of the following statements is the negation of the statements "4 is odd or -9 is positive"?
- a) 4 is even or -9 is not negative.
 - b) 4 is odd or -9 is not negative.
 - c) 4 is even and -9 is negative.
 - d) 4 is odd and -9 is not negative.

Answer: c

Explanation: Using De Morgan's Law $\sim (A \vee B) \leftrightarrow \sim A \wedge \sim B$

18. Which of the following represents: $\sim A$ (negation of A) if A stands for "I like badminton but hate math's"?
- a) I hate badminton and math's
 - b) I do not like badminton or math's
 - c) I dislike badminton but love math's
 - d) I hate badminton or like math's

Answer: d

Explanation: De Morgan's Law $\sim (A \wedge B) \leftrightarrow \sim A \vee \sim B$.

19. The compound statement $A \vee \sim(A \wedge B)$.
- a) True
 - b) False

Answer: a

Explanation: Applying De-Morgan's law we get $A \vee \sim A \equiv$ Tautology.

Discrete Structures and Optimization Unit – 1 MCQs

20. Which of the following is De-

Morgan's law?

a) $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$

b) $\sim(P \wedge R) \equiv \sim P \vee \sim R, \sim(P \vee R) \equiv \sim P \wedge \sim R$

c) $P \vee \sim P \equiv \text{True}, P \wedge \sim P \equiv \text{False}$

d) None of the mentioned

Answer: b

Explanation: Definition of De-Morgan's Law.

21. A compound proposition that is always _____ is called a tautology.

a) True

b) False

Answer: a

Explanation: Tautology is always true.

22. If A is any statement, then which of the following is a tautology?

a) $A \wedge F$

b) $A \vee F$

c) $A \vee \sim A$

d) $A \wedge T$

Answer: c

Explanation: $A \vee \sim A$ is always true.

23. A compound proposition that is neither a tautology nor a contradiction is called a _____

a) Contingency

b) Equivalence

c) Condition

d) Inference

Answer: a

Explanation: Definition of contingency.

24. $\neg (A \vee q) \wedge (A \wedge q)$ is a _____

a) Tautology

b) Contradiction

c) Contingency

d) None of the mentioned

Answer: b

Explanation: $\equiv (\neg A \wedge \neg q) \wedge (A \wedge q)$

$$\equiv (\neg A \wedge A) \wedge (\neg q \wedge q)$$

$$\equiv F \wedge F \equiv F.$$

25. The contra positive of $p \rightarrow q$ is the proposition of _____

a) $\neg p \rightarrow \neg q$

b) $\neg q \rightarrow \neg p$

Discrete Structures and Optimization Unit – 1 MCQs

- c) $q \rightarrow p$
- d) $\neg q \rightarrow p$

Answer: b

Explanation: Definition of contra positive.

26. The inverse of $p \rightarrow q$ is the proposition of _____

- a) $\neg p \rightarrow \neg q$
- b) $\neg q \rightarrow \neg p$
- c) $q \rightarrow p$
- d) $\neg q \rightarrow p$

Answer: a

Explanation: Definition of inverse.

27. The converse of $p \rightarrow q$ is the proposition of _____

- a) $\neg p \rightarrow \neg q$
- b) $\neg q \rightarrow \neg p$
- c) $q \rightarrow p$
- d) $\neg q \rightarrow p$

Answer: c

Explanation: Definition of converse.

28. What is the contra positive of the conditional statement? "The home team misses whenever it is drizzling?"

- a) If it is drizzling, then home team misses
- b) If the home team misses, then it is drizzling
- c) If it is not drizzling, then the home team does not misses
- d) If the home team wins, then it is not drizzling

Answer: d

Explanation: q whenever p contra positive is $\neg q \rightarrow \neg p$.

29. What is the converse of the conditional statement "If it ices today, I will play ice hockey tomorrow."

- a) "I will play ice hockey tomorrow only if it ices today."
- b) "If I do not play ice hockey tomorrow, then it will not have iced today."
- c) "If it does not ice today, then I will not play ice hockey tomorrow."
- d) "I will not play ice hockey tomorrow only if it ices today."

Answer: a

Explanation: If p, then q has converse $q \rightarrow p$.

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30. What are the converse of the conditional statement “When Raj stay up late, it is necessary that Raj sleep until noon.”
- a) “If Raj stay up late, then Raj sleep until noon.”
 - b) “If Raj does not stay up late, then Raj does not sleep until noon.”
 - c) “If Raj does not sleep until noon, then Raj does not stay up late.”
 - d) “If Raj sleep until noon, then Raj stay up late.”

Answer: d

Explanation: Necessary condition for p is q has converse $q \rightarrow p$.

31. The compound propositions p and q are called logically equivalent if _____ is a tautology.
- a) $p \leftrightarrow q$
 - b) $p \rightarrow q$
 - c) $\neg(p \vee q)$
 - d) $\neg p \vee \neg q$

Answer: a

Explanation: Definition of logical equivalence.

32. $p \rightarrow q$ is logically equivalent to _____

- a) $\neg p \vee \neg q$
- b) $p \vee \neg q$
- c) $\neg p \vee q$
- d) $\neg p \wedge q$

Answer: c

Explanation: $(p \rightarrow q) \leftrightarrow (\neg p \vee q)$ is tautology.

33. Let P (x) denote the statement “ $x > 7$.” Which of these have truth value true?
- a) P (0)
 - b) P (4)
 - c) P (6)
 - d) P (9)

Answer: d

Explanation: Put $x=9$, $9 > 7$ which is true.

34. Let Q(x, y) denote “ $M + A = 0$.” What is the truth value of the quantifications $\exists A \forall M Q (M, A)$.
- a) True
 - b) False

Answer: b

Explanation: For each A there exist only one M, because there is no real number A such

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that $M + A = 0$ for all real numbers M .

35. Which rule of inference is used in each of these arguments, "If it is Wednesday, then the Smart-mart will be crowded. It is Wednesday. Thus, the Smart-mart is crowded."
- a) Modus Tollens
 - b) Modus ponens
 - c) Disjunctive syllogism
 - d) Simplification

Answer: b

Explanation: $(M \wedge (M \rightarrow N)) \rightarrow N$ is Modus ponens.

36. Let the statement be "If n is not an odd integer then square of n is not odd.", then if $P(n)$ is " n is not an odd integer" and $Q(n)$ is "(square of n) is not odd." For direct proof we should prove _____

- a) $\forall n P((n) \rightarrow Q(n))$
- b) $\exists n P((n) \rightarrow Q(n))$
- c) $\forall n \sim(P((n)) \rightarrow Q(n))$
- d) $\forall n P((n) \rightarrow \sim(Q(n)))$

Answer: a

Explanation: Definition of direct proof.

37. $\{x: x \text{ is an integer neither positive nor negative}\}$ is _____

- a) Empty set
- b) Non-empty set
- c) Finite set
- d) Non-empty and Finite set

Answer: d

Explanation: Set = $\{0\}$ non-empty and finite set.

38. A _____ is an ordered collection of objects.

- a) Relation
- b) Function
- c) Set
- d) Proposition

Answer: c

Explanation: By the definition of set.

39. The union of the sets $\{1, 2, 5\}$ and $\{1, 2, 6\}$ is the set _____

- a) $\{1, 2, 6, 1\}$
- b) $\{1, 2, 5, 6\}$
- c) $\{1, 2, 1, 2\}$
- d) $\{1, 5, 6, 3\}$

Answer: b

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Explanation: The union of the sets A and B is the set that contains those elements that are either in A or in B.

40. Let the set A is {1, 2, 3} and B is {2, 3, 4}. Then the number of elements in $A \cup B$ is?

- a) 4
- b) 5
- c) 6
- d) 7

Answer: a

Explanation: $A \cup B$ is {1, 2, 3, 4}.

41. In which of the following sets A – B is equal to $B - A$?

- a) $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$
- b) $A = \{1, 2, 3\}$, $B = \{1, 2, 3, 4\}$
- c) $A = \{1, 2, 3\}$, $B = \{2, 3, 1\}$
- d) $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{2, 3, 4, 5, 1\}$

Answer: c

Explanation: $A - B = B - A = \text{Empty set}$.

42. If A has 4 elements B has 8 elements then the minimum and maximum number of elements in $A \cup B$ are _____

- a) 4, 8

b) 8, 12

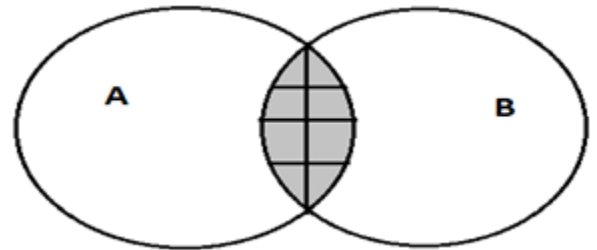
c) 4, 12

d) None of the mentioned

Answer: b

Explanation: Minimum would be when 4 elements are same as in 8, maximum would be when all are distinct.

43. The shaded area of figure is best described by?



a) $A \cap B$

b) $A \cup B$

c) A

d) B

Answer: a

Explanation: The region is A intersection B.

44. If $n(A) = 20$ and $n(B) = 30$ and $n(A \cup B) = 40$ then $n(A \cap B)$ is?

a) 20

b) 30

c) 40

d) 10

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Answer: d

Explanation: $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

45. The relation between sets A, B, C as shown by Venn diagram is _____

- a) A is subset of B and B is subset of C
- b) C is not a subset of A and A is subset of B
- c) C is subset of B and B is subset of A
- d) None of the mentioned

Answer: c

Explanation: As set C is totally inside set B, set B is totally inside set A.

46. If $n(A) = 10$, $n(B) = 30$, $n(C) = 50$ and if set A, B, C are pair-wise disjoint then which of the following is correct?

- a) $n(A \cup B) = 0$
- b) $n(B \cup C) = 0$
- c) $n(A \cup B \cup C) = 90$
- d) All of the mentioned

Answer: d

Explanation: All the statements are true based on definition.

47. Let C and D be two sets then which of the following statements are true?

- i) $C \cup D = D \cup C$
- ii) $C \cap D = D \cap C$
- a) Both of the statements
- b) only I statement
- c) Only ii statement
- d) none of the statements

Answer: a

Explanation: Commutative laws hold good in sets.

48. If set C is $\{1, 2, 3, 4\}$ and $C - D = \Phi$ then set D can be _____

- a) $\{1, 2, 4, 5\}$
- b) $\{1, 2, 3\}$
- c) $\{1, 2, 3, 4, 5\}$
- d) None of the mentioned

Answer: c

Explanation: $C \cap D$ should be equivalent to C for $C - D = \Phi$.

49. Let C and D be two sets then $C - D$ is equivalent to _____

- a) $C' \cap D$
- b) $C' \cap D'$

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- c) $C \cap D'$
d) None of the mentioned

Answer: c

Explanation: Set C-D will be having those elements which are in C but not in D.

50. Let set $A = \{1, 2\}$ and C be $\{3, 4\}$ then $A \times B$ (Cartesian product of set A and B) is?
a) $\{1, 2, 3, 4\}$
b) $\{(1, 3), (2, 4)\}$
c) $\{(1, 3), (2, 4), (1, 4), (2, 3)\}$
d) $\{(3, 1), (4, 1)\}$

Answer: c

Explanation: In set $A \times B$: $\{(c, d) \mid c \in A \text{ and } d \in B\}$.

51. If set A has 4 elements and B has 3 elements then set $n(A \times B)$ is?
a) 12
b) 14
c) 24
d) 7

Answer: a

Explanation: The total elements in $n(A \times B) = n(A) * n(B)$.

52. Which of the following statements regarding sets is

false?

- a) $A \times B = B \times A$
b) $A \times B \neq B \times A$
c) $n(A \times B) = n(A) * n(B)$
d) All of the mentioned

Answer: a

Explanation: The Cartesian product of sets is not commutative.

53. If a set contains 3 elements then the number of subsets is?
a) 6
b) 3
c) 12
d) 8

Answer: d

Explanation: For elements with n elements the numbers of subsets are 2^n .

54. The set containing all the collection of subsets is known as _____
a) Subset
b) Power set
c) Union set
d) None of the mentioned

Answer: b

Explanation: Power set contains all the subsets as its elements.

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55. If a set is empty then number of subsets will be _____

- a) 1
- b) 2
- c) 0
- d) 4

Answer: a

Explanation: The set has zero elements so $2^0 = 1$.

56. What is the base case for the inequality $7^n > n^3$, where $n = 3$?

- a) $652 > 189$
- b) $42 < 132$
- c) $343 > 27$
- d) $42 \leq 431$

Answer: c

Explanation: By the principle of mathematical induction, we have $7^3 > 3^3 \Rightarrow 343 > 27$ as a base case and it is true for $n = 3$.

57. In the principle of mathematical induction, which of the following steps is mandatory?

- a) induction hypothesis
- b) inductive reference
- c) induction set assumption
- d) minimal set representation

Answer: a

Explanation: The hypothesis of Step is a must for

mathematical induction that is the statement is true for $n = k$, where n and k are any natural numbers, which is also called induction assumption or induction hypothesis.

58. For $m = 1, 2, \dots$, $4m+2$ is a multiple of _____

- a) 3
- b) 5
- c) 6
- d) 2

Answer: d

Explanation: For $n = 1$, $4 * 1 + 2 = 6$, which is a multiple of 2.

Assume that $4m+2$ is true for $m=k$ and so $4k+2$ is true based on the assumption. Now, to prove that $4k+2$ is also a multiple of 2 $\Rightarrow 4(k+1) + 2 \Rightarrow 2 * 4k - 4k + 6 \Rightarrow 2 * 4k + 4 - 4k + 2 \Rightarrow 2(4k+2) - 2(2k+1)$. Here, the first term $2(4k+2)$ is true as per assumption and the second term $2(4k+2)$ is must to be a multiple of 2. Hence, $4(k+1)+2$ is a multiple of 2. So, by induction hypothesis, $(4m+2)$ is a multiple of 2, for $m = 1, 2, 3 \dots$

59. A polygon with 7 sides can be triangulated into _____

- a) 7
- b) 14
- c) 5

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d) 10

Answer: c

Explanation: A simple polygon with n sides can be triangulated into $n-2$ triangles, where $n > 2$.

60. Every simple polygon has an interior diagonal.

- a) True
- b) False

Answer: a

Explanation: By using Strong Induction.

61. A polygon with 12 sides can be triangulated into _____

- a) 7
- b) 10
- c) 5
- d) 12

Answer: b

Explanation: A simple polygon with n sides can be triangulated into $n-2$ triangles, where $n > 2$.

62. Which of the following is contained in a recursive grammar?

- a) semantic rules
- b) production rules
- c) recursive language

d) recursive function

Answer: b

Explanation: In natural language semantics, recursive grammar plays a vital role as well as in syntax. A recursive grammar in a context free language is a formal grammar which consists of recursive production rules.

63. _____ is the consequence of dynamic programming.

- a) Bellman equation
- b) Fresenius equation
- c) Linear equation
- d) Boolean expression

Answer: a

Explanation: Dynamic programming can lead to recursive optimization that can restate a multistep optimization problem in its recursive form. The Bellman equation that writes the value of the optimization problem at an earlier time in terms of its value at a later time is the result of dynamic programming.

64. How many types of self-referential recursive data are there in computer programs?

- a) 6

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- b) 2
- c) 10
- d) 4

Answer: b

Explanation: There are two types of self-referential definitions and these are inductive and conductive definitions. An inductively defined recursive data definition must have to specify how to construct instances of the data. For example, linked lists are defined as an inductively recursive data definition

65. How many even 4 digit whole numbers are there?

- a) 1358
- b) 7250
- c) 4500
- d) 3600

Answer: c

Explanation: The thousands digit cannot be zero, so there are 9 choices. There are 10 possibilities for the hundreds digit and 10 possibilities for the tens digit. The units digit can be 0, 2, 4, 6 or 8, so there are 5 choices. By the basic counting principle, the number of even five digit whole numbers is $9 \times 10 \times 10 \times 5 = 45,00$.

66. In a multiple-choice question paper of 15 questions, the answers can be A, B, C or D. The number of different ways of answering the question paper are _____

- a) 65536×4^7
- b) 194536×4^5
- c) 23650×4^9
- d) 11287435

Answer: a

Explanation: There are $4^{15} = 65536 \times 4^7$ different ways of answering the exam paper of 15 MCQs.

67. How many words with seven letters are there that start with a vowel and end with an A? Note that they don't have to be real words and letters can be repeated.

- a) 45087902
- b) 64387659
- c) 12765800
- d) 59406880

Answer: d

Explanation: The first letter must be a vowel, so there are 5 choices. By the basic counting principle, the number of 'words' is $5 \times 26 \times 26 \times 26 \times 26 \times 26 \times 1 = 59406880$.

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68. A drawer contains 12 red and 12 blue socks, all unmatched. A person takes socks out at random in the dark. How many socks must he take out to be sure that he has at least two blue socks?

- a) 18
- b) 35
- c) 28
- d) 14

Answer: d

Explanation: Given 12 red and 12 blue socks so, in order to take out at least 2 blue socks, first we need to take out 12 socks (which might end up red in worst case) and then take out 2 socks (which would be definitely blue). Thus we need to take out total 14 socks.

69. The least number of computers required to connect 10 computers to 5 routers to guarantee 5 computers can directly access 5 routers is

- a) 74
- b) 104
- c) 30
- d) 67

Answer: c

Explanation: Since each 5 computer need directly

connected with each router. So 25 connections + now remaining 5 computer, each connected to 5 different routers, so 5 connections = 30 connections. Hence,

c1→r1, r2, r3, r4, r5

c2→r1, r2, r3, r4, r5

c3→r1, r2, r3, r4, r5

c4→r1, r2, r3, r4, r5

c5→r1, r2, r3, r4, r5

c6→r1

c7→r2

c8→r3

c9→r4

c10→r5

Now, any pick of 5 computers will have a direct connection to all the 5 routers.

70. In a group of 267 people how many friends are there who have an identical number of friends in that group?

- a) 266
- b) 2
- c) 138
- d) 202

Answer: b

Explanation: Now, consider the numbers from 1 to n-1 as holes and the n members as pigeons. Since there is n-1 holes and n pigeons there must exist a hole which must contain more than one pigeon. So, in a group

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of n members there must exist at least two persons having equal number of friends. A similar case occurs when there exist a person having no friends.

71. How many substrings (of all lengths inclusive) can be formed from a character string of length 8? (Assume all characters to be distinct)
- a) 14
 - b) 21
 - c) 54
 - d) 37

Answer: d

Explanation: Total no of substrings possible in n length string (All length inclusive) = $1 + [n(n+1)/2] = 8(8+1)/2 = 37$.

72. The number of diagonals can be drawn in a hexagon is _____
- a) 9
 - b) 32
 - c) 16
 - d) 21

Answer: a

Explanation: A hexagon has 6 sides. We obtain the diagonals by joining the vertices in pairs. Total number of sides and

diagonals = ${}^6C_2 = 6*5/2*1 = 5*3 = 15$. This includes its 6 sides also. So, Diagonals = $15 - 6 = 9$. Hence, the number of diagonals is 9.

73. The number of binary strings of 17 zeros and 8 ones in which no two ones are adjacent is _____
- a) 43758
 - b) 24310
 - c) 32654
 - d) 29803

Answer: a

Explanation: First place 17 zeroes side by side $_0_0_0 \dots 0_$ and 8 1's can be placed in any of the $(17+1)$ available gaps hence the number of ways = ${}^{n+1}C_k = 43758$.

74. How many numbers of three digits can be formed with digits 1, 3, 5, 7 and 9?
- a) 983
 - b) 120
 - c) 345
 - d) 5430

Answer: b

Explanation: Here number of digits, $n = 5$ and number of places to be filled-up $r = 3$.

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Hence, the required number is ${}^5P_3 = 5! / 2! * 3! = 120$.

75. In how many ways 6 pens can be selected from 15 identical black pens?

- a) $9 * 3!$
- b) 21
- c) $14!$
- d) 1

Answer: d

Explanation: Here the pens are identical; the total number of ways of selecting 6 pens is 1.

76. Find the number of ways in which 4 people E, F, G, H, A, C can be seated at a round table, such that E and F must always sit together.

- a) 32
- b) 290
- c) 124
- d) 48

Answer: d

Explanation: E and F can sit together in all arrangements in $2!$ Ways. Now, the arrangement of the 5 people in a circle can be done in $(5 - 1)!$ or 24 ways. Therefore, the total number of ways will be $24 \times 2 = 48$.

77. There are 6 equally spaced points A, B, C, D, E and F marked on a circle with radius R. How many convex heptagons of distinctly different areas can be drawn using these points as vertices?

- a) $7! * 6$
- b) $7C5$
- c) $7!$
- d) same area

Answer: d

Explanation: Since all the points are equally spaced; hence the area of all the convex heptagons will be the same.

78. The number of words of 4 consonants and 3 vowels can be made from 15 consonants and 5 vowels, if all the letters are different is _____

- a) $3! * {}^{12}C_5$
- b) ${}^{16}C_4 * {}^4C_4$
- c) $15! * 4$
- d) ${}^{15}C_4 * {}^5C_3 * 7!$

Answer: d

Explanation: There are 4 consonants out of 15 can be selected in ${}^{15}C_4$ ways and 3

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vowels can be selected in ${}^5C_3$ ways. Therefore, the total number of groups each containing 4 consonants and 3 vowels = ${}^{15}C_4 * {}^4C_3$. Each group contains 7 letters which can be arranged in 7! Ways. Hence, required number of words = ${}^{15}C_4 * {}^5C_3 * 7!$

- c) 3387
- d) 4123

Answer: d

Explanation: The prime factorization of 1800 is $19 * 2^2 * 5^2$ and
 $S(2^2) = 1 + 2 + 4 = 7$
 $S(5^2) = 1 + 5 + 25 = 31$
Therefore, $S(1800) = 19 * 7 * 31 = 4123$.

79. There are six movie parts numbered from 1 to 6. Find the number of ways in which they be arranged so that part-1 and part-3 are never together.

- a) 876
- b) 480
- c) 654
- d) 237

Answer: b

Explanation: The total number of ways in which 6 part can be arranged = $6! = 720$. The total number of ways in which part-1 and part-3 are always together: = $5! * 2! = 240$. Therefore, the total number of arrangements, in which they are not together is = $720 - 240 = 480$.

80. Calculate sum of divisors of $n = 1900$.

- a) 6530
- b) 5346

81. The number of even positive integers of 3200 is _____

- a) 24
- b) 32
- c) 164
- d) 209

Answer: a

Explanation: To find the number of even factors, we can multiply the number of even factors by the power of 2. For 3200, we have $(5 + 1)(1 + 1)(2) = 24$ even factors.

82. What is the sum of divisors of the number 1872?

- a) 12493
- b) 5438
- c) 45862
- d) 654

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Answer: a

Explanation: The prime factorization of 1872 is $13 * 3^2 * 2^4$ and $S(2^4) = 1 + 2 + 4 + 8 + 16 = 31$, $S(5^2) = 1 + 5 + 25 = 31$. Therefore, $S(1872) = 31 * 31 * 13 = 12493$.

83. Calculate sum of divisors of $n = 8620$.

- a) 7549
- b) 54201
- c) 18102
- d) 654

Answer: c

Explanation: The prime factorization of 1800 is $431 * 2^2 * 5$ and $S(2^2) = 1 + 2 + 4 = 7$, $S(5^2) = 1 + 5 = 6$. Therefore, $S(1800) = 6 * 7 * 431 = 18102$.

84. Assume that it is an afternoon. What is the time on the 24 hour clock after 146 hours?

- a) 12:10 pm
- b) 8:30 am
- c) 3 am
- d) 2 pm

Answer: d

Explanation: Divide 146 with 24. The remainder is the time on the 24 hour clock. So, $146 = 6 * 24 + 2$ and the result is 2pm.

85. Suppose, there are 7 of your friends who want to eat pizza (8 distinct people in total). You order a 16-cut pizza (16 identical slices). How many distributions of pizza slices are there if each person gets at least one slice of pizza?

- a) 346
- b) 6435
- c) 3214
- d) 765

Answer: b

Explanation: This problem can be viewed as identical objects distributed into distinct non-empty bins. Using the formula for these kind of distributions ${}^{n-1}C_{r-1} = {}^{15}C_7 = 6435$. Thus, there are distributions of the pizza slices.

86. The linear system $Cx = d$ is known as _____ if $d \neq 0$.

- a) homogeneous
- b) heterogeneous
- c) non-homogeneous
- d) augmented system

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Answer: c

Explanation: A linear system $Cx = d$ is known as a homogeneous system if $d = 0$. The homogeneous linear system $Ax = 0$ is called its corresponding homogeneous linear system.

- 87.2. Every linear equation determines a _____ in n -dimensional space for n variables.
- a) shipshape
 - b) hyper-plane
 - c) cone
 - d) pyramid

Answer: b

Explanation: In an m -dimensional space, every linear equation produces a hyper-plane for n variables. The solution set is the intersection of these hyper-planes and is planar which may have any dimension smaller than m .

88. Determine all possibilities for the solution set of the homogeneous system of 5 equations in 3 unknowns and the rank of the system is 3.
- a) more than two
 - b) only one

- c) zero
- d) infinite

Answer: c

Explanation: Since the rank of this homogeneous system (which is always consistent) and the number of unknowns are equal, the only possible solution is zero and it is a unique solution.

89. Computational complexity of derangements is of _____
- a) NP-complete
 - b) NP-hard
 - c) NP
 - d) P

Answer: a

Explanation: Computational complexity of derangements is NP-complete in order to determine whether a given permutation group consists of any derangements or not.

90. Determine the number of ways In a single competition a singing couple from 5 boys and 5 girls can be formed so that no girl can sing a song with their respective boy?
- a) 123
 - b) 44

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- c) 320
d) 21

Answer: b

Explanation: This is a case of derangement of 5 boys and 5 girls. The required number of ways can be described as $D = 5! (1 - 1/1! + 1/2! - 1/3! + 1/4! - 1/5!) = 120(11/30) = 44$ ways.

91. A fair coin is tossed 15 times. Determine the probability in which no heads turned up.

- a) 2.549×10^{-3}
b) 0.976
c) 3.051×10^{-5}
d) 5.471

Answer: c

Explanation: According to the null hypothesis it is a fair coin and so in that case the probability of flipping at least 59% tails is $= {}^{15}C_0(0.5)^{15} = 3.051 \times 10^{-5}$.

92. Find the value of a_4 for the recurrence relation $a_n = 2a_{n-1} + 3$, with $a_0 = 6$.
- a) 320
b) 221
c) 141
d) 65

Answer: c

Explanation: When $n=1$, $a_1 = 2a_0 + 3$, Now $a_2 = 2a_1 + 3$. By substitution, we get $a_2 = 2(2a_0 + 3) + 3$. Regrouping the terms, we get $a_4 = 141$, where $a_0 = 6$.

93. How many positive integers less than or equal to 100 are divisible by 2, 4 or 5?

- a) 12.3
b) 87.2
c) 45.3
d) 78.2

Answer: d

Explanation: To count the number of integers $= 100/2 + 100/4 + 100/5 - 100/8 - 100/20 + 100/100$
 $= 50 + 25 + 20 - 12.8 - 5 + 1 = 78.2$.

94. How many binary relations are there on a set S with 9 distinct elements?

- a) 2^{90}
b) 2^{100}
c) 2^{81}
d) 2^{60}

Answer: c

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Explanation: S is the set with 9 elements. A relation on S is defined as $S \times S$. There are 9^2 numbers of ordered pairs in relation. So, the number of binary relations is $2^{(9 \times 9)} = 281$.

95. Synonym for binary relation is _____

- a) equivalence relation
- b) dyadic relation
- c) orthogonal relation
- d) one to many relations

Answer: b

Explanation: A binary relation on a set S is a set of ordered pairs of elements of S. It is a subset of the Cartesian product $S^2 = S \times S$. The terms correspondence, dyadic relation and 2-place relation are synonyms for the binary relation.

96. R is a binary relation on a set S and R is reflexive if and only if _____

- a) $r(R) = R$
- b) $s(R) = R$
- c) $t(R) = R$
- d) $f(R) = R$

Answer: a

Explanation: Let reflexive closure of R: $r(R) = R$. If R is reflexive, it satisfies all the condition in the definition of reflexive closure. So, a reflexive closure of a relation is the smallest number of reflexive relation contain in R. Hence, $R = r(R)$.

97. Amongst the properties {reflexivity, symmetry, anti-symmetry, transitivity} the relation $R = \{(a, b) \in N^2 \mid a \neq b\}$ satisfies _____ property.

- a) symmetry
- b) transitivity
- c) anti-symmetry
- d) reflexivity

Answer: a

Explanation: It is not reflexive as aRa is not possible. It is symmetric as if aRb then bRa . It is not anti-symmetric as aRb and bRa are possible and we can have $a \neq b$. It is not transitive as if aRb and bRc then aRc need not be true. This is violated when $c = a$. So the answer is symmetry property.

98. The binary relation $U = \Phi$ (empty set) on a set $A = \{11, 23, 35\}$ is _____

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- a) Neither reflexive nor symmetric
- b) Symmetric and reflexive
- c) Transitive and reflexive
- d) Transitive and symmetric

Answer: d

Explanation: $U = \Phi$ (empty set) on a set $A = \{11, 23, 35\}$ need to be hold Ir-reflexive, symmetric, anti-symmetric, asymmetric and transitive closure property, but it is not Reflexive as it does not contain any self loop in itself.

99. The binary relation $\{(1,1), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2)\}$ on the set $\{1, 2, 3\}$ is

- a) reflective, symmetric and transitive
- b) ir-reflexive, symmetric and transitive
- c) neither reflective, nor ir-reflexive but transitive
- d) ir-reflexive and anti-symmetric

Answer: c

Explanation: Not reflexive $\rightarrow (3,3)$ not present; not ir-reflexive $\rightarrow (1, 1)$ is present; not symmetric $\rightarrow (2, 1)$ is present but not $(1, 2)$; not anti-

symmetric $\rightarrow (2, 3)$ and $(3, 2)$ are present; not asymmetric $\rightarrow$ asymmetry requires both anti-symmetry and ir-reflexivity. So, it is transitive closure of relation.

100. Consider the relation: $R' (x, y)$ if and only if $x, y > 0$ over the set of non-zero rational numbers, then R' is _____

- a) not equivalence relation
- b) an equivalence relation
- c) transitive and asymmetry relation
- d) reflexive and anti-symmetric relation

Answer: b

Explanation: Reflexive: $a, a > 0$
Symmetric: if $a, b > 0$ then both must be +ve or -ve, which means $b, a > 0$ also exists
Transitive: if $a, b > 0$ and $b, c > 0$ then to have b as same number, both pairs must be positive or negative which implies $a, c > 0$. Hence, R' is an equivalence relation.

101. Consider the binary relation, $A = \{(a, b) \mid b = a - 1 \text{ and } a, b \text{ belong to } \{1, 2, 3\}\}$. The reflexive transitive closure of A is?
- a) $\{(a, b) \mid a \geq b \text{ and } a, b \text{ belong to } \{1, 2, 3\}\}$

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- b) $\{(a, b) \mid a > b \text{ and } a, b \text{ belong to } \{1, 2, 3\}\}$
 c) $\{(a, b) \mid a \leq b \text{ and } a, b \text{ belong to } \{1, 2, 3\}\}$
 d) $\{(a, b) \mid a = b \text{ and } a, b \text{ belong to } \{1, 2, 3\}\}$

Answer: a

Explanation: By definition of Transitive closure we have that a is related to all smaller b (as every a is related to $b - 1$) and from the reflexive property a is related to a.

102. The less-than relation, $<$, on a set of real numbers is _____
 a) not a partial ordering because it is not asymmetric and ir-reflexive equals anti-symmetric
 b) a partial ordering since it is asymmetric and reflexive
 c) a partial ordering since it is anti-symmetric and reflexive
 d) not a partial ordering because it is not anti-symmetric and reflexive

Answer: a

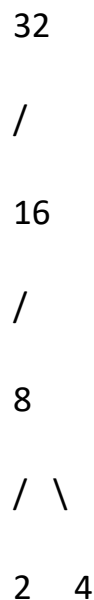
Explanation: Relation less than a set of real numbers is not anti-symmetric and reflexive. Relation is not POSET because it is ir-reflexive. Again, $a R b \neq$

$b R a$ unless $a=b$ and so it is anti-symmetric. A relation may be 'not asymmetric and not reflexive but still anti-symmetric, as $\{(1, 1) (1, 2)\}$. So, the relation is not a partial ordering because it is not asymmetric and ir-reflexive equals anti-symmetric.

103. Let a set $S = \{2, 4, 8, 16, 32\}$ and $\leq$ be the partial order defined by $S \leq R$ if a divides b. Number of edges in the Hasse diagram of is _____
 a) 6
 b) 5
 c) 9
 d) 4

Answer: b

Explanation: Hass Diagram is:



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104. Suppose $X = \{a, b, c, d\}$ and π_1 is the partition of X , $\pi_1 = \{\{a, b, c\}, d\}$. The number of ordered pairs of the equivalence relations induced by _____
- a) 15
 - b) 10
 - c) 34
 - d) 5

Answer: b

Explanation: The ordered pairs of the equivalence relations induced = $\{(a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b), (c, c), (d, d)\}$. Po-set $\rightarrow$ equivalence relations = each partition power set $-\Phi$.

105. Suppose a relation $R = \{(3, 3), (5, 5), (5, 3), (5, 5), (6, 6)\}$ on $S = \{3, 5, 6\}$. Here R is known as _____
- a) equivalence relation
 - b) reflexive relation
 - c) symmetric relation
 - d) transitive relation

Answer: a

Explanation: Here, $[3] = \{3, 5\}$, $[5] = \{3, 5\}$, $[6] = \{6\}$. We can see that $[3] = [5]$ and that S/R will be $\{[3], [6]\}$ which is a partition of S . Thus, we can

choose either $\{3, 6\}$ or $\{5, 6\}$ as a set of representatives of the equivalence classes.

106. Consider the congruence $45 \equiv 3 \pmod{7}$. Find the set of equivalence class representatives.
- a) $\{\dots, 0, 7, 14, 28, \dots\}$
 - b) $\{\dots, -3, 0, 6, 21, \dots\}$
 - c) $\{\dots, 0, 4, 8, 16, \dots\}$
 - d) $\{\dots, 3, 8, 15, 21, \dots\}$

Answer: a

Explanation: Now, for integers n , a and b , we have congruence $a \equiv b \pmod{n}$, then the set of equivalence classes are $\{\dots, -2n, -n, 0, n, 2n, \dots\}$, $\{\dots, 1-2n, 1-n, 1, 1+n, 1+2n, \dots\}$. The required answer is $\{\dots, 0, 7, 14, 28, \dots\}$.

107. Which of the following relations is the reflexive relation over the set $\{1, 2, 3, 4\}$?
- a) $\{(0,0), (1,1), (2,2), (2,3)\}$
 - b) $\{(1,1), (1,2), (2,2), (3,3), (4,3), (4,4)\}$
 - c) $\{(1,1), (1,2), (2,1), (2,3), (3,4)\}$
 - d) $\{(0,1), (1,1), (2,3), (2,2), (3,4), (3,1)\}$

Answer: b

Explanation: $\{(1,1), (1,2), (2,2), (3,3), (4,3), (4,4)\}$ is a reflexive

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relation because it contains set
 $= \{(1,1), (2,2), (3,3), (4,4)\}$.

108. A directed graph or digraph can have directed cycle in which _____

- a) starting node and ending node are different
- b) starting node and ending node are same
- c) minimum four vertices can be there
- d) ending node does not exist

Answer: b

Explanation: If the start node and end node are same in the path of a graph then it is termed as directed cycle i.e., $c_0 = c_n$. For instance, a c b a is a simple cycle in which start and end nodes are same (a). But, a c b b a is not a simple cycle as there is a loop $\langle b, b \rangle$.

109. Let, $D = \langle A, R \rangle$ be a directed graph or di-graph, then $D' = \langle A', R' \rangle$ is a sub-graph if _____

- a) $A' \subset A$ and $R' = R \cap (A' \times A')$
- b) $A' \subset A$ and $R \subset R' \cap (A' \times A')$
- c) $R' = R \cap (A' \times A')$
- d) $A' \subseteq A$ and $R \subseteq R' \cap (A' \times A')$

Answer: a

Explanation: The relation R is called the incidence relation on D. Now, a digraph is a sub-graph of D if

- i) $A' \subset A$
- ii) $R' = R \cap (A' \times A')$. If $D' \subset D$, D' is a proper sub-graph of D.

110. The graph representing universal relation is called _____

- a) complete digraph
- b) partial digraph
- c) empty graph
- d) partial sub-graph

Answer: a

Explanation: Consider, A is a graph with vertices $\{a, b, c, d\}$ and the universal relation is $A \times A$. The graph representing universal relation is called a complete graph and all ordered pairs are present there.

111. Hass diagrams are first made by _____

- a) A.R. Hass
- b) Helmut Hass
- c) Dennis Hass
- d) T.P. Hass

Answer: b

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Explanation: Hass diagrams can be described as the transitive reduction as an abstract directed acyclic graph. This graph drawing techniques are constructed by Helmut Hass (1948).

112. If a partial order is drawn as a Hass diagram in which no two edges cross, its covering graph is called _____
- a) upward planar
 - b) downward planar
 - c) lattice
 - d) bi-connected components

Answer: a

Explanation: In a Hass diagram if no two edges cross each other in the drawing of partial order Hass diagram, then its covering graph called the upward planar.

113. If the partial order of a set has at most one minimal element, then to test whether it has a non-crossing Hass diagram its time complexity _____
- a) NP-complete
 - b) $O(n^2)$
 - c) $O(n+2)$
 - d) $O(n^3)$

Answer: a

Explanation: If the partial order has at most one minimal element, or it has at most one maximal element, then to test whether a partial order with multiple sources and sinks can be drawn as a crossing-free Hass diagram or not it's time complexity is NP-complete.

114. A Po-set in which every pair of elements has both a least upper bound and a greatest lower bound is termed as _____
- a) sub-lattice
 - b) lattice
 - c) trail
 - d) walk

Answer: b

Explanation: A Po-set in which every pair of elements has both a least upper bound and a greatest lower bound is called a lattice. A lattice can contain sub-lattices which are subsets of that lattice.

115. In the Po-set $(Z^+, |)$ (where Z^+ is the set of all positive integers and $|$ is the divides relation) are the integers 9 and 351 comparable?
- a) comparable

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- b) not comparable
- c) comparable but not determined
- d) determined but not comparable

Answer: a

Explanation: The two integers 9 and 351 are comparable since $9 \mid 351$ i.e., 9 divides 351. But 5 and 127 are not comparable since $5 \nmid 127$ i.e. 5 does not divide 127.

116. If every two elements of a Po-set are comparable then the Po-set is called _____
- a) sub ordered Po-set
 - b) totally ordered Po-set
 - c) sub lattice
 - d) semi-group

Answer: b

Explanation: A Po-set $(P, \leq)$ is known as totally ordered if every two elements of the Po-set are comparable. " $\leq$ " is called a total order and a totally ordered set is also termed as a chain.

117. The maximum number of edges in a bipartite graph on 14 vertices is _____
- a) 56
 - b) 14

- c) 49
- d) 87

Answer: c

Explanation: Number of edges in a complete bipartite graph is $a*b$, where a and b are no. of vertices on each side. This quantity is maximum when $a = b$ i.e. when there are 7 vertices on each side. So answer is $7 * 7 = 49$.

118. In a _____ the degree of each and every vertex is equal.
- a) regular graph
 - b) point graph
 - c) star graph
 - d) Euler graph

Answer: c

Explanation: A regular graph has the same degree in each of its vertices. In a regular bipartite graph, if the common degree of each vertex is 1, the two parts are of the same size.

119. The time complexity to test whether a graph is bipartite or not is said to be _____ using depth first search.
- a) $O(n^3)$
 - b) linear time
 - c) $O(1)$
 - d) $O(n \log n)$

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Answer: b

120. In a 7-node directed cyclic graph, the number of Hamiltonian cycle is to be

- a) 728
- b) 450
- c) 360
- d) 260

Answer: c

Explanation: In an n -complete graph, there is $(n-1)! / 2$ Hamiltonian cycles and so the answer is 360. $(7-1)! / 2$

121. If each and every vertex in G has degree at most 23 then G can have a vertex coloring of

- a) 24
- b) 23
- c) 176
- d) 54

Answer: a

Explanation: A vertex coloring of a graph $G = (V', E')$ with m colors is a mapping $f: V' \rightarrow \{1, m\}$ such that $f(u) \neq f(v)$ for every (u, v) belongs to E' . Since in worst case the graph can be complete, $d+1$ color are necessary for graph containing vertices with degree at most's'. So, the required answer is 24.

122. Triangle free graphs have the property of clique number is

- a) less than 2
- b) equal to 2
- c) greater than 3
- d) more than 10

Answer: d

Explanation: In an undirected triangle-free graph no three vertices can form a triangle of edges. It can be described as graphs with clique number less than 2 and the graphs with girth greater than 4.

123. A bridge cannot be a part of

- a) a simple cycle
- b) a tree
- c) a clique with size ≥ 3 whose every edge is a bridge
- d) a graph which contains cycles

Answer: a

Explanation: In a connected graph, a bridge is an edge whose removal disconnects the graph. In a cycle if we remove an edge, it will still be connected. So, bridge cannot be part of a cycle. A clique is any complete sub-graph of a graph.

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124. Any subset of edges that connects all the vertices and has minimum total weight, if all the edge weights of an undirected graph are positive is called _____

- a) sub-graph
- b) tree
- c) Hamiltonian cycle
- d) grid

Answer: b

Explanation: If all the edge weights of an undirected graph are positive, any subset of edges that connects all the vertices and has minimum total weight is termed as a tree. In this case, we need to have a minimum spanning tree need to be exact.

125. G is a simple undirected graph and some vertices of G are of odd degree. Add a node n to G and make it adjacent to each odd degree vertex of G . The resultant graph is _____

- a) Complete bipartite graph
- b) Hamiltonian cycle
- c) Regular graph
- d) Euler graph

Answer: d

126. A graph which has the same number of edges as its

complement must have number of vertices congruent to _____ or _____ modulo 4 (for integral values of number of edges).

- a) $6k, 6k-1$
- b) $4k, 4k+1$
- c) $k, k+2$
- d) $2k+1, k$

Answer: c

Explanation: By using invariant of isomorphism and property of edges of graph and its complement.

127. Every Isomorphic graph must have _____ representation.

- a) cyclic
- b) adjacency list
- c) tree
- d) adjacency matrix

Answer: d

Explanation: Two graphs G_1 and G_2 are said to be isomorphic if $\rightarrow$ 1) their number of components (vertices and edges) are same and 2) their edge connectivity is retained. Isomorphic graphs must have adjacency matrix representation.

128. A cycle on n vertices is isomorphic to its complement. What is the value of n ?

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- a) 5
- b) 32
- c) 17
- d) 8

Answer: a

Explanation: A cycle with n vertices has n edges. Number of edges in cycle = n and number of edges in its complement = $(n*(n-1)/2) - n$.

129. Which algorithm efficiently calculates the single source shortest paths in a Directed Acyclic Graph?

- a) topological sort
- b) hash table
- c) binary search
- d) radix sort

Answer: a

Explanation: For Directed Acyclic graph, single source shortest distances can be calculated in $O(V+E)$ time. Topological Sorting of any graph represents a linear ordering of the graph.

130. The _____ of a graph G consists of all vertices and edges of G .

- a) edge graph
- b) line graph
- c) path complement graph

- d) Eulerian circuit

Answer: d

Explanation: we know that the Eulerian circuit in a graph G is a circuit that includes all vertices and edges of G . A graph that can have Eulerian circuit also can have a Eulerian graph.

131. A _____ in a graph G is a circuit which consists of every vertex (except first/last vertex) of G exactly once.

- a) Euler path
- b) Hamiltonian path
- c) Planar graph
- d) Path complement graph

Answer: b

Explanation: A Hamiltonian path is a walk that contains every vertex of the graph exactly once. Hence, a Hamiltonian path is not a circuit.

132. The chromatic number of a graph is the property of _____

- a) graph coloring
- b) graph ordering
- c) group ordering
- d) group coloring

Answer: b

Explanation: A graph coloring

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is an assignment of labels to the vertices of a graph such that no two adjacent vertices share the same labels is called the colors of the graph. Now, the chromatic number of any graph is the minimal number of colors for which such an assignment is possible.

133. If a graph G is k -colorable and $k < n$, for any integer n then it is

- a) n -colorable
- b) n^2 nodes
- c) $(k + n)$ -colorable
- d) $(k^3 + n^3 + 1)$ nodes

Answer: a

Explanation: The chromatic number of a graph is the minimal number of colors for which a graph coloring is possible. A graph G is termed as k -colorable if there exists a graph coloring on G with k colors. If a graph is k -colorable, then it is n -colorable for any $n > k$.

134. If C_n is the n th cyclic graph, where $n > 3$ and n is odd. Determine the value of $X(C_n)$.
- a) 32572
 - b) 16631
 - c) 3

- d) 310

Answer: c

Explanation: Here n is odd and $X(C_n) = 2$. Since there are two adjacent edges in C_n . Now, a graph coloring for C_n exists where vertices are colored red and blue alternatively and another edge is with a different colour say orange, then the value of $X(C_n)$ becomes 3.

135. A direct product of a group G possesses which of the following characteristics?

- a) a multiplication of subgroups of G
- b) a factorization via subgroups of G
- c) a superset of subgroups of G
- d) a maximal power set of subgroups

Answer: b

Explanation: A direct product of a group G is a factorization via subgroups of G when the intersection is nontrivial, say X and Y , such that $G = XY$, X intersect $Y = 1$, and $[X, Y] = 1$ and X, Y are normal in G .

136. In invariant algebra, some generators of group G that goes

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either into itself or zero under _____ with any other element of the algebra.

- a) commutation
- b) permutation
- c) combination
- d) lattice

Answer: a

Explanation: Some generators of group G in group theory which goes either into itself or zero under commutation with any other element of the whole algebra is called invariant sub-algebra.

137. Which of the following can be embedded in an algebraically closed group?
- a) infinite group
 - b) star-graph
 - c) a countable group
 - d) a semi-lattice

Answer: c

Explanation: We know that any countable group can always be embedded in an algebraically closed group.

138. An undirected graph G which is connected and acyclic is called _____
- a) bipartite graph
 - b) cyclic graph
 - c) tree

d) forest

Answer: c

Explanation: An undirected graph G which is connected and acyclic is termed as a tree. G contains no cycles and if any edge is added to G a simple cycle is formed.

139. An n -vertex graph has _____ edges.
- a) n^2
 - b) $n-1$
 - c) $n*n$
 - d) $n*(n+1)/2$

Answer: b

Explanation: Suppose G is a connected graph which has no cycles. Every sub-graph of G includes at least one vertex with zero or one incident edges. It has n vertices and $n-1$ edges. Generally, the order-zero graph is not considered to be a tree.

140. What is a star tree?
- a) A tree having a single internal vertex and $n-1$ leaves
 - b) A tree having n vertices arranged in a line
 - c) A tree which has 0 or more connected sub-trees
 - d) A tree which contains n

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vertices and $n-1$ cycles

Answer: a

Explanation: A star tree of order n is a tree with as many leaves as possible or in other words a star tree is a tree that consists of a single internal vertex and $n-1$ leaves. However, an internal vertex is a vertex of degree at least 2.

141. If two cycle graphs G_m and G_n are joined together with a vertex, the number of spanning trees in the new graph is _____
- a) $m+n-1$
 - b) $m-n$
 - c) $m*n$
 - d) $m*n+1$

Answer: c

Explanation: As there are n possible edges to be removed from G and m edges to be removed from G and the rest from a spanning tree so the number of spanning tree in the new graph is $m*n$.

142. For an n -vertex undirected graph, the time required to find a cycle is _____
- a) $O(n)$
 - b) $O(n^2)$

- c) $O(n+1)$
- d) $O(\log n)$

Answer: a

Explanation: In an undirected graph, finding any already visited vertex will indicate a back edge. All the back edges which DFS skips over are part of cycles. In the case of undirected graphs, only $O(n)$ time is required to find a cycle in an n -vertex graph, since at most $n - 1$ edges can be tree edges.

143. A binary cycle space forms a _____ over the two element field.
- a) triangular graph
 - b) vector space
 - c) binary tree
 - d) Hamiltonian graph

View Answer

Answer: b

Explanation: The term cycle refers to an element of the cycle space of a graph. There are many cycle spaces. The most common is the binary cycle space, which contains the edge sets that have even degrees at every vertex and it forms a vector space over the two-element field.

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144. In preorder traversal of a binary tree the second step is _____

- a) traverse the right sub-tree
- b) traverse the left sub-tree
- c) traverse right sub-tree and visit the root
- d) visit the root

Answer: b

Explanation: In a preorder traversal of a binary tree first is to visit the root, second traverse the left sub-tree of the tree and third traverse the right sub-tree of the tree.

145. An important application of binary tree is _____

- a) Huffman coding
- b) stack implementation
- c) queue implementation
- d) traverse a cyclic graph

Answer: a

Explanation: A binary tree is used to sort a list of elements; the in-order traversal will do this automatically. Better tree sorting algorithm will involve balancing the trees. The binary coding, in particular for the Huffman coding is an immediate application of binary trees.

146. From the following code identify the which traversal of a binary tree is this _____

```
//if node has left child  
Order (node. left)  
//if node has right child  
Order (node. right)  
Visit (node)
```

- a) In-order traversal
- b) preorder traversal
- c) post-order traversal
- d) Euler tour traversal

Answer: c

Explanation: In a post-order traversal of a binary tree first is to traverse the left sub-tree, second traverse the right sub-tree of the tree and third is to visit the node.

147. Evaluation of expression $a/b+c*d-e$ in postfix notation.

- a) $ab+cd/*-e$
- b) $a\ b/c\ d*+e-$
- c) $a\ b\ c/+d*-e$
- d) $a\ b\ c\ d/+*-e$

Answer: b

Explanation: The expression= $a/b+c*d-e$
 $=\{(a\ b/)+(c\ d*)\}-e$
 $=\{(a\ b/)(c\ d*)+\}-e$

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$=\{(a\ b/)(\ c\ d^*)+\}e-$

So the output is: $a\ b/c\ d^*+e-$

148. Evaluation of $4*5+3/2-9$ in prefix notation.

- a) $*45-/32+9$
- b) $*+453/-29$
- c) $-+*45/329$
- d) $*+/45932$

Answer: c

Explanation: The

expression $=4*5+3/2-9$

$=\{(4*5)+(3/2)-9\}$

$=\{(*45)+(/32)-9\}$

$=\{+(*45)(/32)\}-9$

$=-\{+(*45)(/32)9\}$

So the output is; $-+*45/329$.

149. Infix to prefix conversion can be done using _____

- a) two queues
- b) two stacks
- c) one stack and two queues
- d) one stack

Answer: b

Explanation: In the infix expression, the operator appears between the operands and in infix notation if the operator appears before the operands in the expression. For the conversion between them two stacks are used efficiently. The

idea is to use one stack for operators and other to store operands.

150. Spanning trees have a special class of depth-first search trees named _____

- a) Euclidean minimum spanning trees
- b) Tremaux trees
- c) Complete bipartite graphs
- d) Decision trees

Answer: b

Explanation: A tremaux tree of an undirected graph G is a spanning tree of G which is rooted at one of its vertices with the property that every two adjacent vertices in G are related to each other as an ancestor and descendant in the tree.

151. If the weight of an edge e of cycle C in a graph is larger than the individual weights of all other edges of C , then that edge _____

- a) belongs to an minimum spanning tree
- b) cannot belong to an minimum spanning tree
- c) belongs to all MSTs of the graph
- d) can not belong to the graph

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Answer: b

Explanation: For any cycle C in the graph, if the weight of an edge e of C is larger than the individual weights of all other edges of C, then this edge cannot belong to an MST.

152. For every spanning tree with n vertices and n edges what is the least number of different Spanning trees can be formed?
- a) 2
 - b) 5
 - c) 3
 - d) 4

Answer: c

Explanation: The minimum cycle length can be 3. So, there must be at least 3 spanning trees in any such Graph. Consider a Graph with $n = 4$, then 3 spanning trees possible at maximum (removing edges of cycle one at a time, alternatively). So, any Graph with minimum cycle length '3' will have at least 3 spanning trees.

153. Algebra of logic is termed as

- a) Numerical logic
- b) Boolean algebra

- c) Arithmetic logic
- d) Boolean number

Answer: c

Explanation: The variables that can have two discrete values False (0) and True (1) and the operations of logical significance are dealt with Boolean algebra.

154. Boolean algebra can be used

- a) For designing of the digital computers
- b) In building logic symbols
- c) Circuit theory
- d) Building algebraic functions

Answer: a

Explanation: For designing digital computers and building different electronic circuits Boolean algebra is accepted widely.

155. What is the definition of Boolean functions?
- a) An arithmetic function with k degrees such that $f:Y \rightarrow Y^k$
 - b) A special mathematical function with n degrees such that $f:Y^n \rightarrow Y$
 - c) An algebraic function with n degrees such that $f:X^n \rightarrow X$
 - d) A polynomial function with k

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degrees such that $f: X^2 \rightarrow X^n$

Answer: b

Explanation: A Boolean function is a special mathematical function with n degrees and where $Y = \{0, 1\}$ is the Boolean domain with being a non-negative integer. It helps in describing the way in which the Boolean output is derived from Boolean inputs.

156. What is the use of Boolean identities?

- a) Minimizing the Boolean expression
- b) Maximizing the Boolean expression
- c) To evaluate a logical identity
- d) Searching of an algebraic expression

Answer: a

Explanation: Boolean identities are used for minimizing the Boolean expression and transforming into an equivalent expression.

157. A _____ is a Boolean variable.

- a) Literal
- b) String
- c) Keyword

d) Identifier

Answer: a

Explanation: A literal is a Boolean variable or its complement. A max-term is a sum of n literals and a min-term is a product of n literals.

158. Minimization of function $F(A,B,C) = A * B * (B+C)$ is

- a) AC
- b) $B+C$
- c) B'
- d) AB

Answer: d

Explanation: $AB(B+C)$
 $= ABB + ABC$ [Applying distributive rule]
 $= AB + ABC$ [Applying Idempotent law]
 $= AB(1+C)$
 $= AB * 1$ [As, $1+C=1$]
 $= AB$.

159. Find the simplified expression $A'BC' + AC'$.

- a) B
- b) $A+C$
- c) $(A+B)C'$
- d) $B'C$

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Answer: c

Explanation: Given: $A'BC' + AC'$
 $= C' (A'B + A)$
 $= C' (A + B).$

160. Evaluate the expression: $(X + Z)(X + XZ') + XY + Y.$

- a) $XY + Z'$
- b) $Y + XZ' + Y'Z$
- c) $X'Z + Y$
- d) $X + Y$

Answer: d

Explanation: $(X + Z)(X + XZ') + XY + Y$ [Original Expression]
 $= (x + z)X(1 + Z') + XY + Y$ [Distributive]
 $= (X + Z)X + XY + Y$ [Complement, Identity]
 $= (X+Z)X + Y(X+1)$ [Distributive]
 $= (X+Z)X + Y$ [Idempotent]
 $= XX + XZ + Y$ [Distributive]
 $= X + XZ + Y$ [Identity]
 $= X(1+Z) + Y$
 $= X + Y$ [Idempotent].

161. Simplify the expression: $A'(A + BC) + (AC + B'C).$

- a) $(AB'C + BC')$
- b) $(A'B + C')$
- c) $(A + BC)$
- d) AC

Answer: b

Explanation: Given: $A'(A + BC) + (AC + B'C)$
 $= A'A + A'BC + AC + B'C$
 $= A'BC + C(A + B')$
 $= C(A'B + A + B')$
 $= C(A + B + B')$
 $= C(A + 1)$
 $= AC.$

162. K-map is used for _____

- a) logic minimization
- b) expression maximization
- c) summing of parity bits
- d) logic gate creation

Answer: a

Explanation: K-map (Maurice Karnaugh of Bell labs in 1953) It is more efficient than Boolean algebra. K-map is a diagram made up of squares in which each square represents a min-term or max-term of the logic function.

163. To display time in railway stations which digital circuit is used?

- a) seven segment decoder
- b) eight segment encoder
- c) 8:3 multiplexer
- d) 9 bit segment driver

Answer: a

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Explanation: A seven segment decoder is a digital circuit which is used to construct a common type of digital display device.

164. Simplify the expression using

K-maps: $F(A,B,C,D)=\Sigma$

(1,3,5,6,7,11,13,14).

- a) $AB+BC'D+A'B'C$
- b) $BCD'+A'C'D+BD'$
- c) $A'D+BCD+A'BC+AB'C'$
- d) $AC'D'+BC+A'BD+C'D'$

Answer: c

Explanation: By solving the given expression we have min-terms such as $A'D+BCD+A'BC+AB'C'$.

So, we can get the required expression

$A'D+BCD+A'BC+AB'C'$.

165. In Boolean algebra, the data is a bit-representation of information consists of

- a) 0 and 1
- b) 2 and 5
- c) 1 and 15
- d) 4 and 8

Answer: a

Explanation: The data, in Boolean algebra must be in a bit-representation form which can

be in between two values 0 and 1.

166. How many cells are there for an 8-variable K-Map?

- a) 421
- b) 1048
- c) 256
- d) 375

Answer: c

Explanation: Any Boolean expression or a function comprising of 8 variables can be solved using an 8-variable K-Map. So, an 8-variable K-Map must contain $2^8 = 256$.

167. Determine the number of essential prime implicants of the function $f(a, b, c, d) = \Sigma m(1, 3, 4, 8, 10, 13) + d(2, 5, 7, 12)$, where m denote the min-term and d denotes the don't care condition.

- a) 2^3
- b) 3
- c) 643
- d) 128

Answer: b

Explanation: A prime implicit that cannot be replaced by any other implicit for getting the output is called the essential

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prime implicants. Here, we have 3 essential prime implicants by using the K-map representation.

168. How many number of prime implicants are there in the expression $F(x, y, z) = y' z' + x y + x' z$.
- a) 7
 - b) 19
 - c) 3
 - d) 53

Answer: c

Explanation: An implicit of a function is a product term which is included in the function. Hence, for the given function, $y' z'$, $x y$ and $x' z$ all are prime implicants.

169. A non empty set A is termed as an algebraic structure _____
- a) with respect to binary operation *
 - b) with respect to ternary operation?
 - c) with respect to binary operation +
 - d) with respect to unary operation –

Answer: a

Explanation: A non empty set A is called an algebraic structure

w. r. t binary operation “*” if $(a*b)$ belongs to S for all $(a*b)$ belongs to S. Therefore “*” is closure operation on ‘A’.

170. An algebraic structure _____ is called a semi-group.
- a) $(P, *)$
 - b) $(Q, +, *)$
 - c) $(P, +)$
 - d) $(+, *)$

Answer: a

Explanation: An algebraic structure $(P, *)$ is called a semi-group if $a*(b*c) = (a*b)*c$ for all a, b, c belongs to S or the elements follow associative property under “*”. (Matrix, *) and (Set of integers, +) are examples of semi-group.

171. Condition for monoid is _____
- a) $(a + e) = a$
 - b) $(a*e) = (a + e)$
 - c) $a = (a*(a + e))$
 - d) $(a*e) = (e*a) = a$

Answer: d

Explanation: A Semi-group $(S, *)$ is defined as a monoid if there exists an element e in S such that $(a*e) = (e*a) = a$ for all a in S.

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This element is called identity element of S w. r. t $*$.

172. How many properties can be held by a group?
- a) 2
 - b) 3
 - c) 5
 - d) 4

Answer: c

Explanation: A group holds five properties simultaneously –

- i) Closure
- ii) associative
- iii) Commutative
- iv) Identity element
- v) Inverse element.

173. _____ are called group postulates.
- a) Group lemmas
 - b) Group theories
 - c) Group axioms
 - d) Group

Answer: c

Explanation: The group axioms are also called the group postulates. A group with an identity (that is, a monoid) in which every element has an inverse is termed as semi group.

174. A subgroup has the properties of _____
- a) Closure, associative
 - b) Commutative, associative, closure
 - c) Inverse, identity, associative
 - d) Closure, associative, Identity, Inverse

Answer: d

Explanation: A subgroup S is a subset of a group G (denoted by $S \leq G$) if it holds the four properties simultaneously – Closure, Associative, Identity and Inverse element.

175. If $a * b = a$ such that $a * (b * c) = a * b = a$ and $(a * b) * c = a * b = a$ then _____
- a) $*$ is associative
 - b) $*$ is commutative
 - c) $*$ is closure
 - d) $*$ is aphelion

Answer: a

Explanation: ' $*$ ' can be defined by the formula $a * b = a$ for any a and b in S . Hence, $(a * b) * c = *c = a$ and $a * (b * c) = a * b = a$. Therefore, " $*$ " is associative. Hence $(S, *)$ is a semi-group. On the contrary, $*$ is not associative since, for example, $(b * c) * c = a$

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• $c = c$ but $b \bullet (c \bullet c) = b \bullet a = b$
Thus $(S, \bullet)$ is not a semi-group.

176. Let $(A_7, \otimes_7) = (\{1, 2, 3, 4, 5, 6\}, \otimes_7)$ is a group. It has two subgroups X and Y . $X = \{1, 3, 6\}$, $Y = \{2, 3, 5\}$. What is the order of union of subgroups?
- a) 65
 - b) 5
 - c) 32
 - d) 18

Answer: b

Explanation: Given, $(A_7, \otimes_7) = (\{1, 2, 3, 4, 5, 6\}, \otimes_7)$ and the union of two sub groups X and Y , $X = \{1, 3, 6\}$ $Y = \{2, 3, 5\}$ is $X \cup Y = \{1, 2, 3, 5, 6\}$ i.e., 5. Here, the order of the union cannot be divided by order of the group.

177. A relation $(34 \times 78) \times 57 = 57 \times (78 \times 34)$ can have _____ property.
- a) distributive
 - b) associative
 - c) commutative
 - d) closure

Answer: b

Explanation: For any three elements (numbers) a , b and c associative property describes a $\times (b \times c) = (a \times b) \times c$ [for

multiplication]. Hence associative property is true for multiplication and it is true for multiplication also.

178. If group G has 65 elements and it has two subgroups namely K and L with order 14 and 30. What can be order of K intersection L ?
- a) 10
 - b) 42
 - c) 5
 - d) 35

Answer: c

Explanation: As it is an intersection so the order must divide both K and L . Here 3, 6, 30 does not divide 14. But 5 must be the order of the group as it divides the order of intersection of K and L as well as the order of the group.

179. In a group there must be only _____ element.
- a) 1
 - b) 2
 - c) 3
 - d) 5

Answer: a

Explanation: There can be only one identity element in a group

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and each element in a group has exactly one inverse element. Hence, two important consequences of the group axioms are the uniqueness of the identity element and the uniqueness of inverse elements.

inverse. For an element, a in a group G , an inverse of a is an element b such that $a \cdot b = e$, where e is the identity in the group. The inverse of an element is unique and usually denoted as $-a$.

180. _____ is the multiplicative identity of natural numbers.

- a) 0
- b) -1
- c) 1
- d) 2

Answer: c

Explanation: 1 is the multiplicative identity of natural numbers as $a \cdot 1 = a = 1 \cdot a \forall a \in \mathbb{N}$. Thus, 1 is the identity of multiplication for the set of integers ($\mathbb{Z}$), set of rational numbers ($\mathbb{Q}$), and set of real numbers ($\mathbb{R}$).

182. A trivial subgroup consists of _____

- a) Identity element
- b) Co-set
- c) Inverse element
- d) Ring

Answer: a

Explanation: Let G be a group under a binary operation $*$ and a subset H of G is called a subgroup of G if H forms a group under the operation $*$. The trivial subgroup of any group is the subgroup consisting of only the Identity element.

181. An identity element of a group has _____ element.

- a) associative
- b) commutative
- c) inverse
- d) Homomorphism

Answer: c

Explanation: By the definition of all elements of a group have an

183. Minimum subgroup of a group is called _____

- a) a commutative subgroup
- b) a lattice
- c) a trivial group
- d) a monoid

Answer: c

Explanation: The subgroups of any given group form a complete

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lattice under inclusion termed as a lattice of subgroups. If o is the Identity element of a group (G) , then the trivial group $\{o\}$ is the minimum subgroup of that group and G is the maximum subgroup.

Answer: b

Explanation: Let $(H, *)$ be the semi-group of the group $(G, *)$. Let a belongs to G . $(a * H)$ is the set of a left co-set of H in G and $(H * a)$ be the set of a right co-set of H in G .

184. Let K be a group with 8 elements. Let H be a subgroup of K and $H < K$. It is known that the size of H is at least 3. The size of H is _____
- a) 8
 - b) 2
 - c) 3
 - d) 4

186. $a * H = H * a$ relation holds if _____

- a) H is semi-group of an Abelian group
- b) H is monoid of a group
- c) H is a cyclic group
- d) H is subgroup of an Abelian group

Answer: d

Answer: d

Explanation: For any finite group G , the order (number of elements) of every subgroup L of G divides the order of G . G has 8 elements. Factors of 8 are 1, 2, 4 and 8. Since given the size of L is at least 3 (1 and 2 eliminated) and not equal to G (8 eliminated), the only size left is 4. Size of L is 4.

Explanation: If h is the subgroup of an Abelian group G , then the set of left co-sets of H in G is to be set of right co-sets i.e., $a * H = H * a$. Hence, subgroup is called the normal subgroup.

185. $a * H$ is a set of _____ co-set.
- a) right
 - b) left
 - c) sub
 - d) semi

187. Lagrange's theorem specifies _____

- a) the order of semi-group is finite
- b) the order of the subgroup divides the order of the finite group
- c) the order of an Abelian group is infinite
- d) the order of the semi-group is

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added to the order of the group

Answer: b

Answer: b

Explanation: Lagrange's theorem satisfies that the order of the subgroup divides the order of the finite group.

Explanation: Let C be a cyclic group with a generator $g \in C$. Namely, we have $G = \{g^n \mid n \in \mathbb{Z}\}$. Let x and y be arbitrary elements in C . Then, there exists $n, m \in \mathbb{Z}$ such that $x = g^n$ and $y = g^m$. It follows that $x * y = g^n * g^m = g^{n+m} = g^{m+n} = g^m * g^n = y * x$. Hence, we find that $x * y = y * x$ for any x, y belongs to G . Thus, G is an Abelian group.

188. An infinite cyclic group does not have a _____ series.

- a) AP
- b) GP
- c) Composite
- d) Finite

Answer: c

Explanation: Suppose that any finite group of order less than n has a composition series. Let G be a finite group of order n . If G is simple, then $G \cong \{e\}$, where e is the identity element of G and hence, it is a composition series. However, any infinite cyclic group does not have a composite series.

190. What is an irreducible module?

- a) A cyclic module in a ring with any non-zero element as its generator
- b) A cyclic module in a ring with any positive integer as its generator
- c) An acyclic module in a ring with rational elements as its generator
- d) A linearly independent module in a semigroup with a set of real numbers

189. Every cyclic group is a/an _____

- a) infinite subgroup
- b) Abelian group
- c) monoid
- d) commutative semi-group

Answer: a

Explanation: Let $a \in M$ be any nonzero element and consider the sub-module $\langle a \rangle$ generated by the element a . Since a is a nonzero element, the sub-module $\langle a \rangle$ is non-zero. Since M

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is irreducible, this implies that $M = (a)$. Hence M is a cyclic module generated by a . Since a is any nonzero element, the module M is a cyclic module with any nonzero element as its generator.

191. Consider an integer 23 such that $23 \geq 3p$ for a $2p$ -cycle in a permutation group, then p is

- a) odd prime
- b) even prime
- c) rational number
- d) negative prime

Answer: a

Explanation: Let n an integer such that $n \geq 3p$ and m is a $2p$ -cycle in the permutation group, then p is an odd prime.

192. Which one of the following is true?

- a) The resolvent of two Horn clauses is not a Horn clause
- b) The resolvent of two Horn clauses is a Horn clause
- c) If we resolvent a negated goal G against a fact or rule A to get clause C then C has positive literal or non-null goal
- d) If we resolvent a negated goal G against a fact or rule A to get clause C then C has positive

literal or null goal

Answer: b

Explanation: a) The resolvent of two Horn clauses is not a Horn clause. **False**

b) The resolvent of two Horn clauses is a Horn clause. **True**

c) If we resolve a negated goal G against a fact or rule A to get clause C then C has positive literal or non-null goal. **False**

d) If we resolve a negated goal G against a fact or rule A to get clause C then C has positive literal or null goal. **False**

So, option (B) is correct.

193. Which of the following statements is the negation of the statements “4 is odd or -9 is positive”?

- a) 4 is even or -9 is not negative
- b) 4 is odd or -9 is not negative
- c) 4 is even and -9 is negative
- d) 4 is odd and -9 is not negative

Answer: c

Explanation: Using De Morgan's Law $\sim (A \vee B) \leftrightarrow \sim A \wedge \sim B$.

194. Consider the following LPP :

$$\text{Min } Z = 2x_1 + x_2 + 3x_3$$

Subject to:

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$$x_1 - 2x_2 + x_3 \geq 4$$

$$2x_1 + x_2 + x_3 \leq 8$$

$$x_1 - x_3 \geq 0$$

$$x_1, x_2, x_3 \geq 0$$

The solution of this LPP using Dual Simplex Method is:

- a) $x_1=0, x_2=0, x_3=3$ and $Z=9$
- b) $x_1=0, x_2=6, x_3=0$ and $Z=6$
- c) $x_1=4, x_2=0, x_3=0$ and $Z=8$
- d) $x_1=2, x_2=0, x_3=2$ and $Z=10$

Answer: c

195. Consider the following LPP :

$$\text{Max } Z = 15x_1 + 10x_2$$

Subject to:

$$4x_1 + 6x_2 \leq 360$$

$$3x_1 + 0x_2 \leq 180$$

$$0x_1 + 5x_2 \leq 200$$

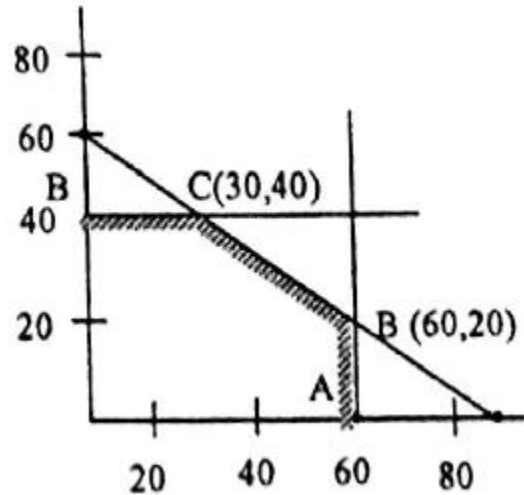
$$x_1, x_2 \geq 0$$

The solution of the LPP using Graphical solution technique is :

- a) $x_1=60, x_2=0$ and $Z=900$
- b) $x_1=60, x_2=20$ and $Z=1100$
- c) $x_1=60, x_2=30$ and $Z=1200$
- d) $x_1=50, x_2=40$ and $Z=1150$

Answer: b

Explanation:



$$x_1/90 + x_2/60 \leq 1$$

$$x_1/60 \leq 1$$

$$x_2/40 \leq 1$$

$$Z_{\max} = 15x_1 + 10x_2$$

$$Z_B = 15 \times 60 + 10 \times 20 = 1100$$

$$Z_C = 15 \times 30 + 10 \times 40 = 850$$

$$Z_A = 15 \times 60 + 10 \times 0 = 900$$

$$Z_D = 15 \times 0 + 10 \times 40 = 400$$

196. Let P, Q, R and S be

Propositions. Assume that the equivalences $P \Leftrightarrow (Q \vee \neg Q)$ and $Q \Leftrightarrow R$ hold. Then the truth value of the formula $(P \wedge Q) \Rightarrow ((P \wedge R) \vee S)$ is always:

- a) True
- b) False
- c) Same as truth table of Q
- d) Same as truth table of S

Answer: a

Explanation: $P \Leftrightarrow (Q \vee \neg Q)$ "P should be true because RHS will be TRUE always"

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$Q \Leftrightarrow R$ "when Q is true R is true" and "when Q is false R is false"

$(P \wedge Q) \Rightarrow ((P \wedge R) \vee S)$ $(P \wedge Q) \Rightarrow ((P \wedge R) \vee S)$ there can be only 2 cases (value of S doesn't matter)

a) $P = \text{True}, Q = \text{True}$ and $R = \text{True}$

$(T \wedge T) \Rightarrow ((T \wedge T) \vee S)$ $(T \wedge T) \Rightarrow ((T \wedge T) \vee S)$ so this case is True

b) $P = \text{True}, Q = R = \text{False}$

$(T \wedge F) \Rightarrow ((P \wedge R) \vee S)$ $(T \wedge F) \Rightarrow ((P \wedge R) \vee S)$

In case implication if the premise is false then whole statement is true this case is also true. The given expression is **true** in both the cases.

197. The relation between sets A, B, C as shown by venn diagram is

a) A is subset of B and B is subset of C

b) C is not a subset of A and A is subset of B

c) C is subset of B and B is subset of A

d) None of the mentioned

Answer: c

Explanation: As set C is totally inside set B, set B is totally inside set A.

198. If in sets A, B, C, the set $B \cap C$ consists of 8 elements, set $A \cap B$ consists of 7 elements and set $C \cap A$ consists of 7 elements then the minimum element in set $A \cup B \cup C$ will be?

a) 8

b) 14

c) 22

d) 15

Answer: a

Explanation: For minimum elements set B and C have 8 elements each and all of the elements are same, also set A should have 7 elements which are already present in B and C. Thus $A \cup B \cup C \equiv A \equiv B$.

199. The set of positive integers is

a) Infinite

b) Finite

c) Subset

d) Empty

Answer: a

Explanation: The set of positive integers is not finite.

200. "Match will be played only if it is not a humid day." The negation of this statement is?

a) Match will be played but it is a

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- humid day
b) Match will be played or it is a humid day
c) All of the mentioned statement are correct
d) None of the mentioned

Answer: a

Explanation: Negation of $P \rightarrow Q$ is $P \wedge \sim Q$.

201. Which of the following statements is the negation of the statements “4 is odd or -9 is positive”?

- a) 4 is even or -9 is not negative
b) 4 is odd or -9 is not negative
c) 4 is even and -9 is negative
d) 4 is odd and -9 is not negative

Answer: c

Explanation: Using De Morgan's Law $\sim(A \vee B) \leftrightarrow \sim A \wedge \sim B$.

202. Which of the following represents: $\sim A$ (negation of A) if A stands for “I like badminton but hate maths”?

- a) I hate badminton and maths
b) I do not like badminton or maths
c) I dislike badminton but love maths
d) I hate badminton or like maths

Answer: d

Explanation: De Morgan's Law $\sim(A \wedge B) \leftrightarrow \sim A \vee \sim B$.

203. The compound statement $A \vee \sim(A \wedge B)$.

- a) True
b) False

Answer: a

Explanation: Applying De-Morgan's law we get $A \vee \sim A \equiv \text{Tautology}$.

204. Which of the following is De-Morgan's law?

- a) $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$
b) $\sim(P \wedge R) \equiv \sim P \vee \sim R, \sim(P \vee R) \equiv \sim P \wedge \sim R$
c) $P \vee \sim P \equiv \text{True}, P \wedge \sim P \equiv \text{False}$
d) None of the mentioned

Answer: b

Explanation: Definition of De-Morgan's Law.

205. What is the dual of $(A \wedge B) \vee (C \wedge D)$?

- a) $(A \vee B) \vee (C \vee D)$
b) $(A \vee B) \wedge (C \vee D)$
c) $(A \vee B) \vee (C \wedge D)$
d) $(A \wedge B) \vee (C \vee D)$

Answer: b

Explanation: In dual $\wedge$ is replaced by $\vee$ and vice – versa.

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206. $\sim A \vee \sim B$ is logically equivalent to?

- a) $\sim A \rightarrow \sim B$
- b) $\sim A \wedge \sim B$
- c) $A \rightarrow \sim B$
- d) $B \vee A$

Answer: c

Explanation: By identity $A \rightarrow B \equiv \sim A \vee B$.

207. Negation of statement $(A \wedge B) \rightarrow (B \wedge C)$ is _____

- a) $(A \wedge B) \rightarrow (\sim B \wedge \sim C)$
- b) $\sim(A \wedge B) \vee (B \vee C)$
- c) $\sim(A \rightarrow B) \rightarrow (\sim B \wedge C)$
- d) None of the mentioned

Answer: a

Explanation: $\sim(A \rightarrow B) \equiv A \wedge \sim B$ using this we can easily fetch the answer.

208. Which of the following satisfies commutative law?

- a) $\wedge$
- b) $\vee$
- c) $\leftrightarrow$
- d) All of the mentioned

Answer: d

Explanation: All of them satisfies commutative law.

209. If the truth value of $A \vee B$ is true, then truth value of $\sim A \wedge B$ can be

- a) True if A is false
- b) False if A is false
- c) False if B is true and A is false
- d) None of the mentioned

Answer: a

Explanation: If A is false then both the condition are obeyed.

210. If P is always against the testimony of Q, then the compound statement $P \rightarrow (P \vee \sim Q)$ is a _____

- a) Tautology
- b) Contradiction
- c) Contingency
- d) None of the mentioned

Answer: a

Explanation: Since either hypothesis is false or both (hypothesis as well as conclusion) are true.

211. Let P (x) denote the statement " $x > 7$." Which of these have truth value true?

- a) P (0)
- b) P (4)
- c) P (6)
- d) P (9)

Answer: d

Explanation: Put $x=9$, $9 > 7$ which is true.

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212. Let $Q(x)$ be the statement " $x < 5$." What is the truth value of the quantification $\forall xQ(x)$, having domains as real numbers.

- a) True
- b) False

Answer: b

Explanation: $Q(x)$ is not true for every real number x , because, for instance, $Q(6)$ is false. That is, $x = 6$ is a counterexample for the statement $\forall xQ(x)$. This is false.

213. Determine the truth value of $\forall n(n + 1 > n)$ if the domain consists of all real numbers.

- a) True
- b) False

Answer: a

Explanation: There are no elements in the domain for which the statement is false.

214. Let $P(x)$ denote the statement " $x = x + 7$." What is the truth value of the quantification $\exists xP(x)$, where the domain consists of all real numbers?

- a) True
- b) False

Answer: b

Explanation: Because $P(x)$ is false for every real number x , the existential

quantification of $Q(x)$, which is $\exists xP(x)$, is false.

215. Let $R(x)$ denote the statement " $x > 2$." What is the truth value of the quantification $\exists xR(x)$, having domain as real numbers?

- a) True
- b) False

Answer: a

Explanation: Because " $x > 2$ " is sometimes true—for instance, when $x = 3$ —the existential quantification of $R(x)$, which is $\exists xR(x)$, is true.

216. The statement, "Every comedian is funny" where $C(x)$ is " x is a comedian" and $F(x)$ is " x is funny" and the domain consists of all people.

- a) $\exists x(C(x) \wedge F(x))$
- b) $\forall x(C(x) \wedge F(x))$
- c) $\exists x(C(x) \rightarrow F(x))$
- d) $\forall x(C(x) \rightarrow F(x))$

Answer: d

Explanation: For every person x , if comedian then x is funny.

217. The statement, "At least one of your friends is perfect". Let $P(x)$ be " x is perfect" and let $F(x)$ be " x is your friend" and let the domain be all people.

- a) $\forall x (F(x) \rightarrow P(x))$

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b) $\forall x (F(x) \wedge P(x))$

c) $\exists x (F(x) \wedge P(x))$

d) $\exists x (F(x) \rightarrow P(x))$

Answer: c

Explanation: For some x, x is friend and funny.

218. "Everyone wants to learn cosmology." This argument may be true for which domains?

a) All students in your cosmology class

b) All the cosmology learning students in the world

c) Both of the mentioned

d) None of the mentioned

Answer: c

Explanation: Domain may be limited to your class or may be whole world both are good as it satisfies universal quantifier.

219. Let domain of m includes all students, P(m) be the statement "m spends more than 2 hours in playing polo". Express $\forall m \neg P(m)$ quantification in English.

a) A student is there who spends more than 2 hours in playing polo

b) There is a student who does not spend more than 2 hours in playing polo

c) All students spends more than 2 hours in playing polo

d) No student spends more than 2 hours in playing polo

Answer: d

Explanation: There is no student who spends more than 2 hours in playing polo.

220. Determine the truth value of statement $\exists n (4n = 3n)$ if the domain consists of all integers.

a) True

b) False

Answer: a

Explanation: For $n=0$, $4n=3n$ hence, it is true.

221. Let $Q(x, y)$ denote " $M + A = 0$."

What is the truth value of the quantifications $\exists A \forall M Q(M, A)$.

a) True

b) False

Answer: b

Explanation: For each A there exist only one M, because there is no real number A such that $M + A = 0$ for all real numbers M.

222. Translate $\forall x \exists y (x < y)$ in English, considering domain as a real number for both the variable.

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a) For all real number x there exists a real number y such that x is less than y

b) For every real number y there exists a real number x such that x is less than y

c) For some real number x there exists a real number y such that x is less than y

d) For each and every real number x and y such that x is less than y

Answer: a

Explanation: Statement is x is less than y . Quantifier used are for each x , there exists a y .

223. "The product of two negative real numbers is not negative." Is given by?

a) $\exists x \forall y ((x < 0) \wedge (y < 0) \rightarrow (xy > 0))$

b) $\exists x \exists y ((x < 0) \wedge (y < 0) \wedge (xy > 0))$

c) $\forall x \exists y ((x < 0) \wedge (y < 0) \wedge (xy > 0))$

d) $\forall x \forall y ((x < 0) \wedge (y < 0) \rightarrow (xy > 0))$

Answer: d

Explanation: For every negative real number x and y , the product of these integer is positive.

224. Let $Q(x, y)$ be the statement " $x + y = x - y$." If the domain for both variables consists of all integers, what is the truth value of $\exists x Q(x, 4)$.

a) True

b) False

Answer: b

Explanation: There exist no integer for which $x+4=x-4$.

225. Let $L(x, y)$ be the statement " x loves y ," where the domain for both x and y consists of all people in the world. Use quantifiers to express, "Joy is loved by everyone."

a) $\forall x L(x, \text{Joy})$

b) $\forall y L(\text{Joy}, y)$

c) $\exists y \forall x L(x, y)$

d) $\exists x \neg L(\text{Joy}, x)$

Answer: a

Explanation: Joy is loved by all the people in the world.

226. Let $T(x, y)$ mean that student x likes dish y , where the domain for x consists of all students at your school and the domain for y consists of all dishes. Express $\neg T(\text{Amit, South Indian})$ by a simple English sentence.

a) All students does not like South Indian dishes.

b) Amit does not like South Indian people.

c) Amit does not like South Indian dishes.

d) Amit does not like some dishes.

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Answer: d

Explanation: Negation of the statement Amit like South Indian dishes.

227. Express, "The difference of a real number and itself is zero" using required operators.

- a) $\forall x(x - x! = 0)$
- b) $\forall x(x - x = 0)$
- c) $\forall x\forall y(x - y = 0)$
- d) $\exists x(x - x = 0)$

Answer: b

Explanation: For every real number x , difference with itself is always zero.

228. Which rule of inference is used in each of these arguments, "If it is Wednesday, then the Smartmart will be crowded. It is Wednesday. Thus, the Smartmart is crowded."

- a) Modus tollens
- b) Modus ponens
- c) Disjunctive syllogism
- d) Simplification

Answer: b

Explanation: $(M \wedge (M \rightarrow N)) \rightarrow N$ is Modus ponens.

229. Which rule of inference is used in each of these arguments, "If it hailstoday, the local office will be closed. The local office is not closed

today. Thus, it did not hailed today."

- a) Modus tollens
- b) Conjunction
- c) Hypothetical syllogism
- d) Simplification

Answer: a

Explanation: $(\neg N \wedge (M \rightarrow N)) \rightarrow \neg M$ is Modus tollens.

230. Which rule of inference is used, "Bhavika will work in an enterprise this summer. Therefore, this summer Bhavika will work in an enterprise or he will go to beach."

- a) Simplification
- b) Conjunction
- c) Addition
- d) Disjunctive syllogism

Answer: c

Explanation: $p \rightarrow (p \vee q)$ argument is 'Addition'.

231. What rule of inference is used here?

"It is cloudy and drizzling now. Therefore, it is cloudy now."

- a) Addition
- b) Simplification
- c) Resolution
- d) Conjunction

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Answer: b

Explanation: $(p \wedge q) \rightarrow p$ argument is Simplification.

232. What rule of inference is used in this argument?

“If I go for a balanced diet, then I will be fit. If I will be fit, then I will remain healthy. Therefore, if I go for a balanced diet, then I will remain healthy.”

- a) Modus tollens
- b) Modus ponens
- c) Disjunctive syllogism
- d) Hypothetical syllogism

Answer: d

Explanation: $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$ argument is ‘Hypothetical syllogism’.

233. What rules of inference are used in this argument?

“All students in this science class has taken a course in physics” and “Marry is a student in this class” imply the conclusion “Marry has taken a course in physics.”

- a) Universal instantiation
- b) Universal generalization
- c) Existential instantiation
- d) Existential generalization

Answer: a

Explanation: $\forall x P(x), \therefore P(c)$ Universal instantiation.

234. What rules of inference are used in this argument?

“It is either colder than Himalaya today or the pollution is harmful. It is hotter than Himalaya today. Therefore, the pollution is harmful.”

- a) Conjunction
- b) Modus ponens
- c) Disjunctive syllogism
- d) Hypothetical syllogism

Answer: c

Explanation: $((p \vee q) \wedge \neg p) \rightarrow q$ argument is Disjunctive syllogism.

235. The premises $(p \wedge q) \vee r$ and $r \rightarrow s$ implies which of the conclusion?

- a) $p \vee r$
- b) $p \vee s$
- c) $q \vee s$
- d) $q \vee r$

Answer: b

Explanation: The premises $(p \wedge q) \vee r$ as two clauses, $p \vee r$ and $q \vee r$. We can also replace $r \rightarrow s$ by the equivalent clause $\neg r \vee s$. using the two clauses $p \vee r$ and $\neg r \vee s$, we can use resolution to conclude $p \vee s$.

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236. $\{x: x \text{ is an integer neither positive nor negative}\}$ is _____

- a) Empty set
- b) Non-empty set
- c) Finite set
- d) Non- empty and Finite set

Answer: d

Explanation: Set = $\{0\}$ non-empty and finite set.

237. $\{x: x \text{ is a real number between 1 and 2}\}$ is an _____

- a) Infinite set
- b) Finite set
- c) Empty set
- d) None of the mentioned

Answer: a

Explanation: It is an infinite set as there are infinitely many real number between any two different real numbers.

238. Write set $\{1, 5, 15, 25, \dots\}$ in set-builder form.

- a) $\{x: \text{either } x=1 \text{ or } x=5n, \text{ where } n \text{ is a real number}\}$
- b) $\{x: \text{either } x=1 \text{ or } x=5n, \text{ where } n \text{ is a integer}\}$
- c) $\{x: \text{either } x=1 \text{ or } x=5n, \text{ where } n \text{ is an odd natural number}\}$
- d) $\{x: x=5n, \text{ where } n \text{ is a natural number}\}$

Answer: c

Explanation: Set should include 1 or an odd multiple of 5.

239. Express $\{x: x = n/(n+1), n \text{ is a natural number less than 7}\}$ in roster form.

- a) $\{1/2, 2/3, 4/5, 6/7\}$
- b) $\{1/2, 2/3, 3/4, 4/5, 5/6, 6/7, 7/8\}$
- c) $\{1/2, 2/3, 3/4, 4/5, 5/6, 6/7\}$
- d) Infinite set

Answer: c

Explanation: $n/(n+1) = 1/(1+1) = 1/2$ and $n > 7$.

240. Number of power set of $\{a, b\}$, where a and b are distinct elements.

- a) 3
- b) 4
- c) 2
- d) 5

Answer: b

Explanation: Power set of $\{a, b\} = \{\emptyset, \{a, b\}, \{a\}, \{b\}\}$.

241. Which of the following is subset of set $\{1, 2, 3, 4\}$?

- a) $\{1, 2\}$
- b) $\{1, 2, 3\}$
- c) $\{1\}$
- d) All of the mentioned

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Answer: d

Explanation: There are total 16 subsets.

242. $A = \{\emptyset, \{\emptyset\}, 2, \{2, \emptyset\}, 3\}$, which of the following is true?

- a) $\{\{\emptyset, \{\emptyset\}\} \in A$
- b) $\{2\} \in A$
- c) $\emptyset \subset A$
- d) $3 \subset A$

Answer: c

Explanation: Empty set is a subset of every set.

243. Subset of the set $A = \{ \}$ is?

- a) A
- b) $\{ \}$
- c) $\emptyset$
- d) All of the mentioned

Answer: d

Explanation: Every set is subset of itself and Empty set is subset of each set.

244. A _____ is an ordered collection of objects.

- a) Relation
- b) Function
- c) Set
- d) Proposition

Answer: c

Explanation: By the definition of set.

245. The set O of odd positive integers less than 10 can be expressed by _____

- a) $\{1, 2, 3\}$
- b) $\{1, 3, 5, 7, 9\}$
- c) $\{1, 2, 5, 9\}$
- d) $\{1, 5, 7, 9, 11\}$

Answer: b

Explanation: Odd numbers less than 10 is $\{1, 3, 5, 7, 9\}$.

246. Power set of empty set has exactly _____ subset.

- a) One
- b) Two
- c) Zero
- d) Three

Answer: a

Explanation: Power set of null set has exactly one subset which is empty set.

246. What is the Cartesian product of $A = \{1, 2\}$ and $B = \{a, b\}$?

- a) $\{(1, a), (1, b), (2, a), (b, b)\}$
- b) $\{(1, 1), (2, 2), (a, a), (b, b)\}$
- c) $\{(1, a), (2, a), (1, b), (2, b)\}$
- d) $\{(1, 1), (a, a), (2, a), (1, b)\}$

Answer: c

Explanation: A subset R of the Cartesian product $A \times B$ is a relation from the set A to the set B.

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247. The Cartesian Product $B \times A$ is equal to the Cartesian product $A \times B$.

- a) True
- b) False

Answer: b

Explanation: Let $A = \{1, 2\}$ and $B = \{a, b\}$. The Cartesian product $A \times B = \{(1, a), (1, b), (2, a), (2, b)\}$ and the Cartesian product $B \times A = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$. This is not equal to $A \times B$.

248. The union of the sets $\{1, 2, 5\}$ and $\{1, 2, 6\}$ is the set

- _____
- a) $\{1, 2, 6, 1\}$
 - b) $\{1, 2, 5, 6\}$
 - c) $\{1, 2, 1, 2\}$
 - d) $\{1, 5, 6, 3\}$

Answer: b

Explanation: The union of the sets A and B , is the set that contains those elements that are either in A or in B .

249. The intersection of the sets $\{1, 2, 5\}$ and $\{1, 2, 6\}$ is the set

- _____
- a) $\{1, 2\}$
 - b) $\{5, 6\}$
 - c) $\{2, 5\}$
 - d) $\{1, 6\}$

Answer: a

Explanation: The intersection of the sets A and B , is the set containing those elements that are in both A and B .

250. Two sets are called disjoint if there _____ is the empty set.

- a) Union
- b) Difference
- c) Intersection
- d) Complement

Answer: c

Explanation: By the definition of the disjoint set.

251. Which of the following two sets are disjoint?

- a) $\{1, 3, 5\}$ and $\{1, 3, 6\}$
- b) $\{1, 2, 3\}$ and $\{1, 2, 3\}$
- c) $\{1, 3, 5\}$ and $\{2, 3, 4\}$
- d) $\{1, 3, 5\}$ and $\{2, 4, 6\}$

Answer: d

Explanation: Two sets are disjoint if the intersection of two sets is the empty set.

252. The difference of $\{1, 2, 3\}$ and $\{1, 2, 5\}$ is the set _____

- a) $\{1\}$
- b) $\{5\}$

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c) $\{3\}$

d) $\{2\}$

Answer: c

Explanation: The difference of the sets A and B denoted by $A-B$, is the set containing those elements that are in A not in B.

253. The complement of the set A is

a) $A - B$

b) $U - A$

c) $A - U$

d) $B - A$

Answer: b

Explanation: The complement of the set A is the complement of A with respect to U.

254. The bit string for the set $\{2, 4, 6, 8, 10\}$ (with universal set of natural numbers less than or equal to 10) is

a) 0101010101

b) 1010101010

c) 1010010101

d) 0010010101

Answer: a

Explanation: The bit string for the set has a one bit in second, fourth, sixth, eighth, tenth positions, and a zero elsewhere.

255. Let $A_i = \{i, i+1, i+2, \dots\}$. Then set $\{n, n+1, n+2, n+3, \dots\}$ is the _____ of the set A_i .

a) Union

b) Intersection

c) Set Difference

d) Disjoint

Answer: b

Explanation: By the definition of the generalized intersection of the set.

256. Let the set A is $\{1, 2, 3\}$ and B is $\{2, 3, 4\}$. Then the number of elements in $A \cup B$ is?

a) 4

b) 5

c) 6

d) 7

Answer: a

Explanation: $A \cup B$ is $\{1, 2, 3, 4\}$.

257. Let the set A is $\{1, 2, 3\}$ and B is $\{2, 3, 4\}$. Then number of elements in $A \cap B$ is?

a) 1

b) 2

c) 3

d) 4

Answer: b

Explanation: $A \cap B$ is $\{2, 3\}$.

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258. Let the set A is $\{1, 2, 3\}$ and B is $\{2, 3, 4\}$. Then the set $A - B$ is?

- a) $\{1, -4\}$
- b) $\{1, 2, 3\}$
- c) $\{1\}$
- d) $\{2, 3\}$

Answer: c

Explanation: In $A - B$ the common elements get cancelled.

259. In which of the following sets $A - B$ is equal to $B - A$?

- a) $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$
- b) $A = \{1, 2, 3\}$, $B = \{1, 2, 3, 4\}$
- c) $A = \{1, 2, 3\}$, $B = \{2, 3, 1\}$
- d) $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{2, 3, 4, 5, 1\}$

Answer: c

Explanation: $A - B = B - A = \text{Empty set}$.

260. Let A be set of all prime numbers, B be the set of all even prime numbers, C be the set of all odd prime numbers, then which of the following is true?

- a) $A \equiv B \cup C$
- b) B is a singleton set.
- c) $A \equiv C \cup \{2\}$
- d) All of the mentioned

Answer: d

Explanation: 2 is the only even prime number.

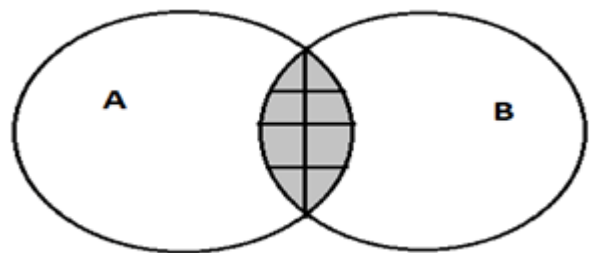
261. If A has 4 elements B has 8 elements then the minimum and maximum number of elements in $A \cup B$ are _____

- a) 4, 8
- b) 8, 12
- c) 4, 12
- d) None of the mentioned

Answer: b

Explanation: Minimum would be when 4 elements are same as in 8, maximum would be when all are distinct.

262. The shaded area of figure is best described by?



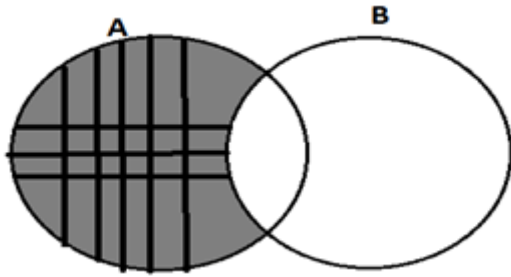
- a) $A \cap B$
- b) $A \cup B$
- c) A
- d) B

Answer: a

Explanation: The region is A intersection B.

263. The shaded area of figure is best described by?

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- a) A' (Complement of A)
- b) $A \cup B - B$
- c) $A \cap B$
- d) B

Answer: b

Explanation: The region is complement of B.

264. If $n(A)=20$ and $n(B)=30$ and $n(A \cup B) = 40$ then $n(A \cap B)$ is?

- a) 20
- b) 30
- c) 40
- d) 10

Answer: d

Explanation: $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

265. A function is said to be _____ if and only if $f(a) = f(b)$ implies that $a = b$ for all a and b in the domain of f .

- a) One-to-many
- b) One-to-one
- c) Many-to-many
- d) Many-to-one

Answer: b

Explanation: A function is one-to-one if and only if $f(a) \neq f(b)$ whenever $a \neq b$.

266. The function $f(x)=x+1$ from the set of integers to itself is onto. Is it True or False?

- a) True
- b) False

Answer: a

Explanation: For every integer "y" there is an integer "x" such that $f(x) = y$.

267. The value of $\lfloor 1/2 \rfloor \cdot \lfloor 5/2 \rfloor$ is _____

- a) 1
- b) 2
- c) 3
- d) 0.5

Answer: a

Explanation: The value of $\lfloor 5/2 \rfloor$ is 2 so, the value of $\lfloor 1/2 \cdot 2 \rfloor$ is 1.

268. Which of the following function $f: \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$ is not onto?

- a) $f(a, b) = a + b$
- b) $f(a, b) = a$
- c) $f(a, b) = |b|$
- d) $f(a, b) = a - b$

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Answer: c

Explanation: The function is not onto as $f(a) \neq b$.

269. The domain of the function that assign to each pair of integers the maximum of these two integers is _____

- a) $\mathbb{N}$
- b) $\mathbb{Z}$
- c) $\mathbb{Z}^+$
- d) $\mathbb{Z}^+ \times \mathbb{Z}^+$

Answer: d

Explanation: The domain of the integers is $\mathbb{Z}^+ \times \mathbb{Z}^+$.

270. How many even 4 digit whole numbers are there?

- a) 1358
- b) 7250
- c) 4500
- d) 3600

Answer: c

Explanation: The thousands digit cannot be zero, so there are 9 choices. There are 10 possibilities for the hundreds digit and 10 possibilities for the tens digit. The units digit can be 0, 2, 4, 6 or 8, so there are 5 choices. By the basic counting principle, the number of even five

digit whole numbers is $9 \times 10 \times 10 \times 5 = 45,00$.

271. In a multiple-choice question paper of 15 questions, the answers can be A, B, C or D. The number of different ways of answering the question paper are _____

- a) 65536×4^7
- b) 194536×4^5
- c) 23650×4^9
- d) 11287435

Answer: a

Explanation: There are $4^{15} = 65536 \times 4^7$ different ways of answering the exam paper of 15 MCQs.

272. How many words with seven letters are there that start with a vowel and end with an A? Note that they don't have to be real words and letters can be repeated.

- a) 45087902
- b) 64387659
- c) 12765800
- d) 59406880

Answer: d

Explanation: The first letter must be a vowel, so there are 5 choices. The second letter can be any one of 26, the third letter can be any one of 26, the fourth letter can be any one of 26

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and fifth and sixth letters can be any of 26 choices. The last letter must be an A, so there is only 1 choice. By the basic counting principle, the number of 'words' is $5 \times 26 \times 26 \times 26 \times 26 \times 26 \times 1 = 59406880$.

273. Neela has twelve different skirts, ten different tops, eight different pairs of shoes, three different necklaces and five different bracelets. In how many ways can Neela dress up?

- a) 50057
- b) 14400
- c) 34870
- d) 56732

Answer: b

Explanation: By the basic counting principle, the number of different ways = $12 \times 10 \times 8 \times 3 \times 5 = 14400$. Note that shoes come in pairs. So she must choose one pair of shoes from ten pairs, not one shoe from twenty.

274. How many five-digit numbers can be made from the digits 1 to 7 if repetition is allowed?

- a) 16807
- b) 54629
- c) 23467
- d) 32354

Answer: a

Explanation: $7^5 = 16807$ ways of making the numbers consisting of five digits if repetition is allowed.

275. For her English literature course, Ruchika has to choose one novel to study from a list of ten, one poem from a list of fifteen and one short story from a list of seven. How many different choices does Rachel have?

- a) 34900
- b) 26500
- c) 12000
- d) 10500

Answer: d

Explanation: By the Basic Counting Principle, the number of different choices is $10 \times 15 \times 7 = 10500$.

276. There are two different Geography books, five different Natural Sciences books, three different History books and four different Mathematics books on a shelf. In how many different ways can they be arranged if all the books of the same subjects stand together?

- a) 353450
- b) 638364
- c) 829440
- d) 768700

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Answer: c

Explanation: There are four groups of books which can be arranged in $4!$ different ways. Among those books, two are Geography books, five are Natural Sciences books, three are History books and four are Mathematics books. Therefore, there are $4! \times 2! \times 5! \times 3! \times 4! = 829440$ ways to arrange the books.

277. The code for a safe is of the form PPPQQQQ where P is any number from 0 to 9 and Q represents the letters of the alphabet. How many codes are possible for each of the following cases? Note that the digits and letters of the alphabet can be repeated.

- a) 874261140
- b) 537856330
- c) 549872700
- d) 456976000

Answer: d

Explanation: $10^3 \times 2^4 = 456976000$ possible codes are formed for the safe with the alphanumeric digits.

278. Amit must choose a seven-digit PIN number and each digit can be chosen from 0 to 9. How many different possible PIN numbers can Amit choose?

- a) 10000000
- b) 9900000
- c) 67285000
- d) 39654900

Answer: a

Explanation: By the basic counting principle, the total number of PIN numbers Amit can choose is $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10,000,000$.

279. A head boy, two deputy head boys, a head girl and 3 deputy head girls must be chosen out of a student council consisting of 14 girls and 16 boys. In how many ways can they be chosen?

- a) 98072
- b) 27384
- c) 36428
- d) 44389

Answer: b

Explanation: There are $16 \times 15 \times 14 + 14 \times 13 \times 12 \times 11 = 27384$ ways to choose from a student council.

280. A drawer contains 12 red and 12 blue socks, all unmatched. A person takes socks out at random in the dark. How many socks must he take out to be sure that he has at least two blue socks?

- a) 18
- b) 35

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c) 28

d) 14

281. How many substrings (of all lengths inclusive) can be formed from a character string of length 8?

(Assume all characters to be distinct)

a) 14

b) 21

c) 54

d) 37

Answer: d

Explanation: Let's consider the given string is CLEAN, so set of string of length 1 = {C,L,E,A,N} ; cardinality of set = 5 set of string of length 2 = {CL,EE,EA,NN}, set of string of length 3 = {CLE,LEE,EAN}, set of strings of length 4 = {CLEN,LEAN}, set of strings of length 5 = {CLEAN} and set of string of length 0 = {} and we cannot have any substring of length 6 as given string has only 5 length. So total no of substrings are possible = 0 length substring + 1 length substring + 2 length substrings + 3 length substrings + 4 length substrings + 5 length substrings = $1 + 5 + 4 + 3 + 2 + 1 = 16$ means for 1 length string to n length substrings it will sum of the n natural no from 1 to n.

so $1+2+3+\dots+n = \frac{n(n+1)}{2}$ so total no

substrings possible = 0 length strings + $\frac{n(n+1)}{2} = 1 + \frac{n(n+1)}{2}$ so total no of substrings possible in n length string (All length inclusive) = $1 + \frac{n(n+1)}{2} = \frac{8(8+1)}{2} = 37$.

282. The number of diagonals can be drawn in a hexagon is _____

a) 9

b) 32

c) 16

d) 21

Answer: a

Explanation: A hexagon has 6 sides. We obtain the diagonals by joining the vertices in pairs.

Total number of sides and diagonals = ${}^6C_2 = \frac{6 \times 5}{2} = 15$. This includes its 6 sides also. So, Diagonals = $15 - 6 = 9$. Hence, the number of diagonals is 9.

283. The number of binary strings of 17 zeros and 8 ones in which no two ones are adjacent is _____

a) 43758

b) 24310

c) 32654

d) 29803

Answer: a

Explanation: First place 17 zeroes side by side _ 0 _ 0 _ 0 ... 0 _ and 8 1's can

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be placed in any of the $(17+1)$ available gaps hence the number of ways $= {}^{n+1}C_k = 43758$.

284. How many words that can be formed with the letters of the word 'SWIMMING' such that the vowels do not come together? Assume that words are of with or without meaning.

- a) 430
- b) 623
- c) 729
- d) 1239

Answer: c

Explanation: The word 'SWIMMING' contains 8 letters. Of which, I occurs twice and M occurs twice. Therefore, the number of words formed by this word $= 8!2!2! = 10080$. In order to find the number of permutations that can be formed where the two vowels I and I come together, we group the letters that should come together and consider that group as one letter. So, the letters are S, W, M, M, N, G, (I, I). So, the number of letters are 7 the number of ways in which 7 letters can be arranged is $7! = 5040$. In I and I, the number of ways in which I and I can be arranged is $2!$. Hence, the total number of ways in which the

letters of the 'SWIMMING' can be arranged such that vowels are always together are $7!2!2! = 5040$ ways. The number of words in which the vowels do not come together is $= (10080 - 5040) = 5040$.

285. In a playground, 3 sisters and 8 other girls are playing together. In a particular game, how many ways can all the girls be seated in a circular order so that the three sisters are not seated together?

- a) 457993
- b) 3386880
- c) 6544873
- d) 56549

Answer: b

Explanation: There are 3 sisters and 8 other girls in total of 11 girls. The number of ways to arrange these 11 girls in a circular manner $= (11-1)! = 10!$. These three sisters can now rearrange themselves in $3!$ ways. By the multiplication theorem, the number of ways so that 3 sisters always come together in the arrangement $= 8! \times 3!$. Hence, the required number of ways in which the arrangement can take place if none of the 3 sisters is seated together: $10! - (8! \times 3!) = 3628800 -$

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$(40320 * 6) = 3628800 - 241920 = 3386880$.

286. How many numbers of three digits can be formed with digits 1, 3, 5, 7 and 9?

- a) 983
- b) 120
- c) 345
- d) 5430

Answer: b

Explanation: Here number of digits, $n = 5$ and number of places to be filled-up $r = 3$. Hence, the required number is ${}^5P_3 = 5!/2!*3! = 120$.

287. The size of a multiset is 6 which is equal to the number of elements in it with counting repetitions (a multiset is an unordered collection of elements where the elements may repeat any number of times). Determine the number of multisets can be grouped from n distinct elements so that at least one element occurs exactly twice?

- a) 326
- b) 28
- c) 45
- d) 62

Answer: c

Explanation: There are six places to

be filled in the multiset using the n distinct elements. At least one element has to occur exactly twice and that would leave 4 more places in the multiset means that at most four elements can occur exactly once. Thus there are two mutually exclusive cases as follows: 1) Exactly one element occurs exactly twice and select this element in n ways. Fill up the remaining four spots using 5 distinct elements from the remaining $n-1$ elements in ${}^{n-1}C_4$ ways. 2) Exactly four elements that occur at least once each. Hence, the total number of ways to form the multiset is ${}^nC_2 + n * {}^{n-1}C_4 = {}^6C_2 + 6 * {}^{6-1}C_4 = 45$.

288. There are 6 equally spaced points A, B, C, D, E and F marked on a circle with radius R . How many convex heptagons of distinctly different areas can be drawn using these points as vertices?

- a) $7! * 6$
- b) $7C5$
- c) $7!$
- d) same area

Answer: d

Explanation: Since all the points are equally spaced; hence the area of all

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the convex heptagons will be the same.

289. There are 2 twin sisters among a group of 15 persons. In how many ways can the group be arranged around a circle so that there is exactly one person between the two sisters?

- a) $15 * 12! * 2!$
- b) $15! * 2!$
- c) ${}^{14}C_2$
- d) $16 * 15!$

Answer: a

Explanation: We know that n objects can be arranged around a circle in $(n-1)!$ ways. If we consider the two sisters and the person in between the brothers as a block, then there will be 12 others and this block of three people to be arranged around a circle. The number of ways of arranging 13 objects around a circle is $12!$ ways. Now the sisters can be arranged on either side of the person who is in between the sisters in $2!$ ways. The person who sits in between the two sisters can be any of the 15 in the group and can be selected in 15 ways. Therefore, the total number of ways is $15 * 12! * 2!$.

290. The number of words of 4 consonants and 3 vowels can be

made from 15 consonants and 5 vowels, if all the letters are different is _____

- a) $3! * {}^{12}C_5$
- b) ${}^{16}C_4 * {}^4C_4$
- c) $15! * 4$
- d) ${}^{15}C_4 * {}^5C_3 * 7!$

Answer: d

Explanation: There are 4 consonants out of 15 can be selected in ${}^{15}C_4$ ways and 3 vowels can be selected in ${}^5C_3$ ways. Therefore, the total number of groups each containing 4 consonants and 3 vowels is ${}^{15}C_4 * {}^4C_3$. Each group contains 7 letters which can be arranged in $7!$ ways. Hence, required number of words is ${}^{15}C_4 * {}^5C_3 * 7!$.

291. How many ways are there to arrange 7 chocolate biscuits and 12 cheesecake biscuits into a row of 19 biscuits?

- a) 52347
- b) 50388
- c) 87658
- d) 24976

Answer: b

Explanation: Consider the situation as having 19 spots and filling them with 7 chocolate biscuits and 12 cheesecake biscuits. Then we just

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choose 7 spots for the chocolate biscuits and let the other 10 spots have cheesecake biscuits. The number of ways to do this job is ${}^{19}C_7 = 50388$.

292. If a, b, c, d and e are five natural numbers, then find the number of ordered sets(a, b, c, d, e) possible such that $a+b+c+d+e=75$.

- a) ${}^{65}C_5$
- b) ${}^{58}C_6$
- c) ${}^{72}C_7$
- d) ${}^{74}C_4$

Answer: d

Explanation: Let assumes that there are 75 identical balls which are to be arranged in 5 different compartments (Since a, b, c, d, e are distinguishable). If the balls are arranged in the row. We have 74 gaps where we can place a ball in each gap since we need 5 compartments we need to place only 4 balls. We can do this in ${}^{74}C_4$ ways.

293. There are 15 people in a committee. How many ways are there to group these 15 people into 3, 5, and 4?

- a) 846
- b) 2468

- c) 658
- d) 1317

Answer: d

Explanation: The number of ways to choose 3 people out of 9 is ${}^{15}C_3$. Then, number of ways to choose 5 people out of $(15-3) = 12$ is ${}^{12}C_5$. Finally, the number of ways to choose 4 people out of $(12-4) = 8$ is ${}^8C_4$. Hence, by the rule of product, ${}^{15}C_3 + {}^{12}C_5 + {}^8C_4 = 1317$.

294. There are six movie parts numbered from 1 to 6. Find the number of ways in which they be arranged so that part-1 and part-3 are never together.

- a) 876
- b) 480
- c) 654
- d) 237

Answer: b

Explanation: The total number of ways in which 6 part can be arranged $= 6! = 720$. The total number of ways in which part-1 and part-3 are always together: $= 5! \cdot 2! = 240$. Therefore, the total number of arrangements, in which they are not together is $= 720 - 240 = 480$.

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295. How many ways are there to divide 4 Indian countries and 4 China countries into 4 groups of 2 each such that at least one group must have only Indian countries?

- a) 6
- b) 45
- c) 12
- d) 76

Answer: a

Explanation: The number of ways to divide $4+4=8$ countries into 4 groups of 2 each is as follows:

$({}^{10}C_2 * {}^{10}C_2 * {}^{10}C_2 * {}^{10}C_2)/4! = 30$. Since it is required that at least one group must have only Indian countries, we need to subtract 30 from the number of possible groupings where all 4 groups have 1 Indian country and 1 China country each. This is equivalent to the number of ways to match each of the 4 Indian countries with one China country: $4! = 24$. Therefore, the answer is $30 - 24 = 6$.

296. Find the number of factors of the product $5^8 * 7^5 * 2^3$ which are perfect squares.

- a) 47
- b) 30
- c) 65
- d) 19

Answer: b

Explanation: Any factor of this number should be of the form $5^a * 7^b * 2^c$. For the factor to be a perfect square a, b, c has to be even. a can take values 0, 2, 4, 6, 8, b can take values 0, 2, 4 and c can take values 0, 2. Total number of perfect squares = $5 * 3 * 2 = 30$.

297. From a group of 8 men and 6 women, five persons are to be selected to form a committee so that at least 3 women are there on the committee. In how many ways can it be done?

- a) 686
- b) 438
- c) 732
- d) 549

Answer: a

Explanation: We may have (2 men and 3 women) or (1 men and 4 woman) or (5 women only). The Required number of ways = $({}^8C_2 * {}^6C_3) + ({}^8C_1 * {}^6C_4) + ({}^6C_5) = 686$.

298. What is the base case for the inequality $7^n > n^3$, where $n = 3$?

- a) $652 > 189$
- b) $42 < 132$
- c) $343 > 27$
- d) $42 \leq 43$

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Answer: c

Explanation: By the principle of mathematical induction, we have $7^3 > 3^3 \Rightarrow 343 > 27$ as a base case and it is true for $n = 3$.

299. In the principle of mathematical induction, which of the following steps is mandatory?

- a) induction hypothesis
- b) inductive reference
- c) induction set assumption
- d) minimal set representation

Answer: a

Explanation: The hypothesis of Step is a must for mathematical induction that is the statement is true for $n = k$, where n and k are any natural numbers, which is also called induction assumption or induction hypothesis.

300. For $m = 1, 2, \dots$, $4m+2$ is a multiple of _____

- a) 3
- b) 5
- c) 6
- d) 2

Answer: d

Explanation: For $n = 1$, $4 * 1 + 2 = 6$, which is a multiple of 2. Assume that $4m+2$ is true for $m=k$ and so $4k+2$ is

true based on the assumption. Now, to prove that $4k+2$ is also a multiple of 2 $\Rightarrow 4(k+1)+2 \Rightarrow 2 * 4k - 4k + 6 \Rightarrow 2*4k+4 - 4k+2 \Rightarrow 2(4k+2) - 2(2k+1)$. Here, the first term $2(4k+2)$ is true as per assumption and the second term $2(4k+2)$ is must to be a multiple of 2. Hence, $4(k+1)+2$ is a multiple of 2. So, by induction hypothesis, $(4m+2)$ is a multiple of 2, for $m = 1, 2, 3, \dots$

301. For any integer $m \geq 3$, the series $2+4+6+\dots+(4m)$ can be equivalent to

- a) m^2+3
- b) $m+1$
- c) m^m
- d) $3m^2+4$

Answer: a

Explanation: The required answer is m^2+3 . Now, by induction assumption, we have to prove $2+4+6+\dots+4(k+1) = (k+1)^2+3$ also can be true, $2+4+6+\dots+4(k+1) = 2+4+6+\dots+(4k+4)$ and by the subsequent steps, we can prove that $(m+1)^2+3$ also holds for $m=k$. So, it is proved.

302. A non empty set A is termed as an algebraic structure _____

- a) with respect to binary operation $*$
- b) with respect to ternary operation ?

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- c) with respect to binary operation +
- d) with respect to unary operation –

Answer: a

Explanation: A non empty set A is called an algebraic structure w.r.t binary operation “*” if $(a*b)$ belongs to S for all $(a*b)$ belongs to S. Therefore “*” is closure operation on ‘A’.

303. An algebraic structure _____ is called a semigroup.

- a) $(P, *)$
- b) $(Q, +, *)$
- c) $(P, +)$
- d) $(+, *)$

Answer: a

Explanation: An algebraic structure $(P, *)$ is called a semigroup if $a*(b*c) = (a*b)*c$ for all a, b, c belongs to S or the elements follow associative property under “*”. (Matrix, *) and (Set of integers, +) are examples of semigroup.

304. Condition for monoid is _____

- a) $(a+e)=a$
- b) $(a*e)=(a+e)$
- c) $a=(a*(a+e))$
- d) $(a*e)=(e*a)=a$

Answer: d

Explanation: A Semigroup $(S, *)$ is defined as a monoid if there exists an element e in S such that $(a*e) = (e*a) = a$ for all a in S. This element is called identity element of S w.r.t *.

305. A monoid is called a group if _____

- a) $(a*a)=a=(a+c)$
- b) $(a*c)=(a+c)$
- c) $(a+c)=a$
- d) $(a*c)=(c*a)=e$

Answer: d

Explanation: A monoid $(B, *)$ is called Group if to each element there exists an element c such that $(a*c)=(c*a)=e$. Here e is called an identity element and c is defined as the inverse of the corresponding element.

306. A group $(M, *)$ is said to be abelian if _____

- a) $(x+y)=(y+x)$
- b) $(x*y)=(y*x)$
- c) $(x+y)=x$
- d) $(y*x)=(x+y)$

Answer: b

Explanation: A group $(M, *)$ is said to be abelian if $(x*y) = (y*x)$ for all x, y belongs to M. Thus Commutative property should hold in a group.

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307. Matrix multiplication is a/an _____ property.

- a) Commutative
- b) Associative
- c) Additive
- d) Disjunctive

Answer: b

Explanation: The set of two $M \times M$ non-singular matrices form a group under matrix multiplication operation. Since matrix multiplication is itself associative, it holds associative property.

308. A cyclic group can be generated by a/an _____ element.

- a) singular
- b) non-singular
- c) inverse
- d) multiplicative

Answer: a

Explanation: A singular element can generate a cyclic group. Every element of a cyclic group is a power of some specific element which is known as a generator 'g'.

309. How many properties can be held by a group?

- a) 2
- b) 3

- c) 5
- d) 4

Answer: c

Explanation: A group holds five properties simultaneously –

- i) Closure
- ii) associative
- iii) Commutative
- iv) Identity element
- v) Inverse element.

310. A cyclic group is always _____

- a) abelian group
- b) monoid
- c) semigroup
- d) subgroup

Answer: a

Explanation: A cyclic group is always an abelian group but every abelian group is not a cyclic group. For instance, the rational numbers under addition is an abelian group but is not a cyclic one.

311. $\{1, i, -i, -1\}$ is _____

- a) semigroup
- b) subgroup
- c) cyclic group
- d) abelian group

Answer: c

Explanation: The set of complex

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numbers $\{1, i, -i, -1\}$ under multiplication operation is a cyclic group. Two generators i and $-i$ will covers all the elements of this group. Hence, it is a cyclic group.

312. A trivial subgroup consists of _____

- a) Identity element
- b) Coset
- c) Inverse element
- d) Ring

Answer: a

Explanation: Let G be a group under a binary operation $*$ and a subset H of G is called a subgroup of G if H forms a group under the operation $*$. The trivial subgroup of any group is the subgroup consisting of only the Identity element.

313. Minimum subgroup of a group is called _____

- a) a commutative subgroup
- b) a lattice
- c) a trivial group
- d) a monoid

Answer: c

Explanation: The subgroups of any given group form a complete lattice under inclusion termed as a lattice of subgroups. If o is the Identity

element of a group(G), then the trivial group(o) is the minimum subgroup of that group and G is the maximum subgroup.

314. Let K be a group with 8 elements. Let H be a subgroup of K and $H < K$. It is known that the size of H is at least 3. The size of H is _____

- a) 8
- b) 2
- c) 3
- d) 4

Answer: d

Explanation: For any finite group G , the order (number of elements) of every subgroup L of G divides the order of G . G has 8 elements. Factors of 8 are 1, 2, 4 and 8. Since given the size of L is at least 3(1 and 2 eliminated) and not equal to G (8 eliminated), the only size left is 4. Size of L is 4.

315. _____ is not necessarily a property of a Group.

- a) Commutativity
- b) Existence of inverse for every element
- c) Existence of Identity
- d) Associativity

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Answer: a

Explanation: Grupoid has closure property; semigroup has closure and associative; monoid has closure, associative and identity property; group has closure, associative, identity and inverse; the abelian group has group property and commutative.

316. A group of rational numbers is an example of _____

- a) a subgroup of a group of integers
- b) a subgroup of a group of real numbers
- c) a subgroup of a group of irrational numbers
- d) a subgroup of a group of complex numbers

Answer: b

Explanation: If we consider the abelian group as a group rational numbers under binary operation + then it is an example of a subgroup of a group of real numbers.

317. Intersection of subgroups is a _____

- a) group
- b) subgroup
- c) semigroup
- d) cyclic group

Answer: b

Explanation: The subgroup property is intersection closed. An arbitrary (nonempty) intersection of subgroups with this property, also attains the similar property.

318. The group of matrices with determinant _____ is a subgroup of the group of invertible matrices under multiplication.

- a) 2
- b) 3
- c) 1
- d) 4

Answer: c

Explanation: The group of real matrices with determinant 1 is a subgroup of the group of invertible real matrices, both equipped with matrix multiplication. It has to be shown that the product of two matrices with determinant 1 is another matrix with determinant 1, but this is immediate from the multiplicative property of the determinant. This group is usually denoted by (n, R) .

319. What is a circle group?

- a) a subgroup complex numbers having magnitude 1 of the group of nonzero complex elements

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- b) a subgroup rational numbers having magnitude 2 of the group of real elements
- c) a subgroup irrational numbers having magnitude 2 of the group of nonzero complex elements
- d) a subgroup complex numbers having magnitude 1 of the group of whole numbers

Answer: a

Explanation: The set of complex numbers with magnitude 1 is a subgroup of the nonzero complex numbers associated with multiplication. It is called the circle group as its elements form the unit circle.

320. Which algorithm efficiently calculates the single source shortest paths in a Directed Acyclic Graph?

- a) topological sort
- b) hash table
- c) binary search
- d) radix sort

Answer: a

Explanation: For Directed Acyclic graph, single source shortest distances can be calculated in $O(V+E)$ time. For that purpose Topological Sorting can be used. Topological

Sorting of any graph represents a linear ordering of the graph.

321. The _____ of a graph G consists of all vertices and edges of G.

- a) edge graph
- b) line graph
- c) path complement graph
- d) eulerian circuit

Answer: d

Explanation: we know that the Eulerian circuit in a graph G is a circuit that includes all vertices and edges of G. A graph that can have Eulerian circuit, also can have a Eulerian graph.

322. A _____ in a graph G is a circuit which consists of every vertex (except first/last vertex) of G exactly once.

- a) Euler path
- b) Hamiltonian path
- c) Planar graph
- d) Path complement graph

Answer: b

Explanation: The Eulerian path in a graph say, G is a walk from one vertex to another, that can pass through all vertices of G as well as traverses exactly once every edge of G. Therefore, an Eulerian path can not be a circuit. A Hamiltonian path is

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a walk that contains every vertex of the graph exactly once. Hence, a Hamiltonian path is not a circuit.

323. A walk has Closed property if _____

- a) $v_0 = v_k$
- b) $v_0 > v_k$
- c) $v < 0$
- d) $v_k > 1$

Answer: a

Explanation: A walk in a graph is said to be closed if the starting vertex is the same as the ending vertex, that is $v_0 = v_k$, it is described as Open otherwise.

324. A trail in a graph can be described as _____

- a) a walk without repeated edges
- b) a cycle with repeated edges
- c) a walk with repeated edges
- d) a line graph with one or more vertices

Answer: a

Explanation: Suppose in a graph G a trail could be defined as a walk with no repeated edges. Suppose a walk can be defined as efgh. There are no repeated edges so this walk is a trail.

325. Let a graph can be denoted as ncfkedn a kind of _____

- a) cycle graph
- b) line graph
- c) hamiltonian graph
- d) path graph

Answer: a

Explanation: In the graph ncfkedn, no edges are repeated in the walk, which makes it a trail and then start and end vertex n is same making it a cycle graph.

326. Determine the edge count of a path complement graph with 14 vertices.

- a) 502
- b) 345
- c) 78
- d) 69

Answer: c

Explanation: Let, an n-path complement graph P_n' is the graph complement of the path graph P_n . Since P_n is self-complementary, P_n' is isomorphic to P_n . Now, P_n' has an edge count $= \frac{1}{2}(n-2)(n-1)$. So, the required edge count is=78.

327. The sum of an n-node graph and its complement graph produces a graph called _____

- a) complete graph
- b) bipartite graph

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- c) star graph
- d) path-complement graph

Answer: a

Explanation: Suppose, the complement G' of a graph G is known as edge-complement graph which consists of with the same vertex set but whose edge set contains the edges not present in G . The graph sum $G+G'$ on an n -node graph G is called the complete graph say, K_n .

328. The maximum number of edges in a bipartite graph on 14 vertices is _____

- a) 56
- b) 14
- c) 49
- d) 87

Answer: c

Explanation: Maximum number of edges occur in a complete bipartite graph when every vertex has an edge to every opposite vertex in the graph. Number of edges in a complete bipartite graph is $a*b$, where a and b are no. of vertices on each side. This quantity is maximum when $a = b$ i.e. when there are 7 vertices on each side. So answer is $7 * 7 = 49$.

329. In a _____ the degree of each and every vertex is equal.

- a) regular graph
- b) point graph
- c) star graph
- d) euler graph

Answer: c

Explanation: A regular graph has the same degree in each of its vertices. In a regular bipartite graph, if the common degree of each vertices is 1, the two parts are of the same size.

330. The time complexity to test whether a graph is bipartite or not is said to be _____ using depth first search.

- a) $O(n^3)$
- b) linear time
- c) $O(1)$
- d) $O(n \log n)$

Answer: b

Explanation: It is possible to test whether a graph is bipartite, and to return either a two-coloring (if it is bipartite) or an odd cycle (if it is not) in linear time i.e, $O(n)$ using depth first search. In case of the intersection of n line segments or other simple shapes in the Euclidean graph, it is possible to test whether the graph is bipartite and it will

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return either a two-coloring or an odd cycle in time $O(n \log n)$, even though the graph itself has up to $O(n^2)$ edges.

331. The partition $V = V_1 \cup V_2$ in a bipartite graph G_1 is called _____

- a) bipartition of G_1
- b) 2-vertex set of G_1
- c) sub bipartite graphs
- d) disjoint vertex set

Answer: b

Explanation: A graph $G_1(V, E)$ is called bipartite if its vertex set $V(G)$ can be decomposed into two non-empty disjoint subsets $V_1(G_1)$ and $V_2(G_1)$ in such a way that each edge $e \in E(G)$ has its one end joint in $V_1(G_1)$ and other endpoint in $V_2(G_1)$. The partition $V = V_1 \cup V_2$ in a bipartite graph G_1 is called bipartition of G_1 .

332. What is the maximum number of edges in a bipartite graph on 14 vertices?

- a) 78
- b) 15
- c) 214
- d) 49

Answer: d

Explanation: By definition, the maximum possible number of edges

in a bipartite graph on 'n' vertices = $(1/4) \times n^2$.

Substituting $n = 14$, we get maximum number of edges in a bipartite graph on 14 vertices, $= (1/4) \times (14)^2$
 $= (1/4) \times 14 \times 14$
 $= 49$

$\therefore$ Maximum number of edges in a bipartite graph on 14 vertices = 49.

333. In a complete bipartite graph, the intersection of two sub graphs is _____

- a) 1
- b) null
- c) 2^{10}
- d) 412

Answer: b

Explanation: In a complete Bipartite graph, there must exist a partition say, $V(G) = X \cup Y$ and $X \cap Y = \emptyset$, that means all edges share a vertex from both set X and Y.

334. Bipartite graphs are used in _____

- a) modern coding theory
- b) colouring graphs
- c) neural networks
- d) chemical bonds

Answer: a

Explanation: All types of cyclic graphs are examples of cyclic graphs. A cyclic

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graph is considered bipartite if all the cycles involved are of even length. Bipartite graphs are widely used in modern coding theory apart from being used in modeling relationships.

335. The chromatic number of a graph is the property of

- a) graph coloring
- b) graph ordering
- c) group ordering
- d) group coloring

Answer: b

Explanation: A graph coloring is an assignment of labels to the vertices of a graph such that no two adjacent vertices share the same labels is called the colors of the graph. Now, the chromatic number of any graph is the minimal number of colors for which such an assignment is possible.

336. If a graph G is k -colorable and $k < n$, for any integer n then it is

- a) n -colorable
- b) n^2 nodes
- c) $(k+n)$ -colorable
- d) (k^3+n^3+1) nodes

Answer: a

Explanation: The chromatic number

of a graph is the minimal number of colors for which a graph coloring is possible. A graph G is termed as k -colorable if there exists a graph coloring on G with k colors. If a graph is k -colorable, then it is n -colorable for any $n > k$.

337. If C_n is the n th cyclic graph, where $n > 3$ and n is odd. Determine the value of $X(C_n)$.

- a) 32572
- b) 16631
- c) 3
- d) 310

Answer: c

Explanation: Here n is odd and $X(C_n) = 2$. Since there are two adjacent edges in C_n . Now, a graph coloring for C_n exists where vertices are colored red and blue alternatively and another edge is with a different colour say orange, then the value of $X(C_n)$ becomes 3.

338. Determine the density of a planar graph with 34 edges and 13 nodes.

- a) $22/21$
- b) $12/23$
- c) 328
- d) 576

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Answer: a

Explanation: The density of a planar graph or network is described as the ratio of the number of edges(E) to the number of possible edges in a network with(N) nodes. So, $D = \frac{E}{N(N-1)/2}$. Hence, the required answer is: $D = \frac{22}{21}$. A completely sparse planar graph has density 0 and a completely dense planar graph has degree 1.

339. If the number of vertices of a chromatic polynomial P_G is 56, what is the degree of P_G ?

- a) 344
- b) 73
- c) 265
- d) 56

Answer: d

Explanation: The chromatic polynomial P_G of a graph G is a polynomial in which every natural number k returns the number $P_G(k)$ of k -colorings of G . Since, the degree of P_G is equal to the number of vertices of G , the required answer is 56.

340. An undirected graph G which is connected and acyclic is called _____

- a) bipartite graph
- b) cyclic graph

- c) tree
- d) forest

Answer: c

Explanation: An undirected graph G which is connected and acyclic is termed as a tree. G contains no cycles and if any edge is added to G a simple cycle is formed.

341. An n -vertex graph has _____ edges.

- a) n^2
- b) $n-1$
- c) $n*n$
- d) $n*(n+1)/2$

Answer: b

Explanation: Suppose G is a connected graph which has no cycles. Every subgraph of G includes at least one vertex with zero or one incident edges. It has n vertices and $n-1$ edges. Generally, the order-zero graph is not considered to be a tree.

342. What is a star tree?

- a) A tree having a single internal vertex and $n-1$ leaves
- b) A tree having n vertices arranged in a line
- c) A tree which has 0 or more connected subtrees

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d) A tree which contains n vertices and $n-1$ cycles

Answer: a

Explanation: A star tree of order n is a tree with as many leaves as possible or in other words a star tree is a tree that consists of a single internal vertex and $n-1$ leaves. However, an internal vertex is a vertex of degree at least 2.

343. A polytree is called _____

- a) directed acyclic graph
- b) directed cyclic graph
- c) bipartite graph
- d) connected graph

Answer: a

Explanation: A directed acyclic graph is known as a polytree whose underlying undirected graph is a tree. In other words, a directed tree is a directed graph which would be tree if the directions on the edges were ignored.

344. The tree elements are called _____

- a) vertices
- b) nodes
- c) points
- d) edges

Answer: b

Explanation: Every tree element is called a node and the lines connecting the elements are called branches. A finite tree structure has a member that has no superior and is called the “root” Or root node. Nodes that have no child are called leaf nodes.

345. In an n -ary tree, each vertex has at most _____ children.

- a) n
- b) n^4
- c) $n*n$
- d) $n-1$

Answer: a

Explanation: An n -ary tree is a rooted tree in which each vertex has at most n children. 2-ary trees are termed as binary trees, 3-ary trees are sometimes called ternary trees.

346. A linear graph consists of vertices arranged in a line.

- a) false
- b) true
- c) either true or false
- d) cannot determined

Answer: b

Explanation: A linear graph also known as a path graph is a graph

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which consists of k vertices arranged in a line, so that vertices from i and $i+1$ are connected by an edge for $i=0, \dots, k-1$.

347. Two labeled trees are isomorphic if _____

- a) graphs of the two trees are isomorphic
- b) the two trees have same label
- c) graphs of the two trees are isomorphic and the two trees have the same label
- d) graphs of the two trees are cyclic

Answer: c

Explanation: The number of labeled trees of k number of vertices is k^{n-2} . Two labeled trees are isomorphic if their graphs are isomorphic and the corresponding points of the two trees have the same labels.

348. A graph which consists of disjoint union of trees is called _____

- a) bipartite graph
- b) forest
- c) caterpillar tree
- d) labeled tree

Answer: b

Explanation: A forest is an undirected acyclic graph in which all the

connected components are individual trees. This graph contains a disjoint union of trees.

349. What is a bipartite graph?

- a) a graph which contains only one cycle
- b) a graph which consists of more than 3 number of vertices
- c) a graph which has odd number of vertices and even number of edges
- d) a graph which contains no cycles of odd length

Answer: d

Explanation: A graph is called a bipartite graph if and only if it contains no cycle of odd length. Every tree is a bipartite graph and a median graph.

350. If two cycle graphs G_m and G_n are joined together with a vertex, the number of spanning trees in the new graph is _____

- a) $m+n-1$
- b) $m-n$
- c) $m*n$
- d) $m*n+1$

Answer: c

Explanation: As there are n possible edges to be removed from G and m edges to be removed from G and the

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rest from a spanning tree so the number of spanning tree in the new graph is $m \cdot n$.

351. For an n -vertex undirected graph, the time required to find a cycle is _____

- a) $O(n)$
- b) $O(n^2)$
- c) $O(n+1)$
- d) $O(\log n)$

Answer: a

Explanation: The existence of a cycle in directed and undirected graphs can be determined by depth-first search (DFS) of the graph finds an edge that points to an ancestor of the current vertex. In an undirected graph, finding any already visited vertex will indicate a back edge. All the back edges which DFS skips over are part of cycles. In the case of undirected graphs, only $O(n)$ time is required to find a cycle in an n -vertex graph, since at most $n - 1$ edges can be tree edges.

352. A binary cycle space forms a _____ over the two element field.

- a) triangular graph
- b) vector space
- c) binary tree
- d) hamiltonian graph

Answer: b

Explanation: The term cycle refers to an element of the cycle space of a graph. There are many cycle spaces. The most common is the binary cycle space, which contains the edge sets that have even degrees at every vertex and it forms a vector space over the two-element field.

1. In preorder traversal of a binary tree the second step is _____

- a) traverse the right subtree
- b) traverse the left subtree
- c) traverse right subtree and visit the root
- d) visit the root

View Answer

Answer: b

Explanation: In a preorder traversal of a binary tree first is to visit the root, second traverse the left subtree of the tree and third traverse the right subtree of the tree.

353. An important application of binary tree is _____

- a) Huffman coding
- b) stack implementation
- c) queue implementation
- d) traverse a cyclic graph

Answer: a

Explanation: A binary tree is used to sort a list of elements; the inorder traversal will do this automatically. Better tree sorting algorithm will

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involve balancing the trees. The binary coding, in particular for the Huffman coding is an immediate application of binary trees.

354. From the following code identify the which traversal of a binary tree is this _____

```
//if node has left child  
order(node.left)  
//if node has right child  
order(node.right)  
visit(node)
```

- a) Inorder traversal
- b) preorder traversal
- c) postorder traversal
- d) Euler tour traversal

Answer: c

Explanation: In a postorder traversal of a binary tree first is to traverse the left subtree, second traverse the right subtree of the tree and third is to visit the node.

355. What is the minimum height for a binary search tree with 60 nodes?

- a) 1
- b) 3
- c) 4
- d) 2

Answer: d

Explanation: If there are k nodes in a binary tree, maximum height of that tree should be k-1, and minimum height should be $\text{floor}(\log_2 k)$. By

using the formula, minimum height must be 2 when there are 60 nodes in a tree.

356. From the following code identify the which traversal of a binary tree is this _____

```
function traversal(node)  
{  
    //Input:root node of the tree  
    //Output:None  
    visitLeft(node)  
    //if node has left child  
    traversal(node.left)  
    visit_Below(node)  
    //if node has right child  
    traversal(node.right)  
    visitRight(node)  
}
```

- a) Inorder traversal
- b) Euler Tour traversal
- c) Post-order traversal
- d) Pre-order Traversal

Answer: b

Explanation: The code signifies Euler Tour traversal which is a generic traversal of a binary tree. In this tree traversal we have to walk around the tree and visit each node three times: 1. On the left (pre-order), 2. From below (in-order), 3. On the right (post-order) and Create subtrees for all the nodes.

357. For the expression $(7 - (4 * 5)) + (9 / 3)$ which of the following is

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the post order tree traversal?

- a) $*745-93/+$
- b) $93/+745*-$
- c) $745*-93/+$
- d) $74*+593/-$

Answer: c

Explanation: First build a binary tree for the expression then find out the postorder traversal of that tree and after that the answer will be $745*-93/+$.

358. The time complexity of calculating the sum of all leaf nodes in an n-order binary tree is

-
- a) $O(n^2)$
 - b) $O(n+1)$
 - c) $O(1)$
 - d) $O(n)$

Answer: d

Explanation: The approach is to traverse the binary tree in any fashion and check if the node is the leaf node(child node) or not. After that, add node data to the sum variable. So, after summing up all leaf nodes, the time complexity of the operation should be $O(n)$.

359. An immediate application of a Depth First Search traversal is

-
- a) count the number of leaf nodes
 - b) perform Inorder traversal in easy way

- c) count number of nodes
- d) implement preorder traversal

Answer: a

Explanation: Given an n-ary binary tree, by performing DFS traversal on that tree number of leaf nodes can be calculated and for that we need to maintain an array for the leaf count.

360. Breadth First Search traversal of a binary tree finds its application in

-
- a) Cloud computing
 - b) Peer to peer networks
 - c) Weighted graph
 - d) Euler path

Answer: b

Explanation: Breadth First Search traversal has diverse applications such as in the peer to peer networks like BitTorrent, BFS traversal is used to find all the neighbour nodes of the network.

361. Worst case complexity of Breadth First Search traversal

-
- a) $O(n*n)$
 - b) $O(n \log n)$
 - c) $O(n^2 \log n)$
 - d) $O(n^3)$

Answer: b

Explanation: In an n-ary binary tree, we must have to visit all nodes from an adjacent node and repeat the

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same for next unvisited nodes.
Hence, in worst case the time complexity should be $O(n \log n)$.

362. Evaluation of expression $a/b+c*d-e$ in postfix notation.

- a) $ab+cd/*-e$
- b) $ab/cd*+e-$
- c) $abc/+d*-e$
- d) $abcd/+*-e$

Answer: b

Explanation: The expression $=a/b+c*d-e$
 $=\{(ab/)+(cd*)\}-e$
 $=\{(ab/)(cd*)+ \}-e$
 $=\{(ab/)(cd*)+ \}e-$
So the output is: $ab/cd*+e-$

363. Evaluation of $4*5+3/2-9$ in prefix notation.

- a) $*45-/32+9$
- b) $*+453/-29$
- c) $-+*45/329$
- d) $*+/45932$

Answer: c

Explanation: The expression $=4*5+3/2-9$
 $=\{(4*5)+(3/2)-9\}$
 $=\{(*45)+(/32)-9\}$
 $=\{+(*45)(/32)\}-9$
 $=- \{+(*45)(/32)9\}$
So the output is; $-+*45/329$.

364. What is the output of the following if $\text{func1}(7)$?

```
{
    int n;
    long int func;
    scanf("%d",&n);
    func=func1 (n)
    printf("%ld!=%ld",n,func);
}
long int func1(int n)
{
    if(n==0)
    {
        Return 1;
    }
    else
    {
        return(n*func1(n-1));
    }
}
```

- a) 128
- b) 4320
- c) 720
- d) 5040

Answer: d

Explanation: This is a factorial function of an integer using recursive approach. By running the function on integer 7 we get 5040.

365. Infix to prefix conversion can be done using _____

- a) two queues
- b) two stacks
- c) one stack and two queues
- d) one stack

Void main()

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Answer: b

Explanation: In the infix expression, the operator appears between the operands and in infix notation if the operator appears before the operands in the expression. For the conversion between them two stacks are used efficiently. The idea is to use one stack for operators and other to store operands.

366. Conversion from prefix to postfix expression can be done

- a) using bubble sort
- b) using radix sort
- c) using two queues
- d) in a direct manner

Answer: d

Explanation: In a postfix expression, the operators appear after the operands. Conversion from prefix to postfix is done directly which is better than converting the prefix expression in infix and then infix to postfix expression. It gives better efficiency.

367. What is the postfix expression of $9+3*5/(10-4)$?

- a) $9\ 3\ +\ *\ 5\ /\ 10\ 4\ -$
- b) $9\ 3\ 5\ +\ *\ /\ 10\ 4\ -$
- c) $9\ 3\ +\ 5\ *\ /\ 10\ 4\ -$
- d) $9\ 3\ 5\ *\ /\ +\ 10\ -\ 4$

Answer: c

Explanation: The expression, $9+3*5/(10-4)$

$$= 9+3*5/(10-4)$$

$$= 9+35/*(10-4)$$

$$= 935/*+(10-4)$$

So the output is: $9\ 3\ 5\ /\ *\ +\ 10\ 4\ -$.

368. What is the postfix expression of $(A+B)-C*(D/E))+F$?

- a) $A\ B\ +\ C\ D\ E\ /\ *\ -\ F\ +$
- b) $A\ B\ C\ D\ E\ +\ /\ *\ F\ -\ +$
- c) $A\ B\ C\ +\ *\ D\ E\ /\ F\ +\ -$
- d) $A\ B\ +\ C\ -\ *\ D\ E\ /\ F\ +$

Answer: a

Explanation: The expression is $(A+B)-C*(D/E))+F$

$$= (A+B)-C*(DE/)+F$$

$$= (A+B)-C*(DE/)F+$$

$$= (A+B)-C(DE/)*F+$$

$$= (A+B)C(DE/)*-F+$$

$$= (AB+)C(DE/)*-F+$$

So the output is: $AB+CDE/*-F+$.

369. Algebra of logic is termed as

- a) Numerical logic
- b) Boolean algebra
- c) Arithmetic logic
- d) Boolean number

Answer: c

Explanation: The variables that can have two discrete values False(0) and True(1) and the operations of logical significance are dealt with Boolean algebra.

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370. Boolean algebra can be used _____

- a) For designing of the digital computers
- b) In building logic symbols
- c) Circuit theory
- d) Building algebraic functions

Answer: a

Explanation: For designing digital computers and building different electronic circuits boolean algebra is accepted widely.

371. What is the definition of Boolean functions?

- a) An arithmetic function with k degrees such that $f:Y \rightarrow Y^k$
- b) A special mathematical function with n degrees such that $f:Y^n \rightarrow Y$
- c) An algebraic function with n degrees such that $f:X^n \rightarrow X$
- d) A polynomial function with k degrees such that $f:X^2 \rightarrow X^n$

Answer: b

Explanation: A Boolean function is a special mathematical function with n degrees and where $Y = \{0,1\}$ is the Boolean domain with n being a non-negative integer. It helps in describing the way in which the Boolean output is derived from Boolean inputs.

372. $F(X,Y,Z,M) = X'Y'Z'M'$. The degree of the function is _____

- a) 2
- b) 5
- c) 4
- d) 1

Answer: c

Explanation: This is a function of degree 4 from the set of ordered pairs of Boolean variables to the set $\{0,1\}$.

373. A _____ value is represented by a Boolean expression.

- a) Positive
- b) Recursive
- c) Negative
- d) Boolean

Answer: d

Explanation: A Boolean value is given by a Boolean expression which is formed by combining Boolean variables and logical connectives.

374. What is the use of Boolean identities?

- a) Minimizing the Boolean expression
- b) Maximizing the Boolean expression
- c) To evaluate a logical identity
- d) Searching of an algebraic expression

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Answer: a

Explanation: Boolean identities are used for minimizing the Boolean expression and transforming into an equivalent expression.

375. _____ is used to implement the Boolean functions.

- a) Logical notations
- b) Arithmetic logics
- c) Logic gates
- d) Expressions

Answer: c

Explanation: To implement a Boolean function logic gates are used. Basic logic gates are AND, OR and NOT.

376. Inversion of single bit input to a single bit output using _____

- a) NOT gate
- b) NOR gate
- c) AND gate
- d) NAND gate

Answer: a

Explanation: A NOT gate is used to invert a single bit input (say A) to a single bit of output ($\sim A$).

377. There are _____ numbers of Boolean functions of degree n.

- a) n
- b) $2^{(2*n)}$

c) n^3

d) $n^{(n*2)}$

Answer: b

Explanation: There are $2n$ different n-tuples of 0's and 1's. A Boolean function is an assignment of 0's or 1's to each of these $2n$ different n-tuples. Hence, there are $2^{(2*n)}$ different Boolean functions.

378. A _____ is a Boolean variable.

- a) Literal
- b) String
- c) Keyword
- d) Identifier

Answer: a

Explanation: A literal is a Boolean variable or its complement. A maxterm is a sum of n literals and a minterm is a product of n literals.

379. Find the simplified expression $A'BC' + AC'$.

- a) B
- b) $A+C$
- c) $(A+B)C'$
- d) $B'C$

Answer: c

Explanation: Given: $A'BC' + AC'$
 $= C'(A'B + A)$
 $= C'(A + B).$

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380. Evaluate the expression: $(X + Z)(X + XZ') + XY + Y$.

- a) $XY + Z'$
- b) $Y + XZ' + Y'Z$
- c) $X'Z + Y$
- d) $X + Y$

Answer: d

Explanation: $(X + Z)(X + XZ') + XY + Y$

[Original Expression]

$= (x + z)X(1 + Z') + XY + Y$ [Distributive]

$= (X + Z)X + XY + Y$ [Complement, Identity]

$= (X + Z)X + Y(X + 1)$ [Distributive]

$= (X + Z)X + Y$ [Idempotent]

$= XX + XZ + Y$ [Distributive]

$= X + XZ + Y$ [Identity]

$= X(1 + Z) + Y$

$= X + Y$ [Idempotent].

381. Simplify the expression: $A'(A + BC) + (AC + B'C)$.

- a) $(AB'C + BC')$
- b) $(A'B + C')$
- c) $(A + BC)$
- d) AC

Answer: b

Explanation: Given: $A'(A + BC) + (AC + B'C)$

$= A'A + A'BC + AC + B'C$

$= A'BC + C(A + B')$

$= C(A'B + A + B')$

$= C(A + B + B')$

$= C(A + 1)$

$= AC$.

382. What is the simplification value of $MN(M + N') + M(N + N')$?

- a) M
- b) $MN + M'N'$
- c) $(1 + M)$
- d) $M + N'$

Answer: b

Explanation: Given: $MN(M + N') + M(N + N')$

$= MN(M + N') + M.1$

$= MNM + MNN' + M$

$= MN + 0 + M$

$= M(N + 1)$

$= M$.

383. Simplify the expression $XZ' + (Y + Y'Z) + XY$.

- a) $(1 + XY')$
- b) $YZ + XY' + Z'$
- c) $(X + Y + Z)$
- d) $XY' + Z'$

Answer: c

Explanation: Given: $XZ' + (Y + Y'Z) + XY$

$= XZ' + (Y + Z) + XY$

$= XZ' + Y + Z + XY$

$= (XZ' + Z) + (Y + XY)$

$= (X + Z) + Y(1 + X)$

$= X + Y + Z$.

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384. How many binary relations are there on a set S with 9 distinct elements?

- a) 2^{90}
- b) 2^{100}
- c) 2^{81}
- d) 2^{60}

Answer: c

Explanation: S is the set with 9 elements. A relation on S is defined as $S \times S$. There are 9^2 number of ordered pairs in relation. So, the number of binary relations is $2^{(9 \times 9)} = 2^{81}$.

385. _____ number of reflexive relations are there on a set of 11 distinct elements.

- a) 2^{110}
- b) 3^{121}
- c) 2^{90}
- d) 2^{132}

Answer: a

Explanation: Let A be a set consists of n distinct elements. There are $2^{(n \times n) - n}$ number of reflexive relations that can be formed. So, here the answer is $2^{(11 \times 11) - 11} = 2^{110}$.

386. The number of reflexive as well as symmetric relations on a set with 14 distinct elements is _____

- a) 4^{120}
- b) 2^{70}
- c) 3^{201}
- d) 2^{91}

Answer: d

Explanation: Let A be a set consists of n distinct elements. There are $2^{(n \times (n-1))/2}$ number of reflexive and symmetric relations that can be formed. So, here the answer is $2^{14 \times (14-1)/2} = 2^{91}$.

387. The number of symmetric relations on a set with 15 distinct elements is _____

- a) 2^{196}
- b) 2^{50}
- c) 2^{320}
- d) 2^{78}

Answer: a

Explanation: Let S be a set consists of n distinct elements. There are $2^{(n-1) \times (n-1)}$ number of reflexive and symmetric relations that can be formed. So, here the answer is $2^{(15-1) \times (15-1)} = 2^{196}$.

388. R is a binary relation on a set S and R is reflexive if and only if _____

- a) $r(R) = R$
- b) $s(R) = R$

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c) $t(R) = R$

d) $f(R) = R$

Answer: a

Explanation: Let reflexive closure of $R: r(R) = R$. If R is reflexive, it satisfies all the condition in the definition of reflexive closure. So, a reflexive closure of a relation is the smallest number of reflexive relation contain in R . Hence, $R = r(R)$.

389. If R_1 and R_2 are binary relations from set A to set B , then the equality _____ holds.

a) $(R^c)^c = R^c$

b) $(A \times B)^c = \Phi$

c) $(R_1 \cup R_2)^c = R_1^c \cup R_2^c$

d) $(R_1 \cup R_2)^c = R_1^c \cap R_2^c$

Answer: c

Explanation: To proof $(R_1 \cup R_2)^c = R_1^c \cup R_2^c$,

if $\langle x, y \rangle$ belongs to $(R_1 \cup R_2)^c$

$$\Leftrightarrow \langle y, x \rangle \in (R_1 \cup R_2)$$

$$\Leftrightarrow \langle y, x \rangle \in R_1 \text{ or } \langle y, x \rangle \in R_2$$

$$\Leftrightarrow \langle x, y \rangle \in R_1^c \text{ or } \langle x, y \rangle \in R_2^c$$

$$\Leftrightarrow \langle x, y \rangle \in R_1^c \cup R_2^c.$$

390. The condition for a binary relation to be symmetric is _____

a) $s(R) = R$

b) $R \cup R = R$

c) $R = R^c$

d) $f(R) = R$

Answer: c

Explanation: If $\langle a, b \rangle \in R$ then $\langle b, a \rangle \in R$, where a and b belong to two different sets and so its symmetric.

R^c also contains $\langle b, a \rangle$

$$R^c = R.$$

391. _____ number of reflexive closure exists in a relation $R = \{(0,1), (1,1), (1,3), (2,1), (2,2), (3,0)\}$ where $\{0, 1, 2, 3\} \in A$.

a) 2^6

b) 6

c) 8

d) 36

Answer: b

Explanation: The reflexive closure of R is the relation, $R \cup \Delta = \{ \langle a, b \rangle \mid \langle a, b \rangle \in R \text{ or } \langle a, a \rangle \mid a \in A \}$. Hence, $R \cup \Delta = \{(0,1), (1,1), (1,3), (2,1), (2,2), (3,0)\}$ and the answer is 6.

392. The binary relation $\{(1,1), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2)\}$ on the set $\{1, 2, 3\}$ is _____

a) reflexive, symmetric and transitive

b) irreflexive, symmetric and transitive

c) neither reflexive, nor irreflexive

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but transitive

d) irreflexive and antisymmetric

Answer: c

Explanation: Not reflexive $\rightarrow (3,3)$ not present; not irreflexive $\rightarrow (1, 1)$ is present; not symmetric $\rightarrow (2, 1)$ is present but not $(1, 2)$; not antisymmetric – $(2, 3)$ and $(3, 2)$ are present; not asymmetric $\rightarrow$ asymmetry requires both antisymmetry and irreflexivity. So, it is transitive closure of relation.

393. Consider the relation: $R' (x, y)$ if and only if $x, y > 0$ over the set of non-zero rational numbers, then R' is _____

- a) not equivalence relation
- b) an equivalence relation
- c) transitive and asymmetry relation
- d) reflexive and antisymmetric relation

Answer: b

Explanation: Reflexive: a, $a > 0$

Symmetric: if a, $b > 0$ then both must be +ve or -ve, which means b, $a > 0$ also exists

Transitive: if a, $b > 0$ and b, $c > 0$ then to have b as same number, both pairs must be +ve or -ve which implies a, $c > 0$. Hence, R' is an equivalence relation.

394. Let S be a set of $n > 0$ elements.

Let B_r be the number of binary relations on S and let B_f be the number of functions from S to S . The expression for B_r and B_f , in terms of n should be _____

- a) n^2 and $2(n+1)^2$
- b) n^3 and $n^{(n+1)}$
- c) n and $n^{(n+6)}$
- d) $2^{(n \cdot n)}$ and n^n

Answer: d

Explanation: For a set with n elements the number of binary relations should be $2^{(n \cdot n)}$ and the number of functions should be n^n . Hence $B_r = 2^{(n \cdot n)}$ and $B_f = n^n$.

395. Spanning trees have a special class of depth-first search trees named _____

- a) Euclidean minimum spanning trees
- b) Tremaux trees
- c) Complete bipartite graphs
- d) Decision trees

Answer: b

Explanation: A tremaux tree of an undirected graph G is a spanning tree of G which is rooted at one of its vertices with the property that every two adjacent vertices in G are related to each other as an ancestor and descendant in the tree.

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396. If the weight of an edge e of cycle C in a graph is larger than the individual weights of all other edges of C , then that edge _____

- a) belongs to an minimum spanning tree
- b) cannot belong to an minimum spanning tree
- c) belongs to all MSTs of the graph
- d) can not belong to the graph

Answer: b

Explanation: For any cycle C in the graph, if the weight of an edge e of C is larger than the individual weights of all other edges of C , then this edge cannot belong to an MST.

397. For every spanning tree with n vertices and n edges what is the least number of different Spanning trees can be formed?

- a) 2
- b) 5
- c) 3
- d) 4

Answer: c

Explanation: If graph is connected and has ' n ' edges, there will be exactly one cycle, if n vertices are there. A different spanning tree can be constructed by removing one edge from the cycle, one at a time. The

minimum cycle length can be 3. So, there must be at least 3 spanning trees in any such Graph. Consider a Graph with $n = 4$, then 3 spanning trees possible at maximum (removing edges of cycle one at a time, alternatively). So, any Graph with minimum cycle length '3' will have at least 3 spanning trees.

398. Time complexity of Prim's algorithm is _____

- a) $O((V+E)\log V)$
- b) $O(E+V)$
- c) $O(E)$
- d) $O(V+1)$

Answer: a

Explanation: In Prim's Algorithm, we will start with an arbitrary node (take any point to start) and mark it. In each iteration, a new vertex is marked that is adjacent to the one that we have already marked. Each vertex is inserted in the priority queue only once and insertion in priority queue take logarithmic time. Hence, the time complexity of Prim's Algorithm is $O((V+E)\log V)$.

399. What is the time complexity of Kruskal's algorithm?

- a) $O(E\log V)$
- b) $O(V+\log E)$

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- c) $O(E+1)$
- d) $O(V^2)$

Answer: a

Explanation: In Kruskal's algorithm, at each iteration, we will select the edge with the lowest weight. So, we will start with the lowest weighted edge first. After that we will select the second lowest weighted edge. In Kruskal's algorithm, most time consuming operation is sorting because the total complexity of the Disjoint-Set operations will be $O(E \log V)$ and it is the overall Time Complexity of the algorithm.

400. An immediate application of minimum spanning tree _____

- a) gesture analysis
- b) handwriting recognition
- c) fingerprint detection
- d) soft computing

Answer: b

Explanation: Minimum spanning tree is the spanning tree where the cost is minimum among all the spanning trees. It is used in network designing, in the algorithms predicting the travelling salesman problem, multi-terminal minimum cut problem and minimum-cost weighted perfect matching. It can also be used in

Handwriting recognition and image segmentation.

401. The spanning tree will be maximally acyclic if _____

- a) one additional edge makes a cycle in the tree
- b) two additional edges make a cycle in the tree
- c) removing one edge makes the tree cycle free
- d) removing two edges make the tree cycle free

Answer: a

Explanation: A connected graph G can have more than one spanning tree. Removing one edge from the spanning tree will make the graph disconnected and the spanning tree is minimally connected. Adding one edge to the spanning tree will create a circuit or loop and the spanning tree is maximally acyclic.

402. In a maximum spanning tree the weighted graph is of _____

- a) maximum number of edges
- b) maximum number of cyclic trees
- c) minimum number of vertices
- d) maximum weight

Answer: d

Explanation: A maximum spanning tree can be computed by negating

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the weights for each edge and applying Kruskal's algorithm. Thus, it is a spanning tree of a weighted graph having maximum weight assigned to all the edges.

403. Prim's algorithm can be implemented using _____

- a) a stack data structure
- b) radix sort
- c) priority queue data structure
- d) bubble sort

Answer: c

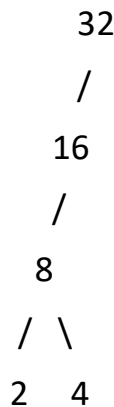
Explanation: The time complexity of Prim's algorithm depends on the data structures used for the graph and for ordering the edges by weight, which can be done using a priority queue. In general, a priority queue will be quicker at finding the vertex in the spanning tree with minimum cost. The choice of data structures for implementation will lead to varying time complexity.

404. Let a set $S = \{2, 4, 8, 16, 32\}$ and $\leq$ be the partial order defined by $S \leq R$ if a divides b . Number of edges in the Hasse diagram of is _____

- a) 6
- b) 5
- c) 9
- d) 4

Answer: b

Explanation: Hasse Diagram is:



So, the number of edges should be: 4.

405. The less-than relation, $<$, on a set of real numbers is _____

- a) not a partial ordering because it is not asymmetric and irreflexive equals antisymmetric
- b) a partial ordering since it is asymmetric and reflexive
- c) a partial ordering since it is antisymmetric and reflexive
- d) not a partial ordering because it is not antisymmetric and reflexive

Answer: a

Explanation: Relation less than a set of real numbers is not antisymmetric and reflexive. Relation is not POSET because it is irreflexive. Again, $aRb \neq bRa$ unless $a=b$ and so it is antisymmetric. A relation may be 'not asymmetric and not reflexive but still antisymmetric, as $\{(1,1) (1,2)\}$. So, the

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relation is not a partial ordering because it is not asymmetric and irreflexive equals antisymmetric.

406. If the longest chain in a partial order is of length l , then the partial order can be written as _____ disjoint antichains.

- a) l^2
- b) $l+1$
- c) l
- d) $l!$

Answer: c

Explanation: If the length of the longest chain in a partial order is l , then the elements in the POSET can be partitioned into l disjoint antichains.

407. Suppose $X = \{a, b, c, d\}$ and π_1 is the partition of X , $\pi_1 = \{\{a, b, c\}, d\}$. The number of ordered pairs of the equivalence relations induced by _____

- a) 15
- b) 10
- c) 34
- d) 5

Answer: b

Explanation: The ordered pairs of the equivalence relations induced = $\{(a,a), (a,b), (a,c), (b,a), (b,b), (b,c),$

$(c,a), (c,b), (c,c), (d,d)\}$. Poset $\rightarrow$ equivalence relations = each partition power set $-\Phi$.

408. A partial order P is defined on the set of natural numbers as follows. Here a/b denotes integer division.

i) $(0, 0) \in P$. ii) $(a, b) \in P$ if and only if $a \% 10 \leq b \% 10$ and $(a/10, b/10) \in P$.

Consider the following ordered pairs:

- i. $(101, 22)$ ii. $(22, 101)$ iii. $(145, 265)$
- iv. $(0, 153)$

The ordered pairs of natural numbers are contained in P are _____ and _____

- a) $(145, 265)$ and $(0, 153)$
- b) $(22, 101)$ and $(0, 153)$
- c) $(101, 22)$ and $(145, 265)$
- d) $(101, 22)$ and $(0, 153)$

Answer: d

Explanation: For ordered pair (a, b) , to be in P , each digit in a starting from unit place must not be larger than the corresponding digit in b . This condition is satisfied by options (iii) $(145, 265) \Rightarrow 5 \leq 5, 4 < 6$ and $1 < 2$; (iv) $(0, 153) \Rightarrow 0 < 3$ and no need to examine further.