THERMODYNAMICS

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THERMODYNAMIC

Open System	closed System	Isolated System
→ System which can exchange both energy & matter with the surrounding	→ System can exchange only energy but not matter with its surrounding	→ System exchange neither energy nor matter with its surroundings
→ Boiling of liquid in open vessel	→ Refrigerator, battery etc.	→ universe = system + surrounding

→ State Variables: (Path independent)

The variable change depends only on initial and final states.

→ exact differential: $\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y}$

→ example: internal energy (U), entropy (S), Pressure (P), volume (V), Temperature (T), Ethalpy (H) etc.

→ Non-State Variable: (Path Dependent)

The variable change depends on initial and final states as well as on path connecting these state

→ not exact differential: $\frac{\partial^2 f}{\partial y \partial x} \neq \frac{\partial^2 f}{\partial x \partial y}$

→ example: Heat, workdone.

Que: which of the following is not exact differential?

a) dQ b) dU c) dS d) df

Soln: dQ → change in Heat

Heat → Path independent

i.e non exact differential

A

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Que 2 which of the following represents the energy in storage?

a) Heat b) work c) internal energy d) none

Sol^n:- Energy in storage is internal energy. C

→ **Intensive:**

If total system is divided into two equal parts, then the variables which are independent of this division are called, intensive variables.

examples: Pressure, Temperature, chemical Potential, Particle density, molar mass, specific Heat, boiling Point etc.

→ **Extensive Variables:**

Variables which are exactly 1/2 after division.

examples: Area, volume, no. of Particles, U, A, G, H, Heat, Heat capacity, mass etc.

→ All extensive properties are additive

→ Ratio of two extensive quantity = Intensive

eg. $n = \frac{N/2}{V/2} = \frac{N}{V}$

Que 3 Imagine dividing a fluid with internal energy U, entropy S, volume V, particle density N, temperature T, pressure P & chemical potential H into two equal parts. The variables that remain unaffected by this division are:

a) U, V, S, N b) U, V, T, P c) T, P, H d) T, P, S

Sol^n:- Intensive: P, T, H, N

Extensive: V, U

C

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→ **Internal Energy**: Total energy stored inside the system is called internal energy.

$$U = U_{K.E}(T) + U_{P.E}(T, V)$$

For ideal gas: $U = U_{K.E}(T) \Rightarrow \left(\frac{\partial U}{\partial V}\right)_T = 0$ Joule's law

→ state function & Extensive variable

→ change in internal energy:

$$dU = \left(\frac{\partial U}{\partial T}\right)_V dT = C_V dT$$

Que: Internal energy of ideal gas depends on:

a) T & V b) V & P c) V only d) T only

Soln: $U_{ideal} = U_{K.E}(T)$ [D]

→ **workdone**: Form of energy that transfer due to change in Pressure.

$$w = \int_i^f F \cdot dr = \int p\,dV$$

→ Path dependent & not-exact differential.

external workdone	Internal workdone
workdone due to interaction between system & its surrounding	workdone due to interaction between different parts of the system

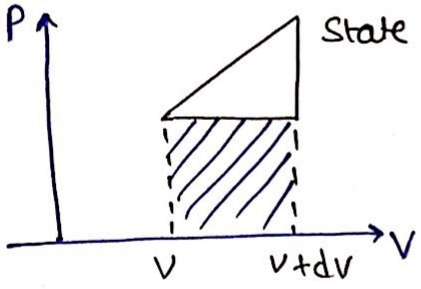
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→ **Heat**: is form of energy that transfer due to temperature difference.

→ Heat can flow spontaneously from:

Hot body $\xrightarrow{\text{to}}$ cold body

→ **Indicator Diagram** (P-V diagram)



$\delta w = PdV$

= Area under the curve

$\delta w = \int_{V_1}^{V_2} PdV$

→ **Cyclic Process**: a process that undergoes the system from one equilibrium state to another & come back to initial state.

→ $\Delta w = \oint PdV$ → workdone in cyclic Process.

→ $dU = 0$ (change in state variable = 0)

Thermodynamic Process:

1) **Isobaric Process**: Pressure ⇒ constant

$w = P(V_2 - V_1)$ & $\Delta P = 0$

2) **Isochoric Process**: volume ⇒ constant

$w = 0$ & $\Delta V = 0$

3) **Isothermal Process** (slow Process): Temperature = Const.

$w = 2.303\, nRT \log\left(\frac{V_2}{V_1}\right)$ & $\Delta T = 0$

4) **Adiabatic Process**: Heat ⇒ constant

$w = \frac{nR}{1-\gamma}(T_2 - T_1)$ & $\Delta Q = 0$

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→ Quasi-Static Process: process in which system is approximately in equilibrium

→ Rate of process is very small.

Reversible Process: process that can proceed in either direction. conditions:

1) Process must be quasi-static

2) Process must be non-dissipative (frichon, resistance, viscosity must be absent).

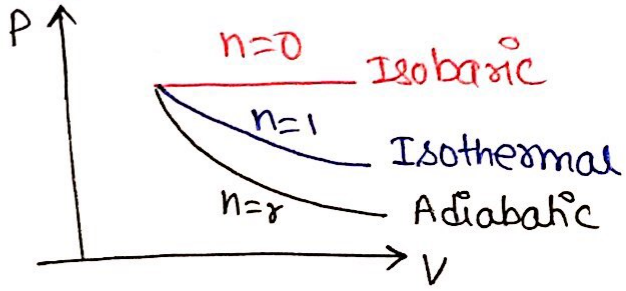
example: Isothermal quash-static expension of gas.

→ Adiabatic equation of Perfect gas:

PV^γ = constant

$TV^{\gamma-1}$ = constant

$Tp^{(1-\gamma)/\gamma}$ = constant



PV^n = Constant

Qu 5 A box, seperated by a movable ball, has two compartments are equal, but the pressure are $3P_0$ & P_0, respectively. when the walls is allowed to move, the final Pressure in the two compartments becomes equal. The final pressure is:

a) $\left(\frac{2}{3}\right)^\gamma P_0$ b) $3\left(\frac{2}{3}\right)^\gamma P_0$ c) $\frac{1}{2^\gamma}(1+3^{1/\gamma})^\gamma P_0$ d) $\left(\frac{3^{1/\gamma}}{1+3^{1/\gamma}}\right)P_0$

Soln:- Adiabatic eqn: PV^γ = Constant

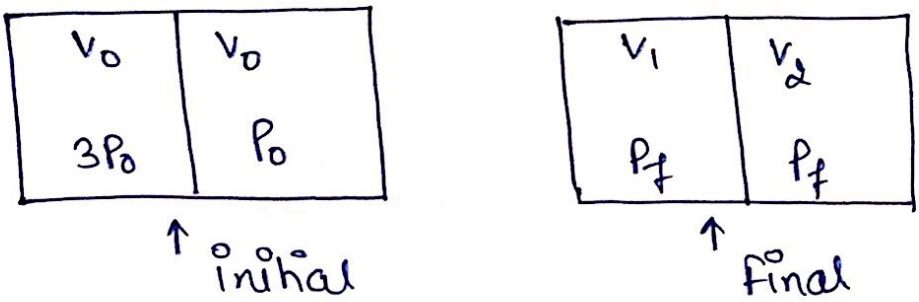
$$P_i V_i^\gamma = P_f V_f^\gamma$$

$$(3P_0)V_0^\gamma = P_f V_1^\gamma \Rightarrow \left(\frac{3P_0 V_0^\gamma}{P_f}\right)^{1/\gamma} = V_1 \quad (1)$$

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Similarly,

$$(P_0)V_0^{\gamma} = P_f V_2^{\gamma} \Rightarrow V_2 = \left(\frac{P_0 V_0^{\gamma}}{P_f}\right)^{1/\gamma}$$



$$V_1 + V_2 = V_0 + V_0$$

$$\left(\frac{3P_0 V_0^{\gamma}}{P_f}\right)^{1/\gamma} + \left(\frac{P_0 V_0^{\gamma}}{P_f}\right)^{1/\gamma} = 2V_0$$

$$\frac{(3P_0)^{1/\gamma} + (P_0)^{1/\gamma}}{(P_f)^{1/\gamma}} = 2$$

C

$$P_f = \frac{1}{2^{\gamma}} P_0 \left[1 + 3^{1/\gamma}\right]^{\gamma}$$

Que 6 Two different thermodynamic systems are described by the following eqn of state:

$$\frac{1}{T^{(1)}} = \frac{3RN^{(1)}}{2U^{(1)}} \quad \& \quad \frac{1}{T^{(2)}} = \frac{5RN^{(2)}}{2U^{(2)}}$$

where $T^{(1,2)}$, $N^{(1,2)}$ & $U^{(1,2)}$ are respectively, the temperatures, the mole numbers & the internal energy of the two systems, and R is the gas constant. Let U_{tot} denote the total energy when these two systems are put in contact and attain thermal equilibrium. The ratio $U^{(1)}/U^{tot}$ is:

a) $\frac{5N^{(2)}}{3N^{(1)}+5N^{(2)}}$ b) $\frac{3N^{(1)}}{3N^{(1)}+5N^{(2)}}$ c) $\frac{N^{(1)}}{N^{(1)}+N^{(2)}}$ d) $\frac{N^{(2)}}{N^{(1)}+N^{(2)}}$

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Soln:- $U_1 = \frac{3}{2} R N^{(1)} T^{(1)}$ & $U_2 = \frac{5}{2} R N^{(2)} T^{(2)}$

$$U_{total} = U^{(1)} + U^{(2)}$$

$$\frac{U^{(1)}}{U_{tot}} = \frac{\frac{3}{2} R N^{(1)} T^{(1)}}{\frac{3}{2} R N^{(1)} T^{(1)} + \frac{5}{2} R N^{(2)} T^{(2)}}$$

in thermal eqm.: $T^{(1)} = T^{(2)}$

$$\frac{U^{(1)}}{U_{tot}} = \frac{3 N^{(1)}}{3N^{(1)} + 5N^{(2)}}$$

B

Que:- when an ideal monoatomic gas is expended adiabatically from an initial volume V_0 to $3V_0$, its temperature changes from T_0 to T. Then the ratio T/T_0 is:

a) $\frac{1}{3}$ b) $\left(\frac{1}{3}\right)^{2/3}$ c) $\left(\frac{1}{3}\right)^{1/3}$ d) 3

Soln:- $TV^{r-1} = \text{constant}$

$T_0 V_0^{r-1} = T(3V_0)^{r-1}$

$$\frac{T}{T_0} = \left(\frac{V_0}{3V_0}\right)^{r-1} = \left(\frac{V_0}{3V_0}\right)^{5/3-1}$$

$$= \left(\frac{1}{3}\right)^{2/3}$$

→ For monoatomic: $r = \frac{C_p}{C_v} = \frac{5}{3}$

B

→ **workdone in cyclic Process:**

1) workdone in clockwise dirn = +ve

2) workdone by the system = +ve

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Que8 Calculate the workdone by the gas if temperature of 2 moles of an ideal gas is increased from T_0 to $2T_0$ through the process:

$$PT^2 = \text{Constant}.$$

a) $2RT_0$ b) $4RT_0$ c) $6RT_0$ d) $8RT_0$

Soln:- let $PT^2 = \gamma$

$$P = \frac{\alpha}{T^2}$$

$PV = nRT$

$$\frac{\alpha}{T^2}V = nRT$$

$$V = \frac{nRT^3}{\gamma}$$

$$dV = \frac{nR}{\alpha} 3T^2 dT$$

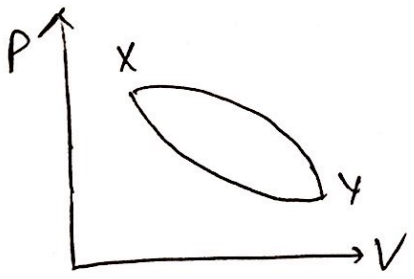
$$\Delta W = \int_{T_0}^{2T_0} P\,dV = \int_{T_0}^{2T_0} \frac{\alpha}{T^2} \frac{nR}{\alpha} 3T^2 dT$$

$$= 3nR\,[T]_{T_0}^{2T_0}$$ & $n = 2$ moles

$$\Delta W = 6RT_0$$

C

Que9 A piston containing an ideal gas is orginally in the state-X (see figure). The gas is taken through a thermal cycle X→Y→X as shown:



The workdone by the gas is positive if the direction of the thermal cycle is:

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a) clockwise

b) counter-clockwise

c) neither clockwise nor counter-clockwise

d) clockwise from X → Y & counter clockwise from Y → X

Soln:- X → Y clockwise (workdone = +ve) **A**

Y → X counter clockwise (workdone = -ve)

Q10 The pressure P of a fluid is related to its number density f by the equation of state $P = af + bf^2$ where a and b are constants. If the initial volume of the fluid is V_0, the workdone on the system when it is compressed so as to increase the number density from an initial value of f_0 to $2f_0$ is:

a) af_0V_0

b) $(a+bf_0)f_0V_0$

c) $\left(\frac{3a}{2}+\frac{7f_0b}{3}\right)f_0V_0$

d) $(a\ln 2+bf_0)f_0V_0$

Soln:-

$$dw = \int P\,dV = \int (af+bf^2)\,dV$$

$f = \frac{N}{V}$, $V = \frac{N}{f}$, $dV = -\frac{N}{f^2}\,df$

$$dw = \int (af+bf^2)\left(-\frac{N}{f^2}\right)df$$

$$dw = -N\int_{f_0}^{2f_0}\left(\frac{a}{f}+b\right)df$$

$N_0 = fV$, $= f_0V_0$

$$= -f_0V_0\left[a\ln f + bf\right]_{f_0}^{2f_0}$$

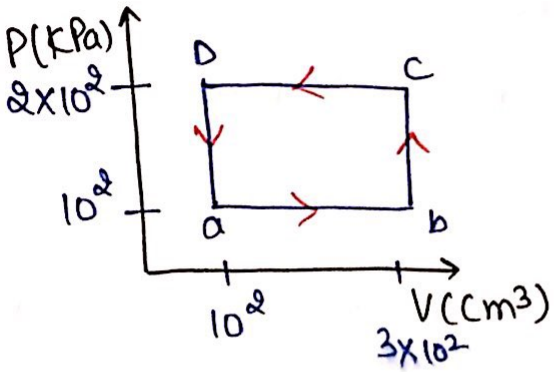
$$dw = -f_0V_0\left[a\ln 2 + bf_0\right]$$

↑ work is done on system

D ⑪

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Que!! A thermodynamic system is taken through the cyclic ABCD as shown in the figure:



calculate the work done during the process CD:

a) 20 J b) 40J

c) -20J d) -40J

Soln:- $\Delta W_{cd} = P(V_d - V_c)$

$= 2\times10^2\times10^3\left[10^2 - 3\times10^2\right]\times10^{-6}$

$= -40J$ D

→ Heat capacity: amount of heat required to change the temperature of sample by 1°c or 1K

$$C = \frac{dQ}{dT} \text{ J/K}$$

→ Specific Heat: amount of heat required to increase (↑) the temperature of 1gm of sample by 1°C or 1K.

$$dQ = CdT = mcdT$$

→ Molar Heat capacity: Heat required to increase the temperature of 1 mole by 1°c or 1K.

$$C_p = \left(\frac{dQ}{dT}\right)_p \quad \& \quad C_v = \left(\frac{dQ}{dT}\right)_v$$

↑ Specific Heat

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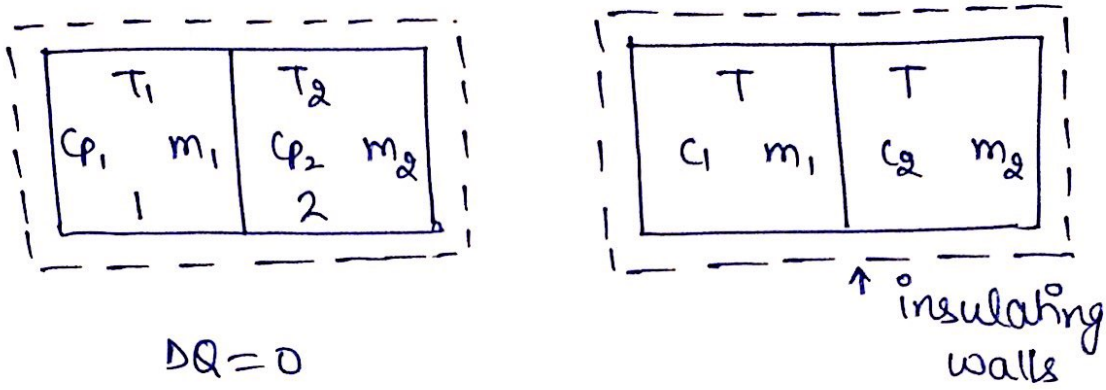
→ $C_p - C_v = nR$ Heat capacity

→ $C_p - C_v = R$ molar Heat capacity / mole

→ $C_p - C_v = k_B$ Atomic Heat capacity / atom

→ **Calorimetry Principle:**

used to find the equilibrium temperature.



$$\Delta Q = 0$$

$$\Delta Q_1 + \Delta Q_2 = 0$$

$$\int_{T_1}^{T} m_1 C_1 dT + \int_{T_2}^{T} m C_2 dT = 0$$

Case I: If C_1 & C_2 are independent of temperature.

$$m_1 C_1 (T - T_1) + m_2 C_2 (T - T_2) = 0$$

$$\boxed{T = \frac{m_1 C_1 T_1 + m_2 C_2 T_2}{m_1 C_1 + m_2 C_2}}$$

Case II: In case of two different samples of same material. i.e $C_1 = C_2 = C$

$$T = \frac{n_1 T_1 + n_2 T_2}{n_1 + n_2} \qquad m_1 = n_1 M \qquad m_2 = n_2 M$$

Case III: If $n_1 = n_2$: $\quad T = \frac{T_1 + T_2}{2}$

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Qu 12 A vessel has two compartments of volume V_1 and V_2, containing an ideal gas at pressure P_1 and P_2, the temperature T_1 & T_2 respectively. If the wall seperating the compartments is removed, the resulting equilibrium temperature is:

a) $\frac{P_1T_1 + P_2T_2}{P_1+P_2}$ b) $\frac{V_1T_1 + V_2T_2}{V_1+V_2}$

c) $\frac{P_1V_1 + P_2V_2}{\left(\frac{P_1V_1}{T}\right) + \left(\frac{P_2V_2}{T_2}\right)}$ d) $\sqrt{T_1T_2}$

Sol^n:- using Principle of calorimetry:

$$T_f = \frac{n_1T_1 + n_2T_2}{n_1+n_2}$$

now, $P_1V_1 = n_1RT_1 \Rightarrow n_1T_1 = \frac{P_1V_1}{R}$

C

$$T_f = \frac{\frac{P_1V_1}{R} + \frac{P_2V_2}{R}}{\frac{P_1V_1}{RT_1} + \frac{P_2V_2}{RT_2}} \Rightarrow \frac{P_1V_1 + P_2V_2}{\left(\frac{P_1V_1}{T_1}\right) + \left(\frac{P_2V_2}{T_2}\right)}$$

Qu 13 Ten grams of Ice at 0°c is added to a beaker containing 30 grams of water at 25°c. what is the final temperature of the system when it comes in thermal equilibrium? (The specific Heat of water is 1cal/gm/°c and the latent ~~Ent~~ Heat of melting of ice is 80 cal/gm).

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a) 0°C b) 7.5°C c) 12.5°C d) −1.25°C

Soln:- Heat required for melting of Ice

$$Q = mxL = 10 \times 80$$
$$= 800 \text{ cal.}$$

amount of heat available for water

$$Q = mcT$$
$$= 30 \times 1 \times 25 = 750 \text{ cal}$$

Heat available < Heat Required.

A

$$T_f = 0°C$$

Ques4 10 gm of water at 70°C is mixed with 5gm of water at 30°C. Find the temperature in the equilibrium?

a) 56.67°C b) 5.67°C c) 28.67°C d) 2.87°C

Soln:- $m_1 = 10$ gm $T_1 = 70°C$

$m_2 = 5$ gm $T_2 = 30°C$

$$T = \frac{m_1T_1 + m_2T_2}{m_1 + m_2} \quad \therefore c_1 = c_2$$

A

$$T = \frac{10 \times 70 + 5 \times 30}{10 + 5} = 56.67°C$$

Ques5 0.5 kg of ice at 0°C is added to a beaker containing 4.5 kg of water at 20°C. what is the final temperature of the system when it comes in thermal equilibrium?

(15)

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a) $0^\circ C$ b) $5^\circ C$ c) $10^\circ C$ d) $20^\circ C$

Soln:-

$\Delta Q_1 = m_1 c_1 T = 4500 \times 1 \times 20 = 90,000$ cal

$\Delta Q_2 = 0.5 \times 80 \times 1000 = 40,000$ cal

$\Delta Q_3 \rightarrow$ Heat required to raise the temperature of $0^\circ C$

$$\Delta Q_1 + \Delta Q_2 + \Delta Q_3 = 0$$

$$0 = 4.5 \times 10^3 \times 1 (t_F - 20) + 0.5 \times 10^3 \times 80 + 0.5 \times 10^3 \times 1 (t_F - 0)$$

$$4500 t_F - 90,000 + 40,000 + 500 T_F = 0$$

$$5000 T_F = 50,000$$

$$T_F = 10^\circ C$$

C

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Laws of Thermodynamics:

→ **Zeroth law**: gives the concept of temperature.

$$A \rightleftharpoons C \;\&\; B \rightleftharpoons C \Rightarrow A \rightleftharpoons B$$

→ **First law**: gives the concept of internal energy.

1) Heat & work are mutually convertiable
2) Heat is form of energy
3) complete conversion of Heat into work is possible but Reverse is not possible.
4) Heat flow : hot body → cold body
5) $dQ = dU + dW = dU + PdV$ (Hydrostatic system)

Cyclic Process	Isothermal Process	Free expension
$\Delta Q = \oint dW$ & $\Delta U = 0$	$\boxed{\Delta Q = nRT \ln\left(\frac{V_f}{V_i}\right)}$ & $dU = 0$ (Joule's law)	$\Delta Q = 0$, $\Delta U = 0$, $\Delta W = 0$

Que! If the equation of state for a gas with internal energy U is:

$$PV = U/3$$

then equation of state in adiabatic process is?

a) $PV^{1/3} = \text{constant}$ b) $PV^{2/3} = \text{constant}$

c) $PV^{4/3} = \text{constant}$ d) $PV^{5/3} = \text{constant}$

Soln. $U = 3PV$

$$dQ = dU + dW \qquad dQ = 0 \text{ (adiabatic)}$$

$$3PdV + 3VdP + PdV = 0$$

$$4PdV = -3VdP$$

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$$\frac{dP}{P} = -\frac{4}{3}\frac{dV}{V}$$

$$\ln P = -4/3 \ln V + \ln C$$ C

$$PV^{4/3} = \text{constant}$$

Ques Let ΔW be the workdone in a quasistatic reversible process. which of the following statements about ΔW is CORRECT?

a) ΔW is a perfect differential if the process is isothermal.

b) ΔW is a perfect differential if if the process is adiabatic

c) ΔW is always a perfect differential

d) ΔW can't be a perfect differential.

Soln:- $\Delta Q = \Delta U + \Delta W$

$\Delta Q = 0$ (adiabatic)

$\Rightarrow \Delta U = -\Delta W$ B

↑

state fxn $\Rightarrow$ Perfect differential

Ques The internal energy of n moles of a gas is given by $E = \frac{3}{2}nRT - \frac{a}{V}$

where V is the volume of the gas at temperature T and a is the constant. one mole of the gas is state (T_1, V_1) is allowed to expend adiabatically into vacuum to a final state (T_2, V_2). The temp. T_2 is:

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a) $T_1 + \frac{a}{R}\left(\frac{1}{V_2} + \frac{1}{V_1}\right)$ b) $T_1 - \frac{2}{3}\frac{a}{R}\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$

c) $T_1 + \frac{2}{3}\frac{a}{R}\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$ d) $T_1 - \frac{1}{3}\frac{a}{R}\left(\frac{1}{V_2} - \frac{V}{V_1}\right)$

Soln:- adiabatically into vacuum ⇒ free expension

$$dE = 0 \Rightarrow E_1 = E_2$$

$$\frac{3}{2}RT_1 - \frac{a}{V_1} = \frac{3}{2}RT_2 - \frac{a}{V_2}$$

$$T_2 = T_1 + \frac{2}{3}\frac{a}{R}\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$$ **C**

Qu4 For an ideal gas the molar heat capacity is given by:

$$C = C_V + 3aT^2$$

where C_V is independent of temperature. Find the equation of state in terms of temperature and volume?

Soln:- $dQ = CdT = (C_V + 3aT^2)\,dT$

$$dQ = C_V dT + 3aT^2 dT$$

$$dQ = dU + 3aT^2 dT \quad \text{— (1)}$$

$$dQ = dU + PdV \rightarrow \text{1st law} \quad \text{— (2)}$$

compare (1) & (2)

$$\int 3aT^2 dT = \int P\,dV$$

$$\frac{3aT^3}{3} = R\ln V + \ln A$$

$\rightarrow P = \frac{RT}{V}$

$\int P dV = \int \frac{RT}{V} dV$ (19)

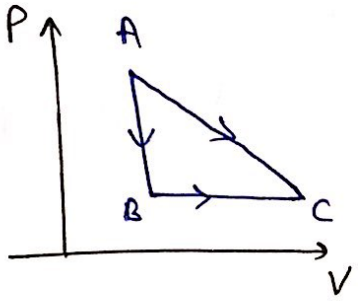
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$$\frac{3aT^2}{2R} = \ln AV$$

$$e^{3aT^2/2R} = AV$$

$$\boxed{V e^{-3aT^2/2R} = \text{constant}}$$ Ans.

Ques A given quantity of gas is taken from the state A→C reversibly, by two paths, A→C directly and A→B→C as shown in the figure below:



During the A→C the work done by the gas is 100 J and the ~~Heat~~ Heat absorbed is 150 J. If during the process A→B→C the work done by the gas is 30 J, the Heat absorbed is:

a) 20 J b) 80 J c) 220 J d) 280 J

Sol^n:- For Path A→C:

$$dQ = dU + dW_1$$

$$150\,J = dU + 100\,J \Rightarrow dU = 50\,J$$

For Path A→B→C:

$$dQ = dU + dW_2$$ B

$$= 50 + 30 = 80\,J$$

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→ Equation of State:

$$\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}$$

f (DoF) → 3 monoatomic, 5 diatomic, 6 triatomic

$\gamma_{mono} > \gamma_{Dia} > \gamma_{tri}$.

Variation of γ with temperature: (e.g. For H_2 molecule)

→ at low temp. (70K): behaves as monoatomic
→ at ordinary temp. (250K): behaves as diatomic
→ at High temp. (750K): behaves as Triatomic

mayer's Formula: $C_p - C_v = R$

$C_v = \frac{DOF}{2} R$ & $C_p = \left(\frac{DOF}{2} + 1\right) R$ → for all gases.

Equipartition Theorem: (For non-Relativistic case)

→ average thermal K.E associated with each DOF is $\frac{1}{2} k_B T$

→ contribution of each quadratic term in Hamiltonian is $\frac{1}{2} k_B T$

Examples:

① Linear Harmonic oscillator:

$$H = \underbrace{\frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2}$$

② quadratic terms

$U = 2 \times \frac{1}{2} k_B T$

$\boxed{U = k_B T}$

For n moles: $U = nRT$

$$\boxed{C_v = \left(\frac{\partial U}{\partial T}\right) = nR}$$

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② monoatomic Ideal gas:

$$U = \frac{n}{2} N K_B T = \frac{n}{2} n R T$$

(n: DOF; n: moles.)

$$\boxed{U = \frac{3}{2} nRT} \quad \& \quad \boxed{C_V = \frac{3}{2} nR} \quad \text{(for 3-Dim.)}$$

③ Diatomic Ideal gas:

$$U = \frac{3}{2} KT + 2 \times \frac{1}{2} KT = \frac{5}{2} KT \quad \text{Rigid molecule}$$

$$U = \frac{3}{2} KT + 2 \times \frac{1}{2} KT + KT = \frac{7}{2} KT \quad \text{non-Rigid}$$

$$\gamma = \frac{C_p}{C_V} \begin{cases} 7/5 \rightarrow \text{Rigid} \\ 9/7 \rightarrow \text{non-Rigid} \end{cases}$$

→ Fraction of Heat:

$$\frac{\Delta W}{\Delta Q} = \frac{\Delta Q - \Delta U}{\Delta Q} = 1 - \frac{\Delta U}{\Delta Q} \Rightarrow 1 - \frac{n C_V dT}{n C_p dT}$$

$$\boxed{\frac{\Delta W}{\Delta Q} = 1 - \frac{C_V}{C_p} = 1 - \frac{1}{\gamma}} \qquad \boxed{\frac{\Delta W}{\Delta Q} = \frac{C_p - C_V}{C_p} = \frac{R}{C_p}}$$

→ For monoatomic: $\frac{\Delta W}{\Delta Q} = 1 - \frac{1}{5/3} = 2/5$

→ For diatomic: $\frac{\Delta W}{\Delta Q} = 1 - \frac{1}{7/5} = \frac{2}{7}$

→ For biatomic: $\frac{\Delta W}{\Delta Q} = 1 - \frac{1}{4/3} = \frac{1}{4}$

($\frac{\Delta W}{\Delta Q}$: Fraction of Heat)

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Que1 The ratio $\gamma = C_p/C_v$ of H_2 gas is 7/5 at room temperature, and 5/3 at low temperature (around 50K). The reason for this change is:

a) Dissociation of H_2 molecule
b) Suppression of rotational degree of freedom
c) formation of random clusters of molecules
d) Liquification of the gas

Soln:- for H_2 gas:

at ordinary temp: DOF = 5 **B**

at low temp ⇒ DOF = 3

i.e suppression of Rotn DOF.

Que2 Mean total energy of a classical 3-Dim. H.O in eqm. with a Real reservoir at temperature T is:

a) k_BT b) $\frac{3}{2}k_BT$ c) $2k_BT$ d) $3k_BT$

Soln:- for 1-D H.O : $U = k_BT$ (2-quadratic terms)

2-D H.O : $U = 2k_BT$ (4-quadratic terms)

3-D H.O : $U = 3k_BT$ (6-quadratic terms)

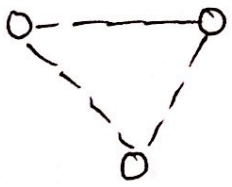
D

$$H = \frac{p_x^2 + p_y^2 + p_z^2}{2m} + \frac{1}{2}m\omega^2(x^2+y^2+z^2)$$

Que3 Consider a triatomic molecule of the shape shown in the figure below in 3-Dim. The Real capacity of this molecule at high temperature (temperature much higher than the vibrational

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and rotational energy scales of the molecule but lower than its bond dissociation energies) is:



a) $\frac{3}{2}k_B$ b) $3k_B$

c) $\frac{9}{2}k_B$ d) $6k_B$

Soln:- $U = \underbrace{\frac{6}{2}k_BT}_{\text{DOF}=6 \text{ equipartition thm}} + \underbrace{3k_BT}_{\text{due to High temp.}} = 6k_BT$

D

$$C_V = \left(\frac{\partial U}{\partial T}\right) = 6k_B$$

Ques The specific heat per molecule of a gas of diatomic molecules at high temperature is:

a) $8k_B$ b) $3.5k_B$ c) $4.5k_B$ d) $3k_B$

Soln:- $U = \underbrace{3\times\frac{1}{2}k_BT}_{\text{Translational}} + \underbrace{2\times\frac{1}{2}k_BT}_{\text{Rotn}} + \underbrace{(3n-5)k_BT}_{\text{Vib.}}$

$$U = \frac{7}{2}k_BT \Rightarrow C_V = \frac{7}{2}k_B$$

$$C_V = 3.5k_B$$

B

Ques For which gas the ratio of specific heats (C_p/C_v) will be largest?

a) monoatomic b) diatomic c) triatomic d) hexatomic

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Soln:- monoatomic > Diatomic > Triatomic

$\gamma = \frac{5}{3} = 1.66$ $\gamma = 1.4$ $\gamma = 1.33$

A

Que 6 Above a temperature T_0, the specific heat of a certain gas is approximate 3.5R and is approximate 2.5R below T_0. The gas is most likely:

a) oxygen b) ozone c) CO_2 d) Ar

Soln:- $\gamma = \frac{3.5}{2.5} = 1.4$ i.e diatomic

Oxygen $\Rightarrow O_2$ (diatomic)

A

Que 7 The molar specific heat of a gas as given from the kinetic theory is $\frac{5}{2}R$. If it is not specified whether it is C_p or C_v, one could conclude that the molecules of the gas

a) are definetly monoatomic
b) are definetly rigid diatomic
c) are definetly non-rigid diatomic
d) can be monoatomic or rigid diatomic

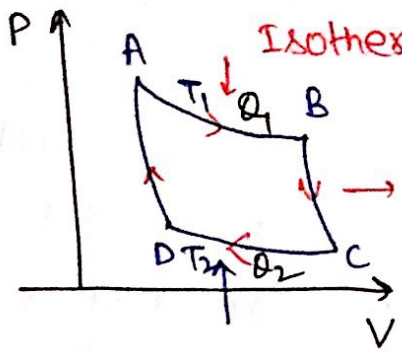
Soln:- monoatomic: $C_v = \frac{3}{2}R$; $C_p = \frac{5}{2}R$

Diatomic { Rigid: $C_v = \frac{5}{2}R$; $C_p = \frac{7}{2}R$
Non-Rigid: $C_v = \frac{7}{2}R$; $C_p = \frac{9}{2}R$

D

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→ Carnot cycle:



Isothermal compression

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1} \quad ***$$

→ A to B: $w_1 = 2.303\ RT_1 \log (V_2/V_1)$

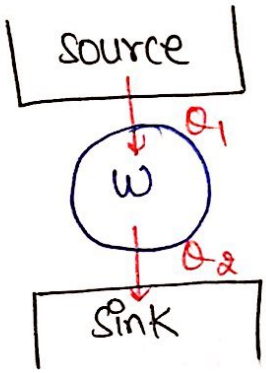
→ B to C: $w_2 = \frac{R}{\gamma - 1}[T_1 - T_2]$

→ C to D: $w_3 = 2303\ RT_2 \log (V_4/V_3)$

→ D to A: $w_4 = \frac{R}{\gamma - 1}[T_2 - T_1]$

$$w_{total} = 2.303\ R(T_1 - T_2) \log (V_2/V_1)$$

Heat engine: convert Heat $\xrightarrow{\text{into}}$ mechanical work



$$\text{efficency } (e) = \left(\frac{\Delta W}{\Delta Q_{taken}}\right)$$

$$e = \frac{Q_1 - Q_2}{Q_1}$$

$$e = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1} \quad ***$$

→ $e = 1$ or 100%. not Possible

→ valid for both real & ideal engine

→ efficency is independent of working substance.

→ To maximise efficency, it is easier to decrease T_2 rather than increase T_1.

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→ **Refrigerator:** Removes heat from a cold body with the help of external work.

$$K = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2}$$

↑ coefficient of Performance.

→ $\boxed{e = 1 - K}$ ***

Que1 An engine absorbs heat at a temperature 1000K and rejects heat at 600K. If the engine operates at maximum possible efficiency, the amount of work performed by the engine, for 2000J heat input is:

a) 1600J b) 1200J c) 800J d) 400J

Sol^{n} $T_1 = 1000K$ $T_2 = 600K$

$Q_1 = 2000J$ $Q_2 = ?$

$$\frac{T_1}{T_2} = \frac{Q_1}{Q_2} \Rightarrow \frac{1000}{2000} = \frac{600}{Q_2}$$

C

$$Q_2 = 1200\ J$$

$$\text{work} = Q_1 - Q_2 = 2000 - 1200 = 800J$$

Que2 A heat pump working on the carnot cycle maintains the inside temperature of a house at 22°C by supplying 450 KJs^{-1}. If the outside temperature is 0°C, the heat taken, in KJ's, from the outside air is approx:

a) 487 b) 470 c) 467 d) 417

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Soln:- $T_1 = 22^\circ C = (22+273)K$

$Q_1 = 450\ KJ\ s^{-1}$

$T_2 = 0^\circ C = 273K$

$$\frac{T_1}{T_2} = \frac{Q_1}{Q_2}$$

$$Q_2 = \left(\frac{T_2}{T_1}\right) Q_1 = \left(\frac{273}{22+273}\right) 450$$

$$Q_2 = 417\ KJ\ s^{-1}$$

D

Que3 A reversible carnot engine is operated between temperature T_1 and T_2 ($T_2 > T_1$) with a photon gas as the working substance. The efficency of the engine is:

a) $1 - \frac{3T_1}{4T_2}$ b) $1 - \frac{T_1}{T_2}$ c) $1 - \left(\frac{T_1}{T_2}\right)^{3/4}$ d) $1 - \left(\frac{T_1}{T_2}\right)^{4/3}$

Soln:- Efficency of carnot engine is independent of working substance

$$e = 1 - \frac{\text{Temp. (Sink)}}{\text{Temp (Source)}}$$

$$e = 1 - \frac{T_1}{T_2}$$

B

Que4 A carnot engine operates blw a heat source at 500K and a sink at 300K. The temperature of the source is increased by 20K. In order that the efficency of the engine remains unchanged, the temperature of the sink should be changed by:

a) 20K b) 12K c) -12K d) -20K

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Soln:- given: $e_1 = e_2 \Rightarrow 1 - \frac{T_2}{T_1} = 1 - \frac{T_2'}{T_1'}$ —①

$T_1 = 500K \qquad T_2 = 300K$

$T_1' = 500 + 20 \qquad T_2' = ?$

By eqn ①

$$\frac{T_2}{T_1} = \frac{T_2'}{T_1'} \Rightarrow T_2' = \left(\frac{T_2}{T_1}\right) T_1'$$

$$T_2' = \frac{300}{500} \times 520 = 312\,K$$

$$\Delta T = T_2' - T_2 = 312 - 300$$

$$\Delta T = 12\,K$$

B

Ques The efficency of a carnot cycle is 1/6. On reducing the temperature of sink by 65°C the efficency increases by 1/3. Find temperature of source & sink?

Soln:- $e_1 = 1 - \frac{T_2}{T_1} = \frac{1}{6}$ —①

$$e_2 = 1 - \left(\frac{T_2 - 65}{T_1}\right) = \frac{1}{3}$$

$$\left(1 - \frac{T_2}{T_1}\right) + \frac{65}{T_1} = \frac{1}{3}$$

$$\frac{1}{6} + \frac{65}{T_1} = \frac{1}{3}$$

$$\frac{65}{T_1} = \frac{1}{3} - \frac{1}{6} \Rightarrow \frac{1}{6}$$

$$T_1 = 65 \times 6$$

$$T_1 = 390\,K$$

Source temp.

now by eqn ①

$$\frac{1}{6} = 1 - \frac{T_2}{390} \Rightarrow T_2 = 325\,K$$

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→ **Second law of Thermodynamics:**

This law defines the direction of the process & condition of process.

1) **Kelvin-Plank Statement:** (Heat engine)

→ we can't generate any device which convert total heat into work

→ Heat exchange from higher temperature to lower temperature.

2) **Clausius Statement:** (Refrigerator)

→ we can't design any device, in which heat transfer from lower Temperature to Higher temperature without doing external work.

Entropy (S):

→ State function i.e Perfect differential

→ measure of disorder.

$$dS = \frac{dQ}{T}$$

$$(\Delta S)_{universe} = (\Delta S)_{system} + (\Delta S)_{surrounding}$$

$(\Delta S)_{surrounding}$ → 0 for isolated system.

$$(\Delta S)_{universe} \geq 0$$

$\Delta S > 0$ Irreversible Process

$\Delta S = 0$ Reversible Process (eg. Reversible carnot engine).

→ entropy of universe (or isolated system) can never be decreases.

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Que: If the entropy of a system remains constant in a thermodynamic process, the process is:

a) isobaric b) isochoric c) isothermal d) Adiabatic

Soln:- entropy(S) = constant

$\Rightarrow dS = 0$

$dQ = TdS \Rightarrow dQ = 0$

↑ Adiabatic

D

Que: A sample of gas (ideal) with initial Pressure P and volume V is taken through an isothermal expansion proceed during which the change in entropy is found to be ΔS. The universal gas constant is R. Then the workdone by the gas is given by:

a) $(PV\Delta S)/(nR)$ b) $nR\Delta S$ c) PV d) $(P\Delta S)/(nRV)$

Soln:- $dQ = C_v dT + dW$ —(1)

$dT = 0$ for isothermal Process

$dQ = TdS$

$dW = PdV \Rightarrow TdS = PdV$

$\Delta W = \int TdS = T\Delta S$ → By (1)

$\Delta W = \frac{PV\Delta S}{nR}$ | → $PV = nRT$

$T = \frac{PV}{nR}$

A

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Ques: which of the following expresses the second law of thermodynamics?

a) $\Delta F \leq W$ b) $\Delta S < \frac{Q}{T}$ c) $\Delta U = Q + W$ d) $\Delta S \leq 0$

Soln:- $\Delta S \geq 0 \rightarrow$ (d) ✗

$dQ \leq T\,dS \Rightarrow \Delta S > \frac{\Delta Q}{T} \rightarrow$ (b) ✗

$dQ = dU + dW \Rightarrow$ (c) ✗ **A**

Ques: A carnot cycle operates as a heat engine b/w two bodies of equal heat capacity until their temperature becomes equal. If the initial temperatures of the bodies are T_1 & T_2 respectively and $T_1 > T_2$ their common final temperature is:

a) $\frac{T_1^2}{T_2}$ b) $\frac{T_2^2}{T_1}$ c) $\sqrt{T_1 T_2}$ d) $\frac{1}{2}(T_1 + T_2)$

Soln:- for carnot cycle:

$\Delta S = 0 \quad \therefore S \rightarrow$ state fx^n.

$$\Delta S = C\int_{T_1}^{T} \frac{dT}{T} + C\int_{T_2}^{T} \frac{dT}{T}$$

$$\ln\left(\frac{T}{T_1}\right) + \ln\left(\frac{T}{T_2}\right) = 0$$

$$\ln\left(\frac{T^2}{T_1 T_2}\right) = 0 \Rightarrow \frac{T^2}{T_1 T_2} = 1$$

$$T = \sqrt{T_1 T_2}$$ **C**

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Ques The internal energy $E(T)$ of a system at a fixed volume is found to depend on the temperature T as $E(T) = aT^2 + bT^4$. Then the entropy $S(T)$, as a fxn of temperature, is:

a) $\frac{1}{2}aT^2 + \frac{1}{4}bT^4$ b) $2aT^2 + 4bT^4$

c) $2aT + \frac{4}{3}bT^3$ d) $2aT + 2bT^3$

Soln- $E(T) = aT^2 + bT^4$

$$C_V = \frac{dE}{dT} = 2aT + 4bT^3$$

$$TdS = C_V dT$$

$$\int dS = \int C_V \frac{dT}{T} = \int \left(\frac{2aT + 4bT^3}{T}\right) dT$$

$$S = 2aT + \frac{4bT^3}{3}$$ C

Ques one mole of an ideal gas undergoes a reversible isothermal expansion from volume V_1 to $2V_1$. what is the change in entropy?

Soln- $$\Delta S = \int \frac{dQ}{T} = \int \frac{dU}{T} + \int \frac{PdV}{T}$$

$\downarrow$ 0 ($\because dU = 0$ for Reversible Process)

$$\Delta S = \int_i^f \frac{PdV}{T} = \int_{V_1}^{2V_1} \frac{nR}{V} dV$$

$$\Delta S = nR \ln 2 \Rightarrow \boxed{R \ln 2} \quad (\because n = 1)$$

Ans.

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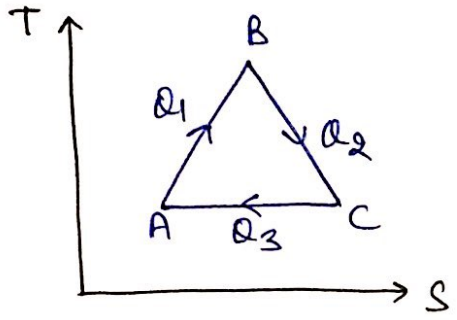
→ **Latent Heat:** amount of heat given to the system at constant temperature to convert phase of one gram is called latent Heat.

Heat given to the system = +ve

Heat rejected by the system = -ve

$$\Delta S = \int \frac{dQ}{T} = \frac{L}{T} \quad \text{(for 1gm)}$$

→ **T-S diagram:**

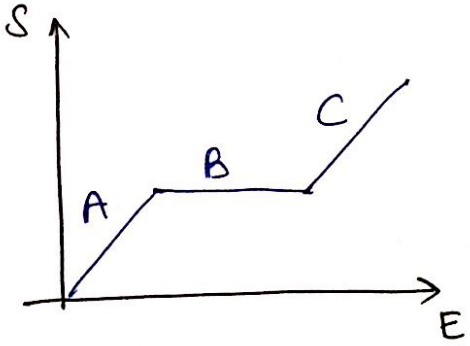


$$e = \frac{\text{net work done}}{\text{total heat taken}}$$

$$e = \frac{Q_1 + Q_2 - Q_3}{Q_1 + Q_2}$$

$$e = \frac{\text{area enclosed by cycle on T-S diagram}}{\text{total area}}$$

Que! The entropy S of a thermodynamic system as a function of Energy E is given by the following graph:



The temperatures of the phases A, B & C denoted by T_A, T_B & T_C respectively, satisfies the following inequalities:

(34)

a) $T_C > T_B > T_A$ b) $T_A > T_C > T_B$ c) $T_B > T_C > T_A$ d) $T_B > T_A > T_C$

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Soln:- $dE = TdS$

$$\frac{1}{T} = \left(\frac{dS}{\partial E}\right)$$

$\frac{dS}{dE} = 0$ for B Phase $\Rightarrow T_B = \infty$ ↑ largest temp.

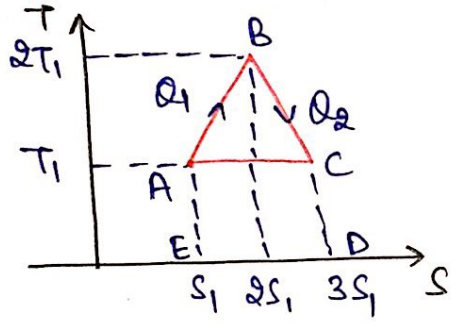
slope: $\left(\frac{dS}{\partial E}\right)_B > \left(\frac{\partial S}{\partial E}\right)_A \Rightarrow T_A > T_C$

$T_B > T_A > T_C$ **D**

Ques A reversible engine cycle is shown in the following T-S diagram. The efficency of the engine is:

a) 1/3 b) 1/2

c) 1/5 d) 1/4



Soln:- $e = \dfrac{\text{area of } \Delta ABC}{\text{area of } \Delta ABC + \text{area of } \square ACDE}$

$$e = \frac{\frac{1}{2}(2S_1 - S_1)(2T_1 - T_1) + \frac{1}{2}(3S_1 - 2S_1)(2T_1 - T_1)}{\frac{1}{2}(2S_1 - S_1)(2T_1 - T_1) + \frac{1}{2}(3S_1 - 2S_1)(2T_1 - T_1) + (3S_1 - S_1)(T_1 - 0)}$$

$$e = \frac{\frac{S_1T_1}{2} + \frac{S_1T_1}{2}}{\frac{S_1T_1}{2} + \frac{S_1T_1}{2} + 2S_1T_1} \Rightarrow e = 1/3$$ **A**

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→ Thermodynamic Potential & Maxwell Relations:

Internal energy	Free energy	Enthalpy	Gibbs Free energy
$dU = dQ - dW$	$A = U - TS$	$H = U + PV$	$G = U + PV - TS$
$dU = TdS - PdV$	$dA = -PdV - SdT$	$dH = TdS + VdP$	$dG = VdP - SdT$
$T = \left(\frac{\partial U}{\partial S}\right)_V$; $P = -\left(\frac{\partial U}{\partial V}\right)_S$	$P = -\left(\frac{\partial A}{\partial V}\right)_T$; $S = -\left(\frac{\partial A}{\partial T}\right)_V$	$T = \left(\frac{\partial H}{\partial S}\right)_P$; $V = \left(\frac{\partial H}{\partial P}\right)_S$	$V = \left(\frac{\partial G}{\partial P}\right)_T$; $S = -\left(\frac{\partial G}{\partial T}\right)_P$
$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$	$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$	$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$	$\left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T$
State fxn: exact diff.	$(dA)_{T,V} = 0$ Rev. < 0 Irrev.	$(dH)_{S,P} \geq 0$	$(dG)_{T,P} \leq 0$

Que! A thermodynamic function

$$G(T,P,N) = U - TS + PV$$

is given in terms of the internal energy U, temperature T, entropy S, pressure P, volume V & number of particles N. which of the following relation is true?

a) $S = -\left.\frac{\partial G}{\partial T}\right|_{N,P}$ b) $S = \left.\frac{\partial G}{\partial T}\right|_{N,P}$

c) $V = -\left.\frac{\partial G}{\partial P}\right|_{V,T}$ d) $\mu = -\left.\frac{\partial G}{\partial N}\right|_{P,T}$

Sol^{n}. $G = U + PV - TS$

$$dG = dU + PdV + VdP - TdS - SdT$$

$$= TdS - PdV + PdV + VdP - TdS - SdT$$

$$= -SdT + VdP$$

$S = -\left(\frac{\partial G}{\partial T}\right)_P$ & $V = \left(\frac{\partial G}{\partial P}\right)_T$ **A**

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$$K_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$$

$V = \frac{N}{n}$

$\partial V = -\frac{N}{n^2}\partial n$

$$= -\frac{1}{V}\left(-\frac{N\,\partial n}{n^2\,\partial P}\right)$$

$$= +\left(\frac{N}{V}\right)\times\frac{1}{n^2}\left(\frac{\partial n}{\partial P}\right)$$

$n = \frac{N}{V}$

$$K_T = \frac{n}{n^2}\left(\frac{\partial n}{\partial P}\right)$$

$$\partial P = \left(\frac{\partial n}{n}\right)\times\frac{1}{K_T} = \frac{1}{100}\times 10^{10}\ Pa$$

$$\partial P = 10^8\ Pa$$

C

Ques The isothermal compressibility, κ of an ideal gas at temperature T_0 & volume V_0 is given by:

a) $-\frac{1}{V_0}\left.\frac{\partial V}{\partial P}\right|_{T_0}$ b) $\frac{1}{V_0}\left.\frac{\partial V}{\partial P}\right|_{T_0}$

c) $-V_0\left.\frac{\partial P}{\partial V}\right|_{T_0}$ d) $V_0\left.\frac{\partial P}{\partial V}\right|_{T_0}$

Soln:-

$$K_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$$

A

$$K = -\frac{1}{V_0}\left.\frac{\partial V}{\partial P}\right|_{T_0}$$

***

$$C_P - C_V = T\left(\frac{\partial P}{\partial T}\right)_V\left(\frac{\partial V}{\partial T}\right)_P$$

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→ **Third law:** (Nernst's Heat theorem)

at absolute zero of temperature, thermal motion is completely disappear.

→ entropy tends to zero
→ C_p & C_V tends to zero
→ at $T \to 0$

→ internal energy & ethalpy of all solids approaches to common value.

i.e $F = h = U$ as $T \to 0$

$$\left(\frac{\partial P}{\partial T}\right)_V \to 0 \text{ \& } \left(\frac{\partial V}{\partial T}\right)_P \to 0 \quad \text{as } T \to 0$$

Isothermal compressibility:

$$\chi_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$$ finite & non zero at $T \to 0$

Bulk modulus:

$$B = V\left(\frac{dP}{dt}\right) = -V\left(\frac{\partial P}{\partial V}\right)$$ finite & non zero at $T \to 0$

Coefficient of expension:

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)$$ zero at $T \to 0$

Que! If the isothermal compressibility of a solid is $K_T = 10^{-10}\ (Pa)^{-1}$, the pressure required to increase its density by 1% approximately.

a) 10^4 Pa b) 10^6 Pa c) 10^8 Pa d) 10^{10} Pa

Soln:- $$K_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T = 10^{-10}\ (Pa)^{-1}$$

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Que 4 (Latent Heat)

When 100 gm ice at 0°c melts, the change in entropy in cal/K is about:

a) 0 b) 0.34 c) 29.3 d) 39.2

Soln:- $L = 80$ cal/gm

C

$$\Delta S = \frac{mL}{T} = \frac{100 \times 80}{273} = 29.3 \text{ cal/K}$$

Que 5 The Helmholtz free energy of a photon gas is given by $F = -\frac{a}{3} V T^4$

The entropy & Pressure of photon gas is?

Soln:- $dF = -PdV - SdT$

$$S = -\left(\frac{\partial F}{\partial T}\right)_V = +\frac{4}{3} a V T^3$$

$$P = -\left(\frac{\partial F}{\partial V}\right)_T = \frac{a}{3} T^4$$

Que 6 If Gibbs free energy of a system is:

$$G(P,T) = RT \ln\left[\frac{aP}{(RT)^{3/2}}\right]$$

where a & R are coefficients. Find entropy?

Soln:- $dG = -SdT + VdP$

$$S = -\left(\frac{\partial G}{\partial T}\right)_P = -R \frac{d}{dT}\left[T \ln\left(\frac{aP}{(RT)^{3/2}}\right)\right]$$

$$= \frac{5}{2} R - R \ln\left[\frac{aP}{(RT)^{3/2}}\right]$$

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Que 2 The Internal energy E of a system is given by $E = \frac{bS^3}{VN}$; where b is a constant and other symbols have their usual meaning. The temperature of this system is equal to:

a) $\frac{bS^2}{VN}$ b) $\frac{3bS^2}{VN}$ c) $\frac{bS^3}{V^3N}$ d) $\left(\frac{S}{N}\right)^2$

Soln:- 1st law: $dQ = dE + dW$

$$TdS = dE + PdV - \mu dN$$ ***

$$\boxed{dE = TdS - PdV + \mu dN}$$

$$T = \left(\frac{\partial E}{\partial S}\right)_{V,N} = \frac{\partial}{\partial S}\left(\frac{bS^3}{VN}\right)_{V,N}$$

$$T = \frac{3bS^2}{VN}$$ [B]

Que 3 For a system with constant volume and temperature, the thermodynamic equilibrium corresponds to the state with minimum?

a) A b) G c) U d) H

Soln:- $A = U - TS$

$dA = -PdV - SdT$ as $dV = 0$, $dT = 0$

$dA = 0$ ← eqm. condn. [A]

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KINETIC THEORY OF GASES

→ Boyle's law : PV = constant

→ Charles law : V/T = constant

→ Maxwell's speed Distribution :

Dim.	Probability Density $P(c)$	most Prob. value c_p	average value $\bar{c}$	RMS value c_{rms}
3-Dim.	$4\pi\left(\frac{m}{2\pi KT}\right)^{3/2} e^{\frac{-mc^2}{2KT}} \cdot c^2$	$\sqrt{\frac{2KT}{m}}$	$\sqrt{\frac{8KT}{\pi m}}$	$\sqrt{\frac{3KT}{m}}$
2-Dim.	$2\pi\left(\frac{m}{2\pi KT}\right) e^{-mc^2/2KT} \cdot c$	$\sqrt{\frac{KT}{m}}$	$\sqrt{\frac{\pi KT}{2m}}$	$\sqrt{\frac{2KT}{m}}$
1-Dim.	$\left(\frac{m}{2\pi KT}\right)^{1/2} e^{-mc^2/2KT}$	0	$\sqrt{\frac{2KT}{m}}$	$\sqrt{\frac{KT}{m}}$

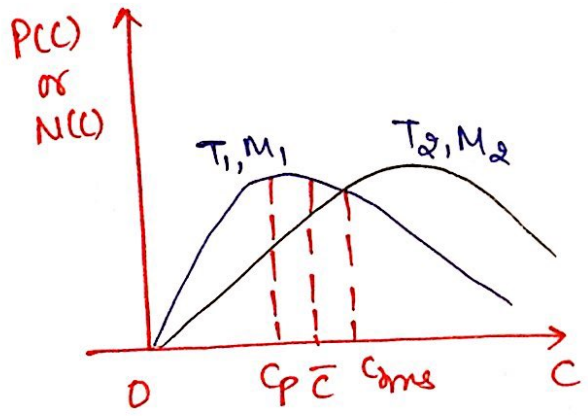


Fig: asymmetric Plots

→ $T_2 > T_1$ & $m_2 > m_1$

→ $c_p < \bar{c} < c_{rms}$

Ration ⇒ 1 : 1.128 : 1.224

→ $\Delta c = \sqrt{\langle c^2 \rangle - \langle c \rangle^2}$

$\Delta c \propto \sqrt{\frac{T}{m}}$

→ Maxwell's velocity distribution :

Dimensions	most Probable velocity	average velocity	RMS value v_{rms}
1-Dim	0	0	$\sqrt{\frac{KT}{m}}$
2-Dim	0	0	$\sqrt{\frac{2KT}{m}}$
3-Dim	0	0	$\sqrt{\frac{3KT}{m}}$

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Que1 Boyle's law can be expressed in differential form as:

a) $\frac{dV}{dP}=1$ b) $\frac{dV}{dP}=\frac{V}{P}$ c) $\frac{dV}{dP}=\frac{P}{V}$ d) $\frac{dV}{dP}=-\frac{V}{P}$

Soln:- $PV = \text{constant}$

Differentiate w.r.t P

$$P\left(\frac{dV}{dP}\right)+V=0 \Rightarrow \frac{dV}{dP}=-\frac{V}{P}$$

[D]

Que2 Electrons of mass m in a thin, long wire at a temperature T follow a one-dimensional maxwell velocity distribution. The most probable speed of these electrons is:

a) $\sqrt{\frac{KT}{2\pi m}}$ b) $\sqrt{\frac{2KT}{m}}$ c) 0 d) $\sqrt{\frac{8KT}{\pi m}}$

Soln:- 1-D maxwell velocity distribution.

most Prob. value $(V_p) = 0$

[C]

Que3 Consider oxygen gas at 300K having the mass of its molecule as 5.31×10^{-26} kg. The root mean square speed of its molecule, is about:

a) 284 m/s b) 248 m/s c) 348 m/s d) 484 m/s

Soln:- $C_{rms}=\sqrt{\frac{3K_BT}{m}}=\sqrt{\frac{3\times1.38\times10^{-23}\times300}{5.31\times10^{-26}}}$

$= 4.836\times10^{2}$ m/s

$\simeq$ 484 m/s

[D]

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Ques 4 A classical gas of molecules each of mass m, is in thermal equilibrium at the absolute temp. T. The velocity components of the molecules along the cartesian axes are V_x, V_y & V_z. The mean value of $(V_x+V_y)^2$ is:

a) $\frac{K_BT}{m}$ b) $\frac{3K_BT}{2m}$ c) $\frac{K_BT}{2m}$ d) $\frac{2K_BT}{m}$

Soln:- $\overline{(V_x+V_y)^2} = \overline{V_x^2} + \overline{V_y^2} + 2\overline{V_xV_y}$

$\overline{V_xV_y} \to 0$

$= \frac{KT}{m} + \frac{KT}{m} \Rightarrow \frac{2KT}{m}$ **D**

Ques 5 The speed v of the molecules of mass m of an ideal gas obeys maxwell's velocity distribution law at an equilibrium temperature T. Let (V_x, V_y, V_z) denote the components of the velocity and k_B the Boltzmann constant. The average value of $(\alpha V_x - \beta V_y)^2$, where α and β are constants, is:

a) $(\alpha^2-\beta^2)\frac{K_BT}{m}$ b) $(\alpha^2+\beta^2)\frac{K_BT}{m}$

c) $(\alpha+\beta)^2\frac{K_BT}{m}$ d) $(\alpha-\beta)^2\frac{K_BT}{m}$

Soln:- $\overline{(\alpha V_x - \beta V_y)^2} = \alpha^2\overline{V_x^2} + \beta^2\overline{V_y^2} - 2\alpha\beta\,\overline{V_x}\,\overline{V_y}$

$\overline{V_x} \to 0$

$= \alpha^2\frac{KT}{m} + \beta^2\frac{KT}{m}$ **B**

$= (\alpha^2+\beta^2)\frac{KT}{m}$

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→ **Vander waal Equation of State for real gas:**

$PV = RT$ for 1 mole of Ideal gas

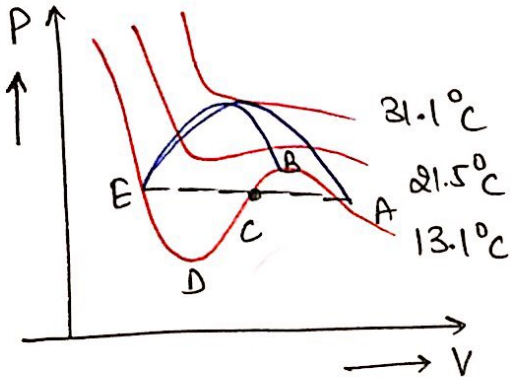
$$\left(P + \frac{n^2 a}{V^2}\right)(V - nb) = nRT$$

$b = 4(na)$ i.e 4 times actual volume of molecule.

Correction:

→ in P is a $\left(P + \frac{a}{V^2}\right)$ for 1-mole.

→ in V is a $(V - b)$



A → B: Supercooling.

D → E: Superheating

BCD: not defined.

critical coefficent:

$V_c = 3b$ | $T_c = \frac{8a}{27Rb}$ | $P_c = \frac{a}{27b^2}$

coefficent of vander waals constants:

$a = \frac{27R^2T_c^2}{64P_c}$ | $b = \frac{RT_c}{8P_c}$ | $\frac{T_c}{P_c} = \frac{8b}{R}$ | $\frac{RT_c}{P_cV_c} = \frac{8}{3}$

→ critical temp. of water = 374°C

→ Latent heat of water is zero at 380°C

→ Density of liquid & vapour are equal at 380°C

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Qu 1 The equation of state for a gas is given by

$$\left[P + \left(\frac{\alpha N}{V}\right)^2\right](V - \beta n) = NK_BT$$

If the critical point corresponds to a point of inflexion of the P-V curve, then the critical volume V_c and critical Pressure P_c for this gas are given by:

a) $V_c = 3\beta N$; $P_c = \frac{\alpha^2}{3\beta^2}$

b) $V_c = 3\beta N$; $P_c = \frac{\alpha}{27\beta^2}$

c) $V_c = 3\beta N$; $P_c = \frac{8\alpha^2}{27\beta}$

d) $V_c = 3\beta N$; $P_c = \frac{\alpha^2}{27\beta^2}$

Sol^{n}:-

$$\left[P + \left(\frac{\alpha^2 N^2}{V^2}\right)\right][V - \beta N] = NK_BT \quad -(1)$$

$$\left[P + \frac{an^2}{V^2}\right][V - bn] = nK_BT \quad -(2)$$

compare (1) & (2)

$a = \alpha^2$ & $\beta = b$

$V_c = 3\beta N$ ← same in all options

$P_c = \frac{a}{27b^2} = \frac{\alpha^2}{27\beta^2}$

D

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→ Clausis clapeyron equation:

$$\frac{dP}{dT} = \frac{L}{T(V_2 - V_1)}$$

1) Effect of Pressure on B.Pt of liquids:

liquid ⟶ gas
V_1 V_2

$(V_2 - V_1) = +ve$

$\frac{dP}{dT} = +ve$

B.Pt ∝ Pressure

i.e liquid will boil at lower temperature under reduced Pressure.

2) Effect of Pressure on M.Pt of solids:

$(V_2 - V_1) = +ve$	$(V_2 - V_1) = -ve$
M.Pt ∝ Pressure	M.Pt ∝ $\frac{1}{\text{Pressure}}$
eg wax, Sulphur etc.	eg. Ice, gallium, Bismuth etc.

Que! A quantity of water is completely converted into steam by boiling.

a) This is a second order phase transition

b) At the boiling point, the chemical potential in the vapour phase is less than in liquid phase

c) The B.Pt temperature would ↑ if boiling were done under ↑ed Pressure.

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d) The entropy of steam is greater than that of water.

Soln:- By clausius clapyron eqn:

liquid ⟶ gas

B.Pt ∝ Pressure → option (C)

$(\Delta S)_{steam} = (\Delta S)_{water} + \frac{L}{T}$ → C & D

$(\Delta S)_{steam} > (\Delta S)_{water}$ → option d

Qu) The B.Pt. of a liquid at Pressure P_0 is T_0. Its molar latent heat of vaporisation is L and molar volume of liquid phase is negligible as compared to the vapour phase. The vapour phase obeys perfect gas equation. The B.Pt. T at Pressure P is given by:

a) $P/P_0 = (L/RT_0)/(1 - T_0/T)$ b) $\ln(P/P_0) = (L/RT_0)/(1 - T_0/T)$

c) $\ln(P/P_0) = (L/RT_0)/(1 - T/T_0)$ d) $P/P_0 = 1 + (L/RT_0)\ln(T_0/T)$

Soln:-

$$\frac{dP}{dt} = \frac{L}{T(V_2 - V_1)} \simeq \frac{L}{TV_2} \quad \therefore V_2 >> V_1$$

ideal gas eqn: $PV_2 = RT \Rightarrow V_2 = \frac{RT}{P}$

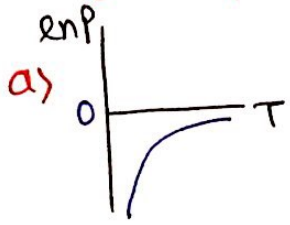
$$\frac{dP}{dT} = \frac{L}{T \cdot RT/P} = \frac{LP}{RT^2}$$

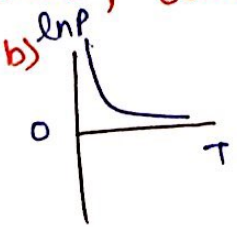
$$\int \frac{dP}{P} = \frac{L}{R}\int \frac{dT}{T^2}$$

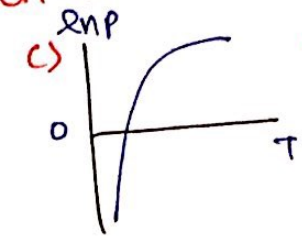
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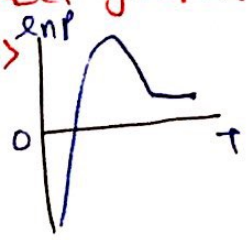
$$\ln(P/P_0) = \frac{L}{R}\left[-\frac{1}{T}\right]_{T_0}^{T} \Rightarrow \frac{L}{RT_0}\left[1-\frac{T_0}{T}\right]$$ **B**

Que 3 The condition for the liquid & vapour phases of a fluid to be in eqm. is given by approximate eqn: $\frac{dP}{dT} \simeq \frac{Q_t}{TV_{vap}}$. If the vapour obeys ideal gas law, which is the CORRECT graph?

a) 

b) 

c) 

d) 

Soln:- $PV_{vap} = nRT$ (ideal gas law).

$$V_{vap} = \frac{RT}{P} \quad (\text{for } n=1) \quad -(1)$$

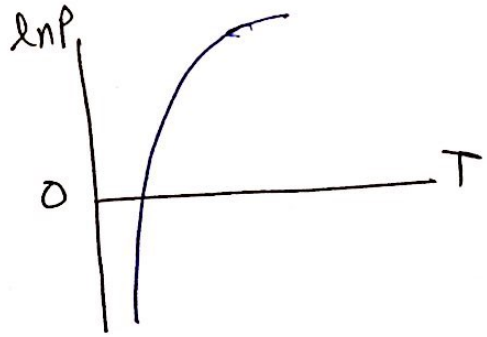
$$\frac{dP}{dt} = \frac{Q_l}{T}\frac{P}{RT} \Rightarrow \int\frac{dP}{P} = \frac{Q_l}{R}\int\frac{dT}{T^2}$$

$$\ln\left(\frac{P}{P_0}\right) = \frac{Q_l}{R}\left(-\frac{1}{T}\right)_{T_0}^{T} \Rightarrow \frac{Q_l}{R}\left(\frac{1}{T_0}-\frac{1}{T}\right)$$

$$\ln P = \ln P_0 + \frac{Q_l}{RT_0} - \frac{Q_l}{RT}$$

$T \to 0 \Rightarrow \ln P \to \infty$

$T \to \infty \Rightarrow \ln P \to C$



C

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→ **Diffusion equation:**

$$\nabla^2 f - \frac{1}{D}\frac{\partial f}{\partial t} = 0$$

$f \rightarrow \dfrac{\text{flow of Particles}}{\text{mass} \times \text{volume}}$

For 3-Dim.:

$$f(\vec{r},t) \propto e^{-r^2/4Dt}$$

$$\int f(\vec{r},t)\, d\tau = \frac{\text{no. of Particles}}{\text{time}} = \frac{N}{t} \Rightarrow N'$$

$$A \int_0^\infty e^{-r^2/4Dt} \cdot 4\pi r^2 dr = N'$$

$$\Rightarrow A = \frac{N'}{(4\pi Dt)^{3/2}}$$

$$\boxed{f(\vec{r},t) = \frac{N'}{(4\pi Dt)^{3/2}} e^{-r^2/4Dt}} \rightarrow \text{3-Dim.}$$

similarly

$$f = \frac{N'}{(4\pi Dt)} e^{-r^2/4Dt} \quad \uparrow \text{2-Dim}$$

$$f = \frac{N'}{(4\pi Dt)^{1/2}} e^{-r^2/4Dt} \quad \uparrow \text{1-Dim}$$

For 1-Dim: $\langle x^2 \rangle = 2Dt$

For 2-Dim: $\langle x^2 + y^2 \rangle = 4Dt$

For 3-Dim: $\langle x^2 + y^2 + z^2 \rangle = 6Dt$

Que! A large number N of Brownian particles in 1-Dimension start their diffusive motion from the origine at time $t=0$. The diffusion coefficient is D. The number of particles crossing a point at a distance L from the origine, per unit time, depends on L & time t as:

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a) $\frac{N}{\sqrt{4\pi Dt}} e^{L^2/4Dt}$ b) $\frac{NL}{\sqrt{4\pi Dt}} e^{4Dt/L^2}$

c) $\frac{N}{\sqrt{16\pi Dt^3}} e^{L^2/4Dt}$ d) $N e^{-4Dt/L^2}$

Soln: eqn of Brownian motion

$$\nabla^2 f - \frac{1}{D}\frac{\partial f}{\partial t} = 0$$

$$f(\vec{r},t) = A \exp(-r^2/4Dt)$$

for 1-Dim. $= A \Rightarrow \frac{N}{(4\pi Dt)^{1/2}}$

$$f(\vec{x},t) = \frac{N}{\sqrt{4\pi Dt}} e^{-L^2/4Dt}$$ [A]

Ques: consider two independent diffusing non-interacting particles in 3-Dim. space, both placed at the origine at time $t=0$. These particles have different diffusion constants D_1 and D_2. The quantity $\langle [\vec{R}_1(t) - \vec{R}_2(t)]^2 \rangle$ where $\vec{R}_1(t)$ and $\vec{R}_2(t)$ are the positions of the particles at time t, behaves as:

a) $6t(D_1+D_2)$ b) $6t(D_1-D_2)$ c) $6t\sqrt{D_1^2+D_2^2}$ d) $6t\sqrt{D_1 D_2}$

Soln: $\langle R_1^2(t) \rangle = 6D_1 t$ & $\langle R_2^2(t) \rangle = 6D_2 t$

$$\langle [\vec{R}_1(t) - \vec{R}_2(t)]^2 \rangle = \langle R_1^2(t) \rangle + \langle R_2^2(t) \rangle - 2\langle R_1(t) \rangle \langle R_2(t) \rangle$$

($\langle R_1(t) \rangle = 0$)

$$= 6t(D_1+D_2)$$ [A]

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STATISTICAL MECHANICS

→ **Classical Statistics**: (maxwell Boltzmann)

1) Particles are identical but distinguishable
2) Utilizes classical result of maxwell's law of distribution of molecular velocity and Boltzmann theorem.
3) Fails at low temperature.

→ **Quantum Statistics**:

for indistinguishable Identical Particles.

Bose-Einstein	Fermi-Dirac Statistics
used in case of indistinguishable identical Particles having integral spin.	used in case of indistinguishable identical Particles having ½ integral spin
wavefunction of the Particle is symmetric & don't follow Pauli exclusion Principle	wavefunction of the Particle is antisymmetric & follow Pauli exclusion Principle
eg. Boson; Photon; graviton etc.	eg. Fermions: $e^{\ominus}$, p, n, τ, μ^-, $\Sigma^{\pm,0}$ etc.

→ at High temperature: all statistics approaches classical statistics.

→ Statistical mechanics solve the problem of macroscopic system by microscopic point of view.

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→ Classical Distribution Law:

Assessible microstate: microstates corresponding to a macrostate.

Thermodynamic Probability: no. of microstates corresponding to a macrostate.

$$\text{Probability} = \frac{\text{no. of microstate for accessible macrostate}}{\text{total no. of microstates.}}$$

Boltzmann entropy Relation: $\boxed{S = k \ln \Omega}$

Stirling Approximation: $\boxed{\ln n! = n \ln n - n}$

number of microstates:

$$\boxed{\Omega_{MB} = \prod_i (g_i)^{n_i}} \qquad \boxed{\Omega_{BE} = \prod_i \frac{(n_i + g_i - 1)!}{n_i! \, (g_i - 1)!}}$$

$$\boxed{\Omega_{FD} = \prod_i {}^{g_i}C_{n_i}}$$

$g_i \rightarrow$ degeneracy

$n_i \rightarrow$ identical Particles

energy distribution function: $\boxed{f(E) = \frac{n_i}{g_i}}$

Que! The number of ways in which N identical bosons can distributed in two energy levels, is:

a) $N+1$ b) $N(N-1)/2$ c) $N(N+1)/2$ d) N

Soln: $\Omega_{BE} = \frac{(N_i + g_i - 1)!}{N_i! \, (g_i - 1)!} = \frac{(N+2-1)!}{N! \, 1!}$

$= \frac{(N+1)!}{N!} = \boxed{N+1}$ Ans. [A]

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Que 2 A system of N distinguishable particles, each of which can be in one of the two energy levels 0 & e, has a total energy ne, where n is an integer. The entropy of the system is proportional to:

a) $N \ln n$ b) $n \ln N$ c) $\ln\left(\frac{N!}{n!}\right)$ d) $\ln\left(\frac{N!}{n!(N-n)!}\right)$

Soln:- $\Omega = {}^{N}C_{n} = \frac{N!}{n!\,(N-n)!}$

n — e

$(N-n)$ — 0

$$\text{entropy } (S) = K \ln \Omega = K \ln \frac{N!}{n!\,(N-n)!}$$

$$S \propto \ln \frac{N!}{n!\,(N-n)!}$$ D

*** Note: Probability will be maximum for equal distribution.

Que 3 Let N_{MB}, N_{BE}, N_{FD} denote the number of ways in which two particles can be distributed in two energy states according to maxwell-Boltzmann, Bose-Einstein & Fermi-Dirac statistics respectively. Then $N_{MB} : N_{BE} : N_{FD}$ is:

a) 4:3:1 b) 4:2:3 c) 4:3:3 d) 4:3:2

Soln:- $\Omega_{MB} = (g_i)^{n_i} = (2)^2 \Rightarrow 4$

$$\Omega_{BE} = \frac{(n_i + g_i - 1)!}{n_i!\,(g_i - 1)!} = \frac{(2+2-1)!}{2!\,(2-1)!} = \frac{3!}{2!} = 3$$

$$\Omega_{FD} = {}^{g_i}C_{n_i} = \frac{2!}{2!\,1!} = 1$$

4:3:1 A

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→ **Relative Probability:**

$$r = \frac{P_1}{P_2} = \frac{\text{Prob. of finding particle in same state}}{\text{Prob. of finding particle in diff. state.}}$$

$$\boxed{r_{BE} : r_{MB} : r_{F.D} = 1 : 1/2 : 0}$$

$$\Rightarrow \quad U_{BE} \leq U_{MB} \leq U_{FD} \Rightarrow P_{BE} \leq P_{MB} \leq P_{FD}$$

Que 4 If N particles are to be distributed into two groups such that group 1 contain n_1 particles & group 2 contains n_2 particles, and $N = n_1 + n_2$. Find the number of ways of distribution?

Soln:- $r = {}^{N}C_{n_1}$ or ${}^{N}C_{n_2}$ & $N = n_1 + n_2$

$${}^{N}C_{n_i} = \frac{N!}{n_1!(N-n)!} = \boxed{\frac{N!}{n_1! n_2!}} \text{ Ans.}$$

Similarly, For N_i Particles.

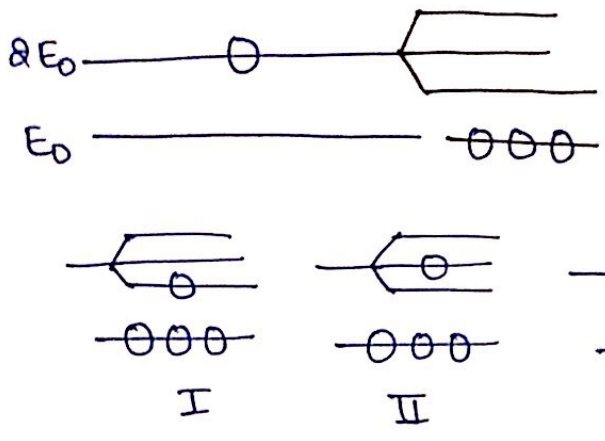
$$r = \frac{N!}{n_1! n_2! \ldots\ldots n_i!}$$

Que 5 A system has energy levels $E_0, 2E_0, 3E_0, \ldots\ldots$ where the excited states are highly degenerate. Four non-interacting bosons are placed in this system. If the total energy of these bosons is $5E_0$ the no. of microstates is:

a) 2 b) 3 c) 4 d) 5

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Soln:-



B

→ **PHASE SPACE:** (Position + momentum) space

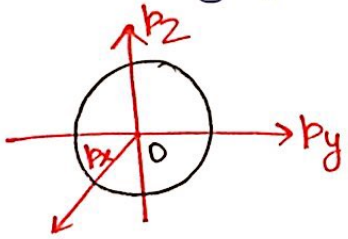
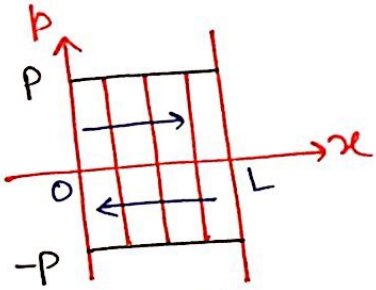
Volume of single Phase cell: $d\tau = \hbar^f$ $\quad f \to$ DOF

allowed vol. of Phase space: $\int d^3q \int d^3p$ (For 3-Dim).

$$V_p \Rightarrow V \times \frac{4}{3}\pi p^3$$

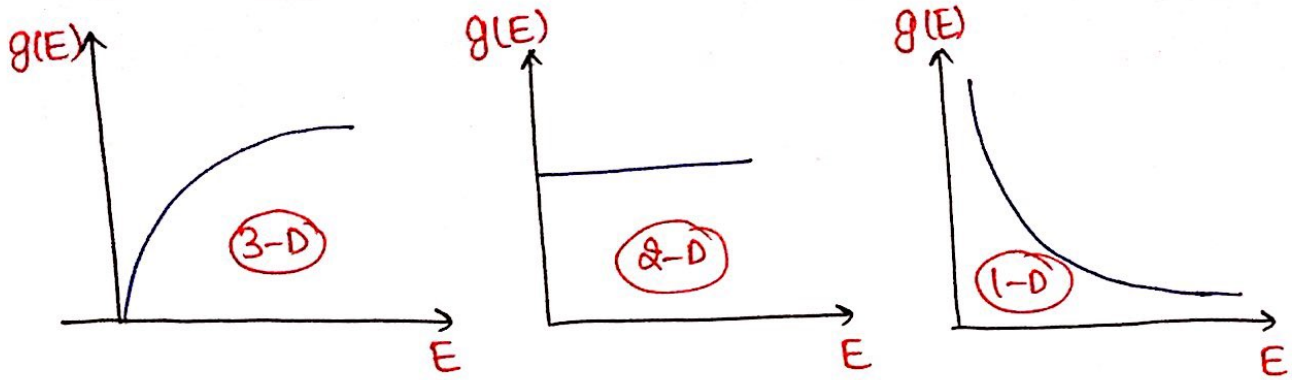
1) **Non-Relativistic Free Particle:** $(E = p^2/2m)$

→

3-Dimensional	2-Dimensional	1-Dimensional
$d\tau = \hbar^3$	$d\tau = \hbar^2$	$d\tau = \hbar$
$V_p = V \times \frac{4}{3}\pi p^3$	$V_p = A \times \pi p^2$	$V_p = L \times 2p$
	$\Omega(p) = \frac{\pi \times \pi p^2}{\hbar^2}$	
$\Omega = \frac{V \cdot \frac{4}{3}\pi p^3}{\hbar^3}$	$\Omega'(p) = \frac{2\pi A p}{\hbar^2} dp$	$\Omega(p) = \frac{L \times 2p}{\hbar}$
$\Omega(E) = \frac{4\pi V (2mE)^3}{3\hbar^3}$	$\Omega(E) = \frac{\pi A (2mE)}{\hbar^2}$	$\Omega(E) = \frac{2L\sqrt{2mE}}{\hbar}$
$\Omega'(E) = \frac{2\pi V (2m)^{3/2} E^{1/2}}{\hbar^3} dE$ ($g(E)$)	$\Omega'(E) = \frac{2\pi m A}{\hbar^2} dE$ (density of states)	$\Omega'(E) = \frac{L (2m)^{1/2} E^{-1/2}}{\hbar} dE$ ($g(E)$)

55

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$g(E) \propto E^{1/2}$ $\quad g(E) \propto E^{0}$ $\quad g(E) \propto E^{-1/2}$

Que! In a classical microcanonical ensemble for a system of N non-interacting particles, the fundamental volume of phase space which is regarded as "equivalent to one-microstate" is:

a) R^{3N} b) R^{2N} c) R^{N} d) R

Sol^{n}:- For N non-interacting particles: DOF $= 3N = f$

Volume of Phase space $= R^{f}$

$= R^{3N}$ **A**

2) **Extremely Relativistic free Particle**: $(E = pc)$

3-Dim.	2-Dimension	1-Dimension
$\Omega(p) = \frac{V \cdot \frac{4}{3}\pi p^3}{R^3}$	$\Omega(p) = \frac{\pi A p^2}{R^2}$	$\Omega(p) = \frac{L \times 2p}{R}$
$\Omega'(p) = \frac{4\pi V p^2 dp}{R^3}$	$\Omega'(p) = \frac{2\pi A p dp}{R^2}$	$\Omega'(p) = \frac{2L dp}{R}$
$\Omega(E) = \frac{4\pi V E^3}{3c^3R^3}$	$\Omega(E) = \frac{A\pi E^2}{c^2R^2}$	$\Omega(E) = \frac{2LE}{Rc}$
$\Omega'(E) = 2 \times \frac{4\pi V E^2}{c^3R^3} dE$ (↑ due to transverse nature of Photon.)	$\Omega'(E) = \frac{2\pi A E}{c^2R^2}$ $g(E) \propto E$	$\Omega'(E) = \frac{2L}{Rc} dE$ $g(E) \propto E^0$

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Que 2 Find density of states at energy E of the quantized radiation (Photon) is:

a) $\frac{8\pi V}{h^3c^3}E^2$ b) $\frac{8\pi V}{h^3c^3}E^{3/2}$ b) $\frac{8\pi V}{h^3c^3}E$ d) $\frac{8\pi V}{h^3c^3}E^{3/2}$

Sol^{n} $\Omega(p) = \frac{V \times \frac{4}{3}\pi p^3}{h^3} \Rightarrow \Omega(E) = \frac{4\pi V E^3}{3c^3h^3}$

$$\Omega'(E) = \left(\frac{4\pi V E^2}{c^3h^3}dE\right) \times 2$$

↑ for Photon.

$$\Omega'(E) = g(E)\,dE$$

$$g(E) = \frac{8\pi V}{c^3h^3}E^2$$ A

3) For free particle (with $E = ap^s$) with d-DOF:

volume of sphere of radius (R): $V_d = \frac{\pi^{d/2}R^d}{(\frac{d}{2})!}$

area of sphere of radius (R): $S_d = \frac{2\pi^{d/2}R^{d-1}}{(\frac{d}{2}-1)!}$

volume of single phase cell: $d\tau = h^d$

$$g(E) = \frac{g_i V_d \pi^{d/2}\left(\frac{1}{a}\right)^{d/s}\left(\frac{d}{s}\right)E^{(d/s-1)}}{h^d\,(d/2)!}$$

$$\boxed{g(E) \propto E^{(d/s-1)}}$$ ***

d = DOF

s = 1 (massless)
2 (massive)

Que 3 For an energy state E of a photon gas, the density of states is proportional to:

a) $\sqrt{E}$ b) E c) $E^{3/2}$ d) E^2

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Soln:- $D(E) \propto E^{\left(\frac{d}{s}-1\right)}$

For proton: $d=3$ & $s=1$

$D(E) \propto E^{3-1} \Rightarrow D(E) \propto E^2$ **D**

Que 2 The number of states for a system of N identical free particles in a three dimensional space having total energy between E & E+δE (δE≪E), is proportional to:

a) $E^{\left(\frac{3N}{2}-1\right)}\delta E$ b) $E^{\frac{N}{2}}\delta E$ c) $NE^{\frac{1}{2}}\delta E$ d) $N^N\delta E$

Soln:- $D(E) \propto E^{\left(\frac{d}{s}-1\right)}$

Here $d = 3N$ & $s = 2$ (massive)

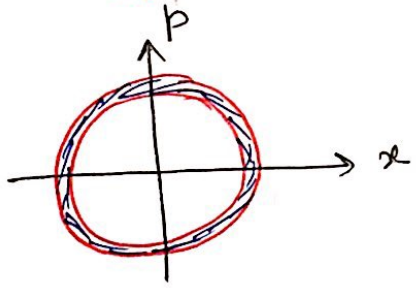
$D(E) \propto E^{\left(\frac{3N}{2}-1\right)}$ **A**

Que 3 Draw phase space trajectory of linear (1-Dim.) Harmonic oscillator & calculate number of micro-state if particle has:

a) sharp value of energy

b) Energy lies b/w E to E+dE

Soln:-



$$E = \frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2$$

$$1 = \frac{p^2}{\sqrt{2mE}} + \frac{x^2}{\left(\sqrt{\frac{2E}{m\omega^2}}\right)^2}$$

area of ellipse $= \pi ab$

Vol. of single phase cell (1-D) $= h$

$$\Omega(E) = \frac{\pi ab}{h} = \frac{\pi \times \sqrt{\frac{2E}{m\omega^2}} \times \sqrt{2mE}}{h}$$

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$$\Omega(E) = \frac{2\pi E}{\hbar\omega} = \frac{E}{\hbar\omega} \quad (a)$$

$$\boxed{\Omega'(E) = \frac{dE}{\hbar\omega}} \quad (b)$$

Note: maximum value of x

on x-axis: $p = 0$

$$E_0 = 0 + \frac{1}{2} m\omega^2 x_0^2$$

$$x_0 = \sqrt{\frac{2E_0}{m\omega^2}}$$

$$\boxed{x_0 \propto \sqrt{E_0}}$$

→ Most Probable Distribution function:

$$\boxed{f(E) = \frac{1}{e^{(E-\mu)/kT} + a}}$$

$a \to 0$ MB

$a \to -1$ BE

$a \to 1$ FD

$f_{unoccupied} = 1 - f_{occupied}$

Fermi-level: is that level at which probability of occupancy is 1/2 for all temperatures.

total no. of particles in the system:

$$N = \int_0^\infty g(E) f(E)\, dE$$

energy of system: $U = \int_0^\infty g(E) f(E) E\, dE$

Que! The probability that an energy level ε at a temperature T is unoccupied by a fermion of chemical potential μ is given by:

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a) $\frac{1}{e^{(E-H)/KT}+1}$ b) $\frac{1}{e^{(E-H)/KT}-1}$ c) $\frac{1}{e^{(H-E)/KT}+1}$ d) $\frac{1}{e^{(H-E)/KT}-1}$

Soln:- $f(E) = \frac{1}{e^{(E-H)/KT}+1}$ For Fermions

$f_{unoccupied} = 1 - f_{occupied}$

$= 1 - \frac{1}{e^{E-H/KT}+1} = \frac{1+e^{E-H/KT}-1}{e^{E-H/KT}+1}$

$= \frac{e^{(E-H)/KT}}{e^{(E-H)/KT}+1} = \frac{1}{1+e^{(H-E)/KT}}$ C

Que:- An ideal Bose gas is confined inside a container that is connected to a particle reservoir. Each particle can occupy a discrete set of single particle quantum states. If the probability that a particle quantum state is unoccupied is 0.1, then the average number of Bosons in that state is:

a) 8 b) 9 c) 10 d) 11

Soln:- $f_{occupied} = 1 - f_{unoccupied}$

$= 1 - 0.1 \Rightarrow 0.9$

Prob. of occupancy = 90%.

If there are total 10 bosons then average no. of bosons in that state = 9

B

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Que 3 The Hamiltonian of a classical particle moving in one-Dimensional is:

$$H = \frac{p^2}{2m} + \alpha q^4$$

where α is a positive constant & p and q are its momentum & position respectively. Given that its total energy $E \leq E_0$ the available volume of phase space depends on E_0 as:

a) $E_0^{3/4}$ b) E_0 c) $\sqrt{E_0}$ d) independent of E_0

Soln:- $H = \frac{p^2}{2m} + \alpha q^4$

Volume of Phase space (1-D) $\propto p \propto \sqrt{2mE} \times \left(\frac{E_0}{\alpha}\right)^{1/4}$

At $q=0$: $H = p_0^2/2m$, $p_0 = \sqrt{2mE}$

At $q=0$: $E_0 = \alpha q_0^4$, $q_0 = (E_0/\alpha)^{1/4}$

$$V \propto (E_0)^{1/2} \times \left(\frac{E_0}{\alpha}\right)^{1/4}$$

$$V \propto (E_0)^{3/4}$$

A

Que 4 The dimension of phase space of ten rigid diatomic molecules is:

a) 5 b) 10 c) 50 d) 100

Soln:- For Rigid molecule = DOF = 5

For 10 molecule: DOF = 5 × 10 = 50

Dim. of Phase space = 2 × DOF = 100

D

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→ Probability Distribution:

1) Coin Related Problem:

ⓐ Tossing a coin:

Total events = H+T

Total no. of events = 2

Total no. of microstate = 2

equally likely probability event = 1/2

ⓑ Two coin are tossed together:

Possible events: HH, HT, TH, TT

equally likely Probability = 1/4

→ Total no. of microstates $= (2)^n$

→ equally likely Prob. $= (1/2)^n$

Prob. of general event $\Rightarrow {}^nC_r (p)^{n-r} (1-p)^r$

$\Rightarrow {}^nC_r \left(\frac{1}{2}\right)^n$

Que! If 10 similar coins are tossed together what is the probability of getting 3 Head?

Soln. $P(3H) = {}^{10}C_3 \left(\frac{1}{2}\right)^{10-3} \left(\frac{1}{2}\right)^3$

$= {}^{10}C_3 \left(\frac{1}{2}\right)^{10} \Rightarrow \frac{10!}{3!\,7!} \left(\frac{1}{2}\right)^{10}$

$= \frac{10 \times 9 \times 8}{3 \times 2} \times \frac{1}{1024} \Rightarrow \frac{15}{128}$ Ans.

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© number of microstates in spin system:

$$\Omega = (2s+1)^N$$

Que 2 The total number of accessible states of N non interacting particles of spin ½ is:

a) 2^N b) N^2 c) $2N^{N/2}$ d) N

Sol$^{n}_{o}$- $\Omega = (2\times\frac{1}{2}+1)^N = 2^N$ (A)

***Note: entropy: $S = k\ln\Omega = k\ln(2)^N \Rightarrow Nk\ln 2$

Que 3 The number of distinct ways of placing four indistinguishable balls into 5 distinguishable boxes is:

a) 20 b) 40 c) 50 d) 70

Sol$^{n}_{o}$- $^{n+r-1}C_r = \frac{8!}{4!4!} = \frac{8\times7\times6\times5}{4\times3\times2\times1}$

$\Rightarrow 70$ (D)

Que 4 A system of 5 indifical but distinguishable Particles having energy 3ε. The single Particle states are available at energies 0, ε, 2ε, 3ε Find the average no. of Particles in each state?

Sol$^{n}_{o}$-

0	ε	2ε	3ε	E_{total}
4	—	—	1	3ε
3	1	1	—	3ε
2	3	—	—	3ε

$\Omega = 5+20+10 \Rightarrow 35$

(P)

$\Omega = \frac{5!}{4!0!0!} = 5$ 5/35

$\Omega = \frac{5!}{3!1!0!} = 20$ 20/35

$\Omega = \frac{5!}{2!3!0!0!} = 10$ 10/35

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*****

2) Random walk:

→ 1-Dimensional Random walk:

let $N \rightarrow$ steps of equal length along a line.

$p \rightarrow$ Probability of taking a step right

$q \rightarrow$ Probability of taking a step left

$n_1 \rightarrow$ no. of steps taken to right

$n_2 \rightarrow$ no. of steps taken to left.

$p+q=1$ & $n_1+n_2=N$

Qu! A one dimensional random walker takes steps to left or right with equal probability. The probability that the random walker starting from origine is back to origine after N even number of steps:

a) $\frac{N!}{(\frac{N}{2})!\,(\frac{N}{2})!}\left(\frac{1}{2}\right)^n$ b) $\frac{N!}{(\frac{N}{2})!\,(\frac{N}{2})!}$

c) $2N!\left(\frac{1}{2}\right)^{2N}$ d) $N!\left(\frac{1}{2}\right)^N$

Soln:- no. of steps $= N$

$P(p)=P(q)=1/2$

$\Rightarrow n_1=N/2$ & $n_2=N/2$

$$\text{Prob.} = {}^{N}C_{N/2}\left(\frac{1}{2}\right)^{N/2}\left(\frac{1}{2}\right)^{N/2}$$

$$= \frac{N!}{(\frac{N}{2})!\,(\frac{N}{2})!}\left(\frac{1}{2}\right)^N$$

[A]

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Que 2 A one dimensional Random walker takes 10 steps in left & Right with equal Probability. find out the probability that the random walker starting from origine is back to origine after taking 10 steps?

Soln:- $P(5,5) = {}^{10}C_5 \left(\frac{1}{2}\right)^{10}$

$= \frac{10!}{5!5!} \times \frac{1}{1024}$

Que 3 A random walker takes a step of unit length in +ve direction with probability 2/3 and a step of unit length in -ve direction with probability 1/3 The net displacement of Random walker after n-steps is?

a) n b) (1/3)n c) (2/3)n d) 2n

Soln:- Displacement $= n_{PL} - n_{BL}$

$= \frac{2}{3}n - \frac{1}{3}n \Rightarrow \frac{1}{3}n$ [B]

Que 4 In a one dimension Random walker takes a step with equal probability to the left or right. what is the probability that the walker returns to starting point after four steps?

a) 1/2 b) 3/4 c) 3/8 d) 3/16

Soln:- $P(2,2) = {}^{4}C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2$

$= \frac{4!}{2!2!} \left(\frac{1}{2}\right)^4 = \frac{3}{8}$ [C]

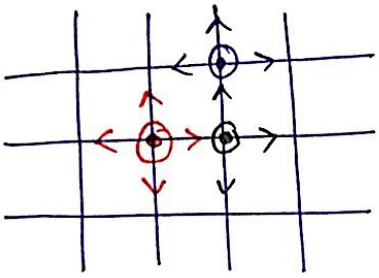
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→ 2-Dimensional Random walk:

Que! Consider a random walker on a square lattice. At each step the walker moves to a nearest neighbour site with equal probability for each of the four sites. The walker starts to the origine & takes 3 steps. The probability that during this walk no site is visited more than once is:

a) 12/27 b) 27/64 c) 3/8 d) 9/16

Soln:-

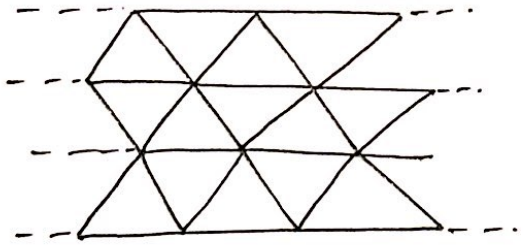


Total ways = $(4)^3 \Rightarrow 64$

favourable = $4 \times 3 \times 3$
$= 36$

$$\text{Prob.} = \frac{36}{64} = \frac{9}{16}$$ [D]

Que2 Consider a random walk on an infinite two dimensional triangular lattice, a part of which is shown in the figure below:

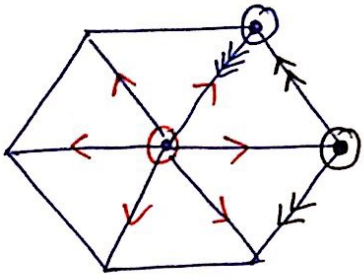


If the probabilities of moving to any of the nearest neighbour sites are equal, what is the probability that the walker returns to the starting position at the end of exactly 3 steps?

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a) $\frac{1}{36}$ b) $\frac{1}{216}$ c) $\frac{1}{18}$ d) $\frac{1}{12}$

Soln$^{n}_{o}$-



$P_1 = \frac{6}{6}$

$P_2 = \frac{2}{6}$

$P_3 = \frac{1}{6}$

C

$P = P_1 P_2 P_3 = \frac{6}{6} \times \frac{2}{6} \times \frac{1}{6} \Rightarrow \frac{1}{18}$

Que 3 A child makes a random walk on a square lattice of lattice constant 'a' taking a step in the north, east, south or west direction with probabilities 0.255, 0.255, 0.245, 0.245, respectively. After a large number of steps, N, the expected position of child w.r.t starting point is at a distance:

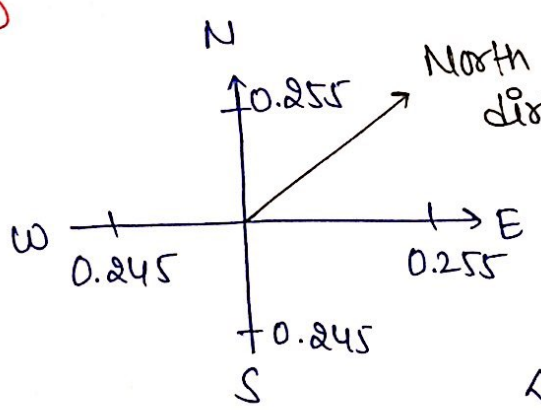
a) $\sqrt{2} \times 10^{-2} Na$ in north-east direction

b) $\sqrt{2N} \times 10^{-2} a$ in north east direction

c) $2\sqrt{2} \times 10^{-2} Na$ in south east direction

d) 0

Soln$^{n}_{o}$-



$\langle x \rangle_N = 0.255\, Na\, \hat{i}$

$+ 0.255\, Na\, \hat{j} - 0.245\, Na\, \hat{i}$

$- 0.245\, Na\, \hat{j}$

$\langle x \rangle_N = 10^{-2} (Na\, \hat{i} + Na\, \hat{j})$

$= 10^{-2} \sqrt{(1)^2 + (1)^2}\, Na$

A

$\langle x \rangle_N = \sqrt{2} \times 10^{-2} Na$ in N-E

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→ Types of Equilibrium:

entropy: $\boxed{S = S_1 + S_2}$ (additive)

no. of microstate:

$$S = S_1 + S_2 \Rightarrow k \ln \Omega = k \ln \Omega_1 + k \ln \Omega_2$$

$$\boxed{\Omega = \Omega_1 \Omega_2}$$ (multiplicative)

*** → in equilibrium: entropy & no. of microstate is maximum.
→ energy is minimum.

1) conducting, rigid & impermeable walls:

$$E_1 + E_2 = E^{(0)} = \bar{E}_1 + \bar{E}_2$$

$$\bar{\beta}_1 = \bar{\beta}_2 \Rightarrow \boxed{\bar{T}_1 = \bar{T}_2}$$ → thermal equilibrium

2) non-conducting, non-rigid & impermeable walls:

$$V_1 + V_2 = V^{(0)} = \bar{V}_1 + \bar{V}_2$$

$$\bar{\eta}_1 = \bar{\eta}_2 \Rightarrow \boxed{\frac{\bar{P}_1}{\bar{T}_1} = \frac{\bar{P}_2}{\bar{T}_2}}$$

if walls are conducting: $\bar{T}_1 = \bar{T}_2$

$$\Rightarrow \boxed{\bar{P}_1 = \bar{P}_2}$$ → mechanical equilibrium

3) non-conducting, rigid & Permeable:

$$N_1 + N_2 = N^{(0)} = \bar{N}_1 + \bar{N}_2$$

$$\mu_1 = \mu_2 \Rightarrow \frac{\bar{\mu}_1}{\bar{T}_1} = \frac{\bar{\mu}_2}{\bar{T}_2}$$

if walls are conducting: $\bar{T}_1 = \bar{T}_2 \Rightarrow \boxed{\bar{\mu}_1 = \bar{\mu}_2}$

chemical equilibrium

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Thermodynamic equilibrium:

→ If wall is conducting, non rigid & permeable.

$$\bar{T}_1 = \bar{T}_2 \; ; \; \bar{P}_1 = \bar{P}_2 \; ; \; \bar{\mu}_1 = \bar{\mu}_2$$

Postulate: Equal a priori Probability

→ All microstates corresponding to a given macrostate are equally probable.

→ Ensemble: collection of large number of systems that are macroscopically identical but microscopically different.

microcanonical	canonical	Grand canonical
E, V, N	T, V, N	T, V, μ → fixed values
insulating, Rigid & impermeable	conducting, Rigid & impermeable	conducting, rigid & permeable
isolated system	thermal eqm.	thermal + chemical eqm.
universe, thermal flask etc.		

Que! The entropy of a system, S, is related to the accessible phase space volume Γ by

$$S = k_B \ln \Gamma (E, V, N)$$

where E, N & V are energy, no. of particles & volume respectively. From this one can conclude that Γ. :

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a) doesn't change during evolution to eqm.

b) Oscillates during evolution to eqm.

c) Is a maximum in eqm.

c) Is a minimum in eqm.

Soln: E, V, N → micro canonical ensemble

$$S = k \ln \Gamma(E, V, N)$$

↑

maximum when Γ is maximum.

in eqm.: $S \rightarrow$ maximum [C]

$\Rightarrow \Gamma \rightarrow$ maximum

Que 2 In a canonical ensemble description of a system, which of the following quantities remain fixed?

a) energy of system b) square of energy

c) volume of system d) Number of Particles

Soln: Canonical ensemble:

Fixed values are T, V, N [C, D]

Que 3 A microcanonical ensemble represents:

a) a system in contact with a heat reservoir.

b) an isolated system in eqm.

c) a system can exchange particle with surroundings

d) a system under constant external Pressure.

Soln: microcanonical: E, V, N fixed [B]

isolated system in eqm

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1) **Microcanonical Ensemble:**

For classical Ideal gas:

a) **non-Relativistic case:**

$$E = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mE}$$

For N-particle: DOF = 3N

Volume of position space = V^N

Volume of momentum space = $\frac{\pi^{3N/2} p^{3N}}{(3N/2)!}$

$$\rightarrow \Omega = \frac{V^N . \pi^{3N/2} (\sqrt{2mE})^{3N}}{h^{3N} (3N/2)!}$$

$$\rightarrow E = U = \frac{3}{2} NKT = \frac{3}{2} nRT$$

$$\rightarrow P = \frac{1}{d}\left(\frac{E}{V}\right) = \frac{2}{3}\left(\frac{E}{V}\right)$$

$$\rightarrow PV^{5/3} = \text{constant}$$

$$\rightarrow S = K \ln \Omega$$

$$S = N_i K \ln V_i + \frac{3}{2} N_i K \ln\left(\frac{4\pi m_i E}{3N_i h^2}\right) + \frac{3}{2} N_i K$$

Mixing of two gases:

→ Volume density (Particle density) may be same but particles on both sides after division not same.

1) mixing of two different gases will not be reversible.

2) mixing of two different samples of same gas may be reversible or irreversible.

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Reversible mixing: (Gibbs Paradox):

entropy of mixing : $\Delta S = S_1 + S_2 + 2NK\ln 2$

↑ Gibbs Paradox.

Removal of gibbs Paradox:

$$\Omega_{indis.} = \frac{\Omega_{dis.}}{N!}$$

$$S_{indis.} = S_{dis} - K \ln N!$$

$$S = NK\ln\left(\frac{V}{N}\right) + \frac{3}{2}NK\ln\left(\frac{2\pi mKT}{h^2}\right) + \frac{5}{2}nK$$

↑ Sackur-Tetrode eqn

b) For exteme relativistic classical gas:

$E = PC$

$$\rightarrow \Omega = \frac{V^N T^{3N/2}}{(3N/2)!}\left(\frac{E}{Ch}\right)^{3N}$$

$$\rightarrow P = \frac{1}{3}\left(\frac{U}{V}\right)$$

$$\rightarrow PV^{4/3} = \text{constant}$$

- $C_V;\ S;\ N \propto T^{d/s}$
- $\Omega \propto E^{d/s}$
- $H = U + PV$
- $U;\ P \propto T^{d/s}$
- $P = \frac{s}{d}\left(\frac{U}{V}\right)$

Que: The pressure for a non-interacting fermi gas with internal energy U at temp. T is:

a) $P = \frac{3}{2}\left(\frac{U}{V}\right)$ b) $P = \frac{2}{3}\frac{U}{V}$ c) $P = \frac{3}{5}\frac{U}{V}$ d) none

Soln: non-interacting fermi gas = ideal classical gas.

$$U = \frac{3}{2}NKT = \frac{3}{2}PV$$

$$P = \frac{2}{3}\left(\frac{U}{V}\right)$$

B

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2) Canonical Ensemble:

$$\rightarrow P_i = \frac{g_i e^{-\beta \varepsilon_i}}{\sum g_i e^{-\beta \varepsilon_i}}$$

P_i ← Prob. for degenerate

$$Q_1 \rightarrow Z = \sum g_i e^{-\beta \varepsilon_i}$$

Z ← Partition fxn

a) For continuous energy:

$$Q_N(T,V) = (Q_1(T,V))^N$$

b) For discrete energy:

$$Q_N(T,V) = [Q_1(T,V)]^N \rightarrow \text{distinguishable}$$

$$= \frac{[Q_1(T,V)]^N}{N!} \rightarrow \text{indistinguishable.}$$

average value of Different quantities:

1) average energy:

$$\langle E_i \rangle = \sum E_i P_i \Rightarrow \frac{\sum \varepsilon_i g_i e^{-\beta \varepsilon_i}}{\sum g_i e^{-\beta \varepsilon_i}} \quad -(1)$$

$$\langle E_i \rangle = -\frac{\partial}{\partial \beta} \ln Z = +kT^2 \frac{\partial}{\partial T} \ln Z \quad -(2)$$

2) average Pressure:

$$\langle P \rangle = \sum p \frac{g e^{-\beta \varepsilon}}{Z} = \frac{1}{\beta} \frac{\partial}{\partial V} \ln Z \quad -(3)$$

3) average chemical Potential:

$$\langle \mu \rangle = \sum \frac{\mu g e^{-\beta \varepsilon_i}}{Z} = -\frac{1}{\beta} \frac{\partial}{\partial N} \ln Z \quad -(4)$$

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4) average entropy:

$$\langle S\rangle = k_B \ln Z + \frac{\langle E\rangle}{T} \Rightarrow k_B \ln Z - \frac{1}{T}\frac{\partial}{\partial \beta}\ln Z$$

5) average free energy:

$$\langle A\rangle = \langle E\rangle - T\langle S\rangle \Rightarrow \boxed{-k_B T \ln Z}$$

Que 2 The partition function of two Bose particles each of which can occupy any of the two energy levels of ε is:

a) $1+e^{-2\varepsilon/kT}+2e^{-\varepsilon/kT}$ b) $1+e^{-2\varepsilon/kT}+e^{-\varepsilon/kT}$

c) $2+e^{-2\varepsilon/kT}+e^{-\varepsilon/kT}$ d) $e^{-2\varepsilon/kT}+e^{-\varepsilon/kT}$.

Soln:- two bose particles: a,a

0	ε	E_{total}
aa	–	0
a	a	ε
–	aa	2ε

$$Z = \sum g_i e^{-\beta\varepsilon_i}$$

$$Z = 1e^{0} + e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}$$

$$Z = 1 + e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}$$

Here $g_i = 1$ **B**

Que 3 consider a system of two non-interacting classical particles which can occupy any of the three energy levels with energy values $E=0, \varepsilon \& 2\varepsilon$ having degeneracies $g(\varepsilon)=1,2 \& 4$ respectively. The mean energy of the system is:

a) $\varepsilon\left(\frac{4\exp(-\varepsilon/k_BT)+8\exp(-2\varepsilon/k_BT)}{1+2\exp(-\varepsilon/k_BT)+4\exp(-2\varepsilon/k_BT)}\right)$

b) $\varepsilon\left(\frac{2\exp(-\varepsilon/k_BT)+8\exp(-2\varepsilon/k_BT)}{1+2\exp(-\varepsilon/k_BT)+4\exp(-2\varepsilon/k_BT)}\right)$

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c) $\varepsilon\left[\frac{2\exp(-\varepsilon/k_BT)+4\exp(-2\varepsilon/k_BT)}{1+2\exp(-\varepsilon/k_BT)+4\exp(-2\varepsilon/k_BT)}\right]^2$

d) $\varepsilon\left[\frac{\exp(-\varepsilon/k_BT)+2\exp(-2\varepsilon/k_BT)}{1+\exp(-\varepsilon/k_BT)+\exp(-2\varepsilon/k_BT)}\right]^2$

soln:-

$$\langle E_i\rangle = \sum E_i p_i$$

$$= \frac{\sum \varepsilon_i g_i e^{-\beta\varepsilon_i}}{\sum g_i e^{-\beta\varepsilon_i}}$$

$$= \frac{0+2\varepsilon e^{-\beta\varepsilon}+3\varepsilon e^{-2\beta\varepsilon}}{1e^0+2e^{-\beta\varepsilon}+4e^{-2\beta\varepsilon}}$$

$$= \varepsilon\left(\frac{2e^{-\beta\varepsilon}+3e^{-2\beta\varepsilon}}{1+2e^{-\beta\varepsilon}+4e^{-2\beta\varepsilon}}\right)$$ **B**

Qu 4 The free energy of a gas, N particles in a volume V and at a temperature T is

$$F = NK_BT \ln\left(\frac{a_0V(K_BT)^{5/2}}{N}\right)$$

where a_0 is a constant & K_B denotes the Boltzmann constant. The internal energy of a gas is:

a) $\frac{3}{2}NK_BT$ b) $\frac{5}{2}NK_BT$

c) $NK_BT\ln\left[\frac{a_0V(K_BT)^{5/2}}{N}\right]-\frac{3}{2}NK_BT$ d) $NK_BT\ln\left[\frac{a_0V}{(K_BT)^{5/2}}\right]$

soln:-

$$F = NK_BT\ln\left[\frac{a_0V(K_BT)^{5/2}}{N}\right]^N \quad -(1)$$

$$F = -NK_BT\ln Z \quad -(2)$$

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compare ① & ②, we get

$$\ln Z = -\ln\left[\frac{a_0 V (K_B T)^{5/2}}{N}\right]^N$$

$$\langle U\rangle = -\frac{\partial}{\partial\beta}\ln Z = +KT^2\frac{\partial}{\partial T}\ln Z$$

$$\langle U\rangle = NK_BT^2\frac{\partial}{\partial T}\left[-\ln\left(\frac{a_0 V (K_BT)^{5/2}}{N}\right)\right]$$

$$= NK_BT^2\frac{\partial}{\partial T}\left[-\ln T^{5/2} - \ln\left(\frac{a_0 V K_B}{N}\right)\right]$$

$\Downarrow_0$

$$= -\frac{5}{2}NK_BT^2 \times \frac{1}{T}$$

$$= -\frac{5}{2}NK_BT$$ **B**

Ques A system has two energy levels with energies ε & 2ε. The lower level is 4-fold degenerate while the upper level is double degenerate. If there are N non-interacting classical particles in the system, which is in thermodynamic eqm. at a temperature T, the fraction of particles in the upper level is:

a) $\frac{1}{1+e^{\varepsilon/K_BT}}$ b) $\frac{1}{1+2e^{\varepsilon/K_BT}}$

c) $\frac{1}{2e^{\varepsilon/K_BT}+4e^{2\varepsilon/K_BT}}$ d) $\frac{1}{2e^{\varepsilon/K_BT}-4e^{2\varepsilon/K_BT}}$

Soln- $\varepsilon_1 = \varepsilon$ $g_1 = 4$

$\varepsilon_2 = 2\varepsilon$ $g_2 = 2$

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Fraction of Particles in upper level

$$P_2 = \frac{N_2}{N} = \frac{2e^{-2\beta\varepsilon}}{4e^{-\beta\varepsilon} + 2e^{-2\beta\varepsilon}}$$

$$P_2 = \frac{1}{2e^{\beta\varepsilon}+1}$$ **B**

Que6 A 1-Dim. chain consists of a set of N rods each of length a. when stretched by a load, each rod can align either parallel or antiparallel to length. The energy of a rod is $-\varepsilon$ when aligned parallel to length of the chain is $+\varepsilon$ when lar to it. when the chain is in thermal eqm. at temp. T its average length is:

a) $\frac{Na}{2}$ b) Na c) $\frac{Na}{1+e^{2\varepsilon/k_BT}}$ d) $\frac{Na}{1+e^{\varepsilon/k_BT}}$

Soln:- $\langle L \rangle = \sum_i L_i P_i$

$$= \frac{Na\,P_{\parallel} + 0 \cdot P_{\perp ar}}{P_{\parallel} + P_{\perp}} = \frac{Na\,e^{\beta\varepsilon}}{e^{\beta\varepsilon} + e^{-\beta\varepsilon}}$$

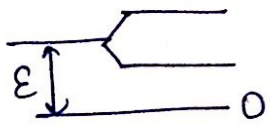
$$= \frac{Na}{1+e^{2\beta\varepsilon}}$$ **C**

Que7 An atom has a non-degenerate ground state and a ~~be~~ doubly-degenerate excited state. The energy difference between the two states is ε. The specific heat at very low temperature ($\beta\varepsilon >> 1$) is given by:

a) $k_B(\beta\varepsilon)$ b) $k_B e^{-\beta\varepsilon}$ c) $2k_B(\beta\varepsilon)^2 e^{-\beta\varepsilon}$ d) k_B (17)

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Soln.



$$Z = 1e^{0\beta} + 2e^{-\beta\varepsilon}$$
$$= 1 + 2e^{-\beta\varepsilon}$$

$$\langle E \rangle = \frac{0 + 2\varepsilon e^{-\beta\varepsilon}}{1 + 2e^{-\beta\varepsilon}} = \frac{2\varepsilon e^{-\beta\varepsilon}}{1 + 2e^{-\beta\varepsilon}}$$

$$C_V = \left(\frac{\partial E}{\partial T}\right)_V =$$
$$= \frac{-1}{K_B T^2}\left(\frac{\partial E}{\partial \beta}\right)_V$$

$$\beta = \frac{1}{K_B T}$$
$$\partial\beta = -\frac{\partial T}{K_B T^2}$$
$$\frac{1}{\partial T} = -\frac{1}{K_B T^2}\frac{1}{\partial \beta}$$

$$C_V = -K_B \beta^2 \left(\frac{\partial E}{\partial \beta}\right)_V$$

$$C_V = -K_B \beta^2 \frac{(1 + 2e^{-\beta\varepsilon})(-2\varepsilon^2 e^{-\beta\varepsilon}) - (2\varepsilon e^{-\beta\varepsilon})(-2\varepsilon e^{-\beta\varepsilon})}{(1 + 2e^{-\beta\varepsilon})^2}$$

$$C_V = -K_B \beta^2 \left(\frac{-2\varepsilon^2 e^{-\beta\varepsilon}}{(1 + 2e^{-\beta\varepsilon})^2}\right)$$ **C**

$$C_V = 2K_B (\beta\varepsilon)^2 e^{-\beta\varepsilon} \qquad (\because \beta\varepsilon >> 1)$$

Ques consider a system maintained at temperature T, with two available energy states E_1 & E_2 each with degeneracies g_1 & g_2. If P_1 & P_2 are probabilities of occupancy of the energy states, what is the entropy of the system:

a) $S = -K_B \left[P_1 \ln\left(\frac{P_1}{g_1}\right) + P_2 \ln\left(\frac{P_2}{g_2}\right)\right]$

b) $S = -K_B \left[P_1 \ln(P_1 g_1) + P_2 \ln(P_2 g_2)\right]$

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c) $S = -k_B [P_1 \ln(P_1^{g_1}) + P_2 \ln(P_2^{g_2})]$

d) $S = -k_B \left[\left(\frac{1}{P_1}\right) \ln\left(\frac{P_1}{g_1}\right) + \left(\frac{1}{P_2}\right) \ln\left(\frac{P_2}{g_2}\right)\right]$

Soln:- $S = -K \sum_i P_i \ln\left(\frac{P_i}{g_i}\right)$

$$S = -k_B \left[P_1 \ln\left(\frac{P_1}{g_1}\right) + P_2 \ln\left(\frac{P_2}{g_2}\right)\right]$$ [A]

Common linked Data Question:

There are four energy levels E, 2E, 3E & 4E (where E>0)
The canonical partition function of two particles is,
if these particles are:

a) Two identical fermions

b) Two distinguishable particle.

Soln:-

a)

E	2E	3E	4E	Total	
a	a	–	–	3E	
a	–	a	–	4E	
a	–	–	a	5E	→ Double degenerate
–	a	a	–	5E	
–	a	–	a	6E	
–	–	a	a	7E	

Partition fxn: $e^{-3\beta\varepsilon} + e^{-4\beta\varepsilon} + 2e^{-5\beta\varepsilon} + e^{-6\beta\varepsilon} + e^{-7\beta\varepsilon}$

(19)

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b) two distinguishable Particles.

$$z = (g_i)^{n_i} = (4)^2 \Rightarrow 16$$

$$z = \left(e^{-\beta\varepsilon} + e^{-2\beta\varepsilon} + e^{-3\beta\varepsilon} + e^{-4\beta\varepsilon}\right)^2$$

$\xrightarrow{***}$ $e^{-x} + e^{x} = 2\cosh x$

$\longrightarrow$ $e^{-x} = (1-x)$ when $(x \ll 1)$

in low Temp limit: $S \to 0$ as $T \to 0$

in High Temp limit: $S \to k \ln z$ as $T \to \infty$

all statistics approaches classical statistics

3) Classical Ideal gas:

Partition function: $z_1 = \frac{V}{h^3}(2\pi m k T)^{3/2} = \frac{V}{\lambda^3}$

mean thermal wavelength: $\lambda = \frac{h}{\sqrt{2\pi m k T}}$

$z_n \rightarrow \frac{(z_i)^N}{N!}$ indistinguishable

$z_n \rightarrow (z_1)^N$ distinguishable Particle.

Que 10

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→ Harmonic Oscillator:

Classical L.H.O	Quantum linear H.O
$Z_1 = \frac{k_B T}{\hbar\omega} = \left(\frac{1}{\beta\hbar\omega}\right)$	$Z_1 = e^{-\beta\hbar\omega/2}\left[\frac{1}{1-e^{\beta\hbar\omega}}\right]$
$Z_N = \left(\frac{KT}{\hbar\omega}\right)^N$	$Z_N = \left[2\sin h\left(\frac{\beta\hbar\omega}{2}\right)\right]^{-N}$
$\langle U\rangle = NKT$	$\langle U\rangle = \frac{\hbar\omega}{2} N \coth\left(\frac{\beta\hbar\omega}{2}\right)$
$\langle P\rangle = 0 \Rightarrow$ (G = A)	$\langle P\rangle = 0 \Rightarrow$ (G = A)

Que! If the Partition function of a Harmonic oscillator with frequency ω at a temperature T is $KT/\hbar\omega$, then the free energy of N such independent oscillators is:

a) $\frac{3}{2}NKT$ b) $KT \ln\frac{\hbar\omega}{KT}$ c) $NKT \ln\frac{\hbar\omega}{KT}$ d) $NKT \ln\frac{\hbar\omega}{2KT}$

Soln:- $Z_N = \left(\frac{KT}{\hbar\omega}\right)^N$

$$\langle A\rangle = -KT \ln Z_N$$

$$= -KT \ln\left(\frac{KT}{\hbar\omega}\right)^N$$

$$= NKT \ln\left(\frac{\hbar\omega}{KT}\right)$$

C

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Que 2 For a system of independent non-interacting 1-Dim oscillators, the value of the free energy per oscillator, in the limit $T \to 0$ is:

a) $\frac{1}{2}\hbar\omega$ b) $\hbar\omega$ c) $\frac{3}{2}\hbar\omega$ d) 0

Soln:- ~~Partion~~ $Z_N = \left[2\sinh\left(\beta\frac{\hbar\omega}{2}\right)\right]^{-N}$

$$U = \frac{\hbar\omega}{2} N \coth\left(\beta\frac{\hbar\omega}{2}\right)$$

as $T \to 0 \Rightarrow \beta \to \infty$

$$U = \frac{N\hbar\omega}{2} \Rightarrow \frac{U}{N} = \frac{1}{2}\hbar\omega \quad \boxed{A}$$

$$F = U - TS = U \quad \text{as } T = 0$$

$$F = U$$

$$\frac{F}{N} = \frac{1}{2}\hbar\omega$$

Que 3 For a quantum mechanical Harmonic oscillator with energy $E_n = \left(n+\frac{1}{2}\right)\hbar\omega$. The Partition fxn is:

a) $\frac{e^{\hbar\omega/k_BT}}{e^{\hbar\omega/k_BT}-1}$ b) $e^{\hbar\omega/2k_BT}-1$

c) $e^{\hbar\omega/2k_BT}-1$ d) $\frac{e^{\hbar\omega/2k_BT}}{e^{\hbar\omega/k_BT}-1}$

Soln:- $E_n = \left(n+\frac{1}{2}\right)\hbar\omega \rightarrow$ 1-Dim Q.H.O

$$Z_1 = \sum_n e^{-\beta E}$$

$$Z_1 = e^{-\beta\left(n+\frac{1}{2}\right)\hbar\omega}$$

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$$Z_1 = e^{-\beta\hbar\omega/2}\left[1 + e^{-\beta\hbar\omega} + e^{-2\beta\hbar\omega} + \cdots\right]$$

$$Z_1 = \frac{e^{-\beta\hbar\omega/2}}{1 - e^{-\beta\hbar\omega}} = \left[2\sinh\left(\frac{\beta\hbar\omega}{2}\right)\right]^{-1}$$

$$Z_1 = \frac{e^{-\beta\hbar\omega/2}}{1 - e^{-\beta\hbar\omega}} \times \frac{e^{\beta\hbar\omega}}{e^{\beta\hbar\omega}}$$

$$Z_1 = \frac{e^{\beta\hbar\omega/2}}{e^{\beta\hbar\omega} - 1}$$

$$Z_1 = \frac{e^{\hbar\omega/2K_BT}}{e^{\hbar\omega/K_BT} - 1}$$

$\beta = \frac{1}{KT}$ ↑ Statistical Parameter.

D

Ques The probability of a system to be in a state with energy E at temperature T is proportional to e^{-E/K_BT}. If the system in question is a 1-Dim. quantum H.O of frequency ω the average energy at temperature T is:

a) $\frac{\hbar\omega}{2}$ b) $\hbar\omega\left(e^{-\hbar\omega/K_BT} + \frac{1}{2}\right)$

c) $\hbar\omega\left(\frac{1}{e^{\hbar\omega/KT} - 1} + \frac{1}{2}\right)$ d) $\hbar\omega\left(\frac{1}{e^{\hbar\omega/KT} + 1} + \frac{1}{2}\right)$

Soln:- For 1-D Q.H.O:

$$Z_1 = \frac{e^{-\beta\hbar\omega/2}}{1 - e^{\beta\hbar\omega}}$$

$$U = -\frac{\partial}{\partial\beta}\ln Z_1 = -\frac{\partial}{\partial\beta}\left[-\frac{\beta\hbar\omega}{2} - \ln\left(1 - e^{-\beta\hbar\omega}\right)\right.$$

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$$U = \frac{\hbar\omega}{2} + \frac{e^{-\beta\hbar\omega}}{1+(-e^{-\beta\hbar\omega})}\hbar\omega$$

$$U = \hbar\omega\left(\frac{e^{-\beta\hbar\omega}}{1-e^{-\beta\hbar\omega}} + \frac{1}{2}\right)$$

C

also $U = \frac{\hbar\omega}{2}\coth\left(\frac{\beta\hbar\omega}{2}\right)$

Ques 4 A system has two normal modes of vibration, with frequencies ω_1 & $\omega_2 = 2\omega_1$. What is the probability that at temperature T, the system has an energy less than $4\hbar\omega_1$? (use $x = e^{-\beta\hbar\omega}$ & z is as Partition fxn)

a) $x^{3/2}(x+2x^2)/z$ b) $x^{3/2}(1+x+x^2)/z$

c) $x^{3/2}(1+2x^2)/z$ d) $x^{3/2}(1+x+2x^2)/z$

Soln:- $E = (n+\frac{1}{2})\hbar\omega_1 + (n+\frac{1}{2})\hbar(2\omega_1)$

$$E = (3n+\frac{3}{2})\hbar\omega_1$$

deg (2)

$E_{0,0} = \frac{3}{2}\hbar\omega_1$; $E_{1,0} = \frac{5}{2}\hbar\omega_1$; $E_2 = \frac{7}{2}\hbar\omega_1$

$(n_2=0, n_1=2)$
$(n_2=1, n_1=0)$

$$P_i = \frac{g_i e^{-\beta\varepsilon_i}}{z}$$

$$P_i = \left(e^{-3/2\,\beta\hbar\omega} + e^{-\frac{5}{2}\beta\hbar\omega} + 2e^{-\frac{7}{2}\beta\hbar\omega}\right)/z$$

Put $x = e^{-\beta\hbar\omega}$

$$P_i = \frac{x^{3/2} + x^{5/2} + 2x^{7/2}}{z} = \frac{x^{3/2}(1+x+2x^2)}{z}$$

D

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→ Paramagnetic System:

Spin	energy	magnetic moment (energy)
1/2	ε	$-\mu_0 B$
$-1/2$	$-\varepsilon$	$\mu_0 B$

$B = \mu_0 H$

$H = B/\mu_0$

1) Partition Function: $Z = e^{\beta\varepsilon} + e^{-\beta\varepsilon} \Rightarrow 2\cosh\beta\varepsilon$

2) Free energy: $\langle A \rangle = -KT \ln Z^N$

$= -NKT \ln\left[2\cosh\left(\frac{\mu_0 B}{KT}\right)\right]$

3) Internal energy: $\langle U \rangle = -N\varepsilon \tanh\beta\varepsilon$

$\langle U \rangle = -N\mu_0 B \tanh\left(\frac{\mu_0 B}{K_B T}\right)$

$M = -\left(\frac{\partial A}{\partial H}\right)$

$\langle U \rangle = -MB$

Que: A system of N localized, non-interacting spin 1/2 ions of magnetic moment μ each is kept in an external magnetic field H. If the system is in equilibrium at temperature T, then Helmholtz free energy of the system is:

a) $N K_B T \ln\left(\cosh\frac{\mu H}{K_B T}\right)$ b) $-N K_B T \ln\left(2\cosh\frac{\mu H}{K_B T}\right)$

c) $N K_B T \ln\left(2\cosh\frac{\mu H}{K_B T}\right)$ d) $-N K_B T \ln\left(2\sin\frac{\mu H}{K_B T}\right)$

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Soln:- $Z = e^{\beta\varepsilon} + e^{-\beta\varepsilon}$

$$Z = 2\cos h\,\beta\varepsilon = 2\cos h\left(\frac{\mu H}{K_B T}\right)$$

$$Z_N = \left[2\cos h\left(\frac{\mu H}{K_B T}\right)\right]^N$$

$$\langle A\rangle = -KT \ln Z_N$$

$$\langle A\rangle = -kT \ln\left(2\cos h\,\frac{\mu H}{K_B T}\right)^N$$

$$\langle A\rangle = -NKT \ln\left(2\cos h\,\frac{\mu H}{K_B T}\right)$$ **B**

Que 2 Consider a system of N non-interacting spin ½ particles, each having a magnetic moment μ, is in a magnetic field $\vec{B} = B\hat{z}$. If E is the total energy, then no. of accessible micro-states Ω is:

a) $\Omega = \dfrac{N!}{\frac{1}{2}(N-\frac{E}{\mu B})!\,\frac{1}{2}(N+\frac{E}{\mu B})!}$

b) $\Omega = \dfrac{(N-\frac{E}{\mu B})!}{(N+\frac{E}{\mu B})!}$

c) $\Omega = \frac{1}{2}(N-\frac{E}{\mu B})!\,\frac{1}{2}(N+\frac{E}{\mu B})!$

d) $\Omega = \dfrac{N!}{(N+\frac{E}{\mu B})!}$

Soln:- $N = N_1 + N_2$

$$E = -N_1 \mu B + N_2 \mu B \Rightarrow \frac{E}{\mu B} = -N_1 + N_2$$

$$N_2 = \frac{1}{2}\left[N + \frac{E}{\mu B}\right] \;\&\; N_1 = \frac{1}{2}\left[N - \frac{E}{\mu B}\right]$$

$$\Omega = \frac{N!}{\frac{1}{2}(N-\frac{E}{\mu B})!\,\frac{1}{2}(N+\frac{E}{\mu B})!}$$ **A**

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→ Energy Fluctuation:

$$U = \langle E_i \rangle = \sum E_i P_i$$

$$\frac{\partial U}{\partial \beta} = \frac{\partial}{\partial \beta}\langle E_i \rangle = \underbrace{\langle E \rangle^2 - \langle E \rangle^2}_{***}$$

$$(\Delta E)^2 = -\frac{\partial U}{\partial \beta} = KT^2 C_V$$

System	mean square energy	RMS energy	Relative mean square	Relative RMS
Non-Rel. Ideal gas	$\frac{3}{2}NK^2T^2$	$\sqrt{\frac{3N}{2}}KT$	$\frac{2}{3N}$	$\sqrt{\frac{2}{3N}}$
classical L.H.O	NK^2T^2	$\sqrt{N}\,KT$	$\frac{1}{N}$	$\sqrt{\frac{1}{N}}$
	$\uparrow$ $(\Delta E)^2$	$\uparrow$ (ΔE)	$\uparrow$ $\left(\frac{\Delta E}{\langle E_i \rangle}\right)^2$	$\uparrow$ $\left(\frac{\Delta E}{\langle E_i \rangle}\right)$

→ Difference b/w classical & quantum statistics:

Degenerate discrimnant factor $= n\lambda^3$

classical statistics:

$n\lambda^3 \rightarrow$ very small

$$\lambda^3 = \left(\frac{N}{V}\right)\left(\frac{h}{\sqrt{2\pi m K T}}\right)^3$$

(n)

$$n\lambda^3 << 1$$

$$\lambda << (n)^{1/3}$$

or

$$\lambda << \left(\frac{N}{V}\right)^{1/3}$$

Quantum statistics:

$$n\lambda^3 \geq 1$$

$$\lambda \geq \left(\frac{N}{V}\right)^{1/3}$$

or

$$\lambda \geq (n)^{1/3}$$

→ $n\lambda^3 = \infty$

completely degenerate.

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Que 1 which of the following relations between the particle number density n & temperature T must hold good for a gas consisting of non-interacting particles to be described by quantum statistics?

a) $n/T^{1/2} \ll 1$ b) $n/T^{3/2} \ll 1$

c) $n/T^{3/2} \gg 1$ d) $n/T^{1/2}$ & $n/T^{3/2}$ can have any value

Sol^{n}. $n\lambda^3 \gg 1$ quantum statistics.

$$\lambda \propto \frac{1}{\sqrt{T}}$$

$$nT^{3/2} \gg 1$$

C

→ at High Temp. & low Pressure:
system behaves as classical ideal gas.

Que 2 In a statistical description of a system in the canonical ensemble, the relative uncertainity in the energy of the system, $\Delta E/E$, varies ~~the~~ with the number of particles N in the system as:

a) N^{-1} b) $N^{-1/2}$ c) N^0 d) $N^{1/2}$

Sol^{n}.
$$\frac{\Delta E}{E} = \sqrt{\frac{2}{3N}} \Rightarrow \frac{\Delta E}{E} \propto N^{-1/2}$$

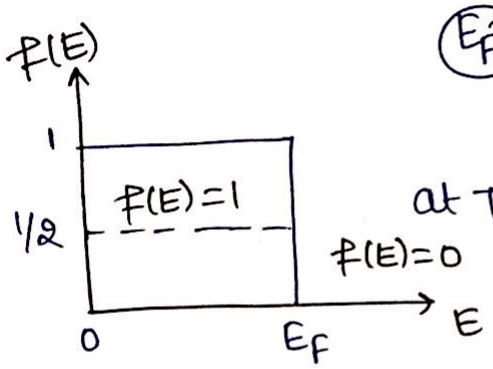
B

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→ Fermi-Dirac Statistics:

$$f(E) = \frac{1}{e^{(E-H)/kT} + 1} \quad \& \quad N = g_i C_{n_i}$$

E_F → Fermi level.



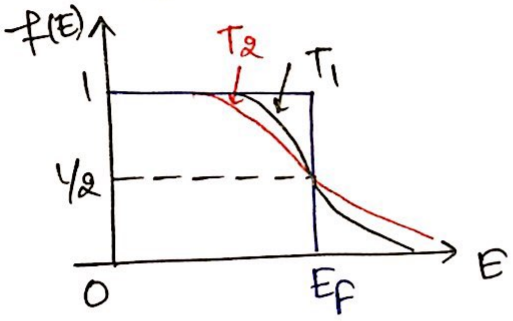
*** at T=0: $f(E)$ → 1 at $E \leq E_f$; → 0 at $E > E_f$

Fermi level: at 0°K

is the energy level below which all states are completelly filled & above which all states are completely empty.

at T≠0:

as temp. ↑es the states below E_f becomes empty & states above E_f start filling.



$f(E) = 1/2$ at $T \neq 0$

Fermi energy	zero Point energy (average energy)	Dimension
$\frac{\hbar^2}{2m}(3\pi^2 n)^{2/3}$	$\frac{3}{5}E_f$	3-Dim
$\frac{\hbar^2}{2m}(2\pi n)$	$\frac{1}{2}E_f$	2-Dim
$\frac{\hbar^2}{2m}(\quad)^2$	$\frac{1}{3}E_f$	1-Dim

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i.e $E_f \propto (f)^{s/d}$

$s = 2$ (Non-Rel.)

d = Dimension.

$E_F \propto (f)^{s/d}$

$P = \frac{s}{d}\left(\frac{E}{V}\right)$

$P = \frac{2}{3} \times \frac{3}{5}\left(\frac{N}{V}\right) E_f$

$\frac{2}{3}$ ↑ N.Rel. $\quad \frac{1}{3}$ ↑ Rel.

$P_0 \propto f \cdot f^{2/3}$

$P_0 \propto f^{5/3}$

Qu 1 If the volume of a 3-Dimensional free fermi gas is decreases by a factor of 8 at zero temperature. Its average energy changes by a factor of:

a) 8 b) 4 c) 2 d) 1/8

Sol^{n} $\bar{E} = \frac{3}{5} E_f$ (3-Dim.)

$\bar{E} \propto \frac{3}{5} (n)^{2/3}$ — (1)

$\bar{E'} \propto \frac{3}{5}\left(\frac{N}{V/8}\right)^{2/3}$

$\bar{E'} \propto \frac{3}{5} (2)^2 \left(\frac{N}{V}\right)^{2/3}$

$\bar{E'} = 4\bar{E}$ B

Qu 2 An ideal gas of non-Relativistic fermion in 3-D is at 0 K. Both the no. density & mass of the Particle are doubled. The energy per particle is amplified by a factor: (90)

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a) $2^{1/2}$ b) 1 c) $2^{1/3}$ d) $2^{-1/3}$

Soln:- $E_f = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3} \longrightarrow$ 3-D

$$E_f \propto \frac{n^{2/3}}{m} \quad -(1)$$

$$E_f' \propto \frac{1}{(2m)}(2n)^{2/3}$$

$$E_f' \propto 2^{-1/3} E_f$$

D

Que3 Consider N non-interacting electrons ($N \sim N_A$) in a box of sides L_x, L_y, L_z. Assume that the dispersion relation is $\varepsilon(k) = Ck^4$, where C is a constant, the ratio of the ground state energy per particle to the fermi energy is:

a) 3/7 b) 7/3 c) 3/5 d) 5/7

Soln:- $\varepsilon(k) = Ck^4$

$s = 4$ & $d = 3$

$$\frac{U_0}{N} = \frac{\int g(\varepsilon)\,\varepsilon\,d\varepsilon}{\int g(\varepsilon)\,d\varepsilon} = \frac{\int \varepsilon^{d/s-1}\,\varepsilon\,d\varepsilon}{\int \varepsilon^{d/s-1}\,d\varepsilon}$$

$$= \frac{\varepsilon^{d/s+1}}{\frac{d}{s}+1} \Big/ \frac{\varepsilon^{d/s}}{d/s}$$

$$\frac{U_0}{N} = \frac{\varepsilon_f^{7/4}}{\varepsilon_f^{3/4}} \times \frac{3}{4} \times \frac{4}{7} = \frac{3}{7}E_f$$

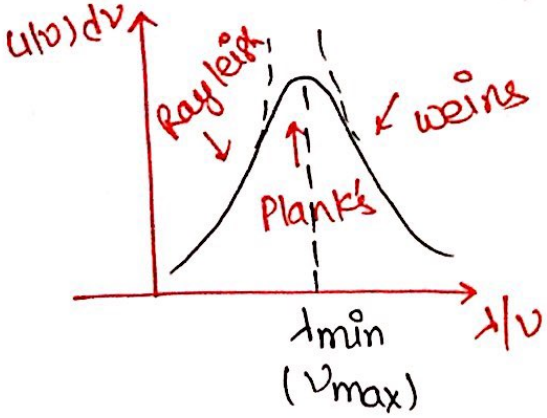
A

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→ **BLACK BODY RADIATION:**

Plank Radiation law for energy distribution:

energy density in Range $\nu \longrightarrow \nu + d\nu$



$$u(\nu)\,d\nu = \frac{8\pi R}{c^3}\,\frac{d\nu}{e^{h\nu/kT}-1}$$

now, $\nu = c/\lambda$

$$d\nu = -\frac{c}{\lambda^2}\,d\lambda$$

$$u(\lambda)\,d\lambda = \frac{8\pi Rc}{\lambda^5}\,\frac{d\lambda}{(e^{hc/\lambda kT}-1)}$$

1) For low wavelength limit (High freq.)

$$\frac{Rc}{\lambda k_B T} >> 1 \Rightarrow \boxed{u(\lambda) \propto \frac{1}{\lambda^5}}$$

↑ weins law.

2) For High wavelength limit (low freq.)

$$\frac{Rc}{\lambda k_B T} << 1$$

$$u(\lambda)\,d\lambda = \frac{8\pi Rc}{\lambda^5}\left(\frac{d\lambda}{1+\frac{hc}{\lambda k_B T}-1}\right)$$

$\boxed{u(\lambda) \propto \frac{1}{\lambda^4}}$ → Rayleigh Jeans law. (UV Region)

3) Wein's displacement law:

$$\boxed{\lambda_m T = 2897.77\ \mu m K = \text{constant}}$$

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4) Stefen's law: $E = \frac{8\pi^5 (KT)^4}{15(hc)^3}$

$$\boxed{E = \sigma T^4} \qquad \sigma = 5.67 \times 10^{-8}\ W/m^2 K^4$$

***

1) $E(T)$ or $U(T) \propto T^4$
2) $C_V = \frac{dU}{dT} \Rightarrow C_V \propto T^3$
3) $P = \frac{1}{3}\left(\frac{U}{V}\right) \Rightarrow P \propto T^4$
4) $S \propto T^3$, $N \propto T^3$

→ Reversible adiabatic expension: $TV^{1/3} = \text{constant}$

→ free expansion: $TV^{1/4} = \text{constant}$

Que: Consider black body radiation in a cavity maintained at 2000K. If the volume of the cavity is reversibly & adiabatically increased from 10 cm^3 to 640 cm^3, the temperature of the cavity changes to:

a) 800K b) 700K c) 600K d) 500K

Sol^n- Reversible adiabatic: $TV^{1/3} = \text{constant}$

$$T_1 V_1^{1/3} = T_2 V_2^{1/3}$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{1/3} \Rightarrow \left(\frac{640}{10}\right)^{1/3}$$

$$\frac{T_1}{T_2} = 4 \Rightarrow T_2 = \frac{T_1}{4}$$ D

$$= T_2 = \frac{2000}{4} = 500K$$

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Que 2 A cavity contains blackbody radiation in eqm. at temperature T. The specific heat per unit volume of the photon gas in the cavity is of the form $C_V = rT^3$ where r is a constant. The cavity is expanded to twice its original volume & then allowed to equilibrate at the same temperature T. The new internal energy per unit volume is:

a) $4rT^4$ b) $2rT^4$ c) $\frac{rT^4}{4}$ d) rT^4

Soln:- $$\frac{\text{Specific Heat}}{\text{Volume}} = C_V = rT^3$$

$$C = rVT^3$$

$$U = \int C dT = \frac{rVT^4}{4} \quad -(1)$$

when volume is doubled i.e $V' = 2V$

$$U' = \frac{r(2V)T^4}{4} = \frac{rVT^4}{2}$$

$$\Delta U = U' - U = \frac{rVT^4}{2} - \frac{rVT^4}{4} \Rightarrow \frac{rVT^4}{4}$$

$$\frac{\Delta U}{V} = \frac{rT^4}{4}$$ **C**

Que 3 Consider black body radiation contained in a cavity whose walls are at temperature T. The radiation is in equilibrium with the walls of the cavity. If the temperature of the walls is increased to 2T, & radiation is allowed to equilibrium

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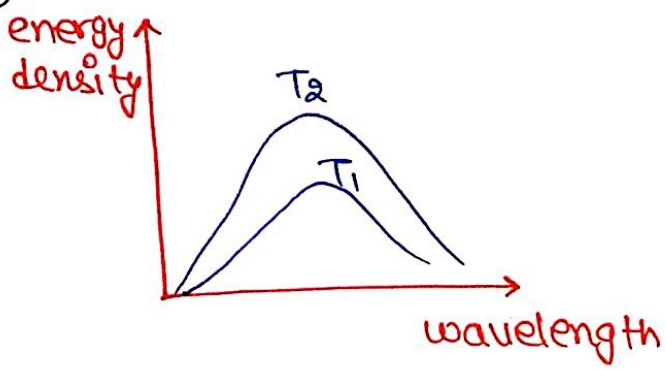
at the new temperature, the entropy of the radiation increases by a factor of:

a) 2 b) 4 c) 8 d) 16

Sol$^{n}_{o}$- $S \propto T^3$

$S' \propto (2T)^3 \Rightarrow S' = 8S$ C

① important:



Wein's displacement law.

$\lambda_m T = \text{const.}$

$\frac{\lambda_{m_1}}{\lambda_{m_2}} = \frac{T_2}{T_1}$

$T_2 > T_1$ & $\lambda_{m_1} > \lambda_{m_2}$

②

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→ BOSE-EINSTEIN CONDENSATION:

Chemical Potential:

μ → −ve in M.B
μ → +ve or −ve in F.D (−ve at Room T)
μ → 0 or −ve in B.E

$\mu = 0$ → For Photon.

Distribution Function of Bosons:

$$f(E) = \frac{1}{e^{(E-\mu)/kT} - 1}$$

→ $\frac{N}{V} = n$ = constant; but in blackbody n ↑es in each interaction.

→ large number of Particles in ground state is called BEC.

→ average no. of Particles:

$$n_i = \frac{g_i}{e^{\beta(E-\mu)} - 1}$$

→ $\frac{N}{V} = n \propto T_B^{3/2}$ T_B → condensation Temperature.

$$\boxed{T_B \propto n^{2/3}}$$

→ Population increases in ground state below T_B for BE gas is called BEC.

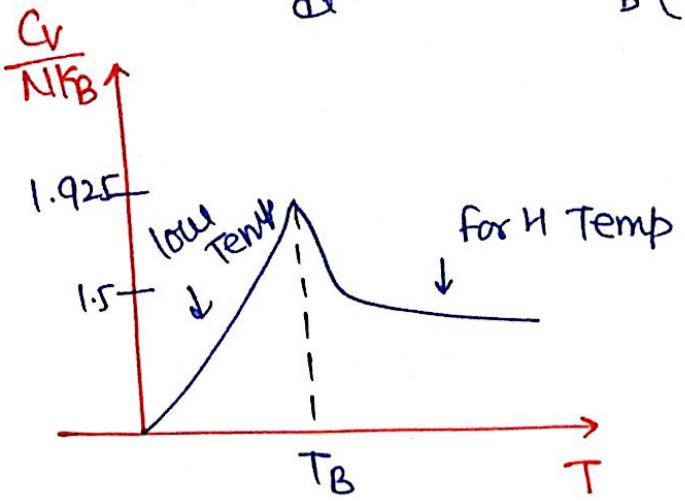
→ energy of Bosonic system:

$$\langle E \rangle = 0.77\, NK \frac{T^{5/2}}{T_B^{3/2}} \Rightarrow \boxed{\langle E \rangle \propto T^{5/2}}$$

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→ Speific Heat:

$$C_V = \frac{dE}{dt} = 1.925\, N K_B \left(\frac{T}{T_B}\right)^{3/2}$$



$$\frac{C_V}{NK_B} = 1.925 \left(\frac{T}{T_B}\right)^{3/2}$$

→ at High temp: quantum particles behaves like a classical particle.

→ Condensation of Photon gas is not Possible:

Condⁿ: $E \propto p^s$ $\quad s>1 \rightarrow$ condensation Possible.

$s=1 \rightarrow$ not Possible.

$\therefore \frac{N}{V} = n$ not constant in case of Photon.

Que! Bose-Einstein condensation occurs in liquid He^4 kept at 2.17K. at which temperature will Bose condensation occurs. He^4 in gaseous state. The density of which is 1000 times smaller than that of liquid He^4:

a) 2.17K b) 2.17×10^{-3} c) 21.7K d) 21.7×10^{-3}K

Solⁿ:- $n \propto T_B^{3/2} \Rightarrow \left(\frac{T_{B_1}}{T_{B_2}}\right)^{3/2} = \frac{n_1}{n_2}$

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$$\left(\frac{2.17}{T_{B_2}}\right) = \left(\frac{1000 n_1}{n_1}\right)^{2/3}$$

D

$$\frac{2.17}{T_{B_2}} = (10)^2 \Rightarrow T_{B_2} = 21.7 \times 10^{-3}\ K$$

Ques The distribution function $f(\varepsilon)$ for a photon gas is given by:

a) $\exp(-\varepsilon/KT)$ b) $1/\exp(\varepsilon/KT)+1$

c) $1/\exp(\varepsilon/KT)-1$ d) $1/\exp(-\varepsilon/KT)-1$

Soln:- $$f(\varepsilon) = \frac{1}{\exp^{(\varepsilon-H)KT} - 1}$$

For photon $H=0$

C

$$f(\varepsilon) = \frac{1}{\exp(\varepsilon/KT)-1}$$

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→ ISING MODEL:

Ising model was set up to investigate the behaviour of substances whose molecules possess a magnetic moment.

e.g Ferromagnetic substances.

$$E = -J\sum \sigma_i \sigma_j - \mu_B H \sum \sigma_i$$

Total energy — interaction energy — interaction energy associated with magnetic field H.

σ_i & σ_j are ising spins $= \pm 1$

Partition function:

$$E_{int.} = \begin{cases} -J & \text{if } \sigma_i = \sigma_j = \pm 1 \quad (\text{deg} = 2) \\ +J & \text{if } \sigma_i = 1, \sigma_j = -1 \text{ & } \sigma_i = -1, \sigma_j = +1 \quad (\text{deg} = 2) \end{cases}$$

$$Z = \sum g_i e^{-\beta \varepsilon_i}$$

$$= 2e^{\beta J} + 2e^{-\beta J}$$

$$= 2.2(\cos h \beta J) \Rightarrow 2^2 (\cos h \beta J)^1$$

For N-Particles:

$$Z_{N_1} = 2^N (\cos h \beta J)^{N-1} \quad \text{—(1)}$$

$$E_H = \begin{cases} -H\mu ; & \sigma_i = 1 \\ H\mu ; & \sigma_i = -1 \end{cases}$$

$$Z = e^{H\mu\beta} + e^{-H\mu\beta} = 2\cos h\, H\beta\mu$$

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For N-Particles:

$$Z_{N_2} = 2^N (\cosh \mu\beta H)^N$$

$$Z_{total} = Z_{N_1} \times Z_{N_2}$$

$$Z_t = 2^N (\cosh \beta J)^{N-1} \times 2^N (\cosh \mu\beta H)^N$$

Que 1 Consider a system of three spins S_1, S_2, S_3 each of which can take value 1 & -1. The energy of the system is given by:

$$E = -J [S_1S_2 + S_2S_3 + S_3S_1]$$

The minimum energy and corresponding number of sign configuration are respectively:

a) J & 1 b) -3J & 2 c) -3J & 1 d) -6J & 2

Soln:- $E = -J [S_1S_2 + S_2S_3 + S_3S_1]$

S_1	S_2	S_3	E
-1	-1	-1	-3J
1	1	1	-3J

→ deg. (2). ↑ min.

B

Que 2 Consider a system of two spins S_1 & S_2 taking values ±1 with internal energy $-JS_1S_2$. When it is in thermal equilibrium at temp. T, for largest temperature, the average energy of the

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system varies as C/k_BT, with C given by:

a) $-2J^2$ b) $-J^2$ c) J^2 d) $4J$

Soln:-

S_1	S_2	$E = -JS_1S_2$	
1	1	$-J$	→ deg. 2
−1	−1	$-J$	
1	−1	J	→ deg 2
−1	1	J	

Partition Fxn: $Z = 2e^{\beta J} + 2e^{-\beta J}$

$$\ln Z = \ln 2 + \ln\left(e^{\beta J} + e^{-\beta J}\right)$$

$$\langle E \rangle = -\frac{\partial}{\partial \beta}\left[\ln 2 + \ln\left(e^{\beta J} + e^{-\beta J}\right)\right]$$

$$= \frac{-J\left(e^{\beta J} - e^{-\beta J}\right)}{e^{\beta J} + e^{-\beta J}}$$

$$= \frac{-J\left[(1+\beta J) - (1-\beta J)\right]}{1+\beta J + 1 - \beta J}$$

$$= -\beta J^2$$

$$\langle E \rangle = \frac{-J^2}{k_BT} = \frac{C}{k_BT}$$

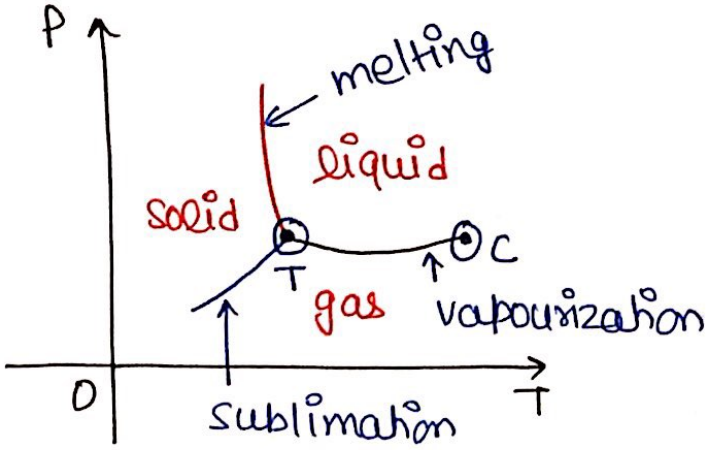
$$\Rightarrow C = -J^2$$

B

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→ **PHASE SPACE TRANSITON:**

Conversion of Phase of the material.



Triple Point (T): 3 Phase co-exist in equilibrium.
→ Unique.

Critical Point (C): terminal Point of liquid-vapour equilibrium line.

First order Phase Transition:

→ Gibb's free energy is continuous

→ First derivative of gibb's free energy is discontinuous.

i.e $S, V, n = \frac{N}{V}, M$ discontinuous at Transition Point.

$$M = -\left(\frac{\partial h}{\partial T}\right)_P$$

example:

1) solid to liquid & vice-versa
2) solid to vapour & vice-versa
3) liquid to vapour away from critical Point.
4) Clausius-Clapryon latent Heat equation.

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→ C, β, K_T diverges at transition (1st order).

Second order Phase Transition:

→ Gibbs free energy and its first derivative is continuous

→ But 2nd derivative is not continuous.

i.e $S, V, n, M \longrightarrow$ continuous

$C, \beta, K_T, \chi_M = \frac{\partial M}{\partial H} \longrightarrow$ discontinuous at transition Point.

examples:

1) Paramagnetic state to Ferromagnetic state
2) normal state to superconducting state.
3) He-I to He-II for a common isotope $_2He^4$. (λ-Point transition)
4) Liquid to vapour transition at critical Point.

→ **Mean Free Path:**

$$\lambda = \frac{1}{\sqrt{2}\pi n d^2}$$

$n \rightarrow$ Particle density

$d \rightarrow$ diameter of molecule

also $PV = NKT$

$$\frac{P}{KT} = \left(\frac{N}{V}\right) \rightarrow n$$

$$\boxed{\lambda = \frac{1}{\sqrt{2}\pi n d^2} = \frac{KT}{\sqrt{2}\pi P d^2}}$$

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Important Points:

If $E \propto a p^s$ (given Relation)

1) $g = \frac{N}{V} \propto E^{d/s}$

2) $\Omega \propto E^{d/s}$

3) $E_f \propto n^{s/d}$

4) $P_0 \propto n^{(s/d)+1}$

5) $g(E) \propto E^{(d/s)-1}$

6) $\frac{E_0}{N} = \bar{E} = \left(\frac{d}{d+s}\right) E_f$

7) $P = \frac{s}{d}\left(\frac{E}{V}\right) \propto T^{(d/s)+1}$

8) $U \propto T^{(d/s)+1}$

9) N OR C_V OR $S \propto T^{d/s}$

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_V \quad ***$$

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